The Merits of Pooling Claims: Mutual vs. Stock Insurer

Working Paper by Carolina Orozco-Garcia and Hato Schmeiser

18.01.2018
Structure

1. Motivation

2. Introduction and research question

3. Model description

4. Numerical illustration: Case A and B

5. Conclusion
1. Motivation (I)

Text books and lectures in insurance theory typically start with risk pooling as the «production law» of the business model (diversification of unsystematic risk)

For instance, for i.i.d. risks we have

A) For a fixed (and small) ruin probability, the premium for each risk decreases and reaches its expected value for $n \to \infty$

B) For a fixed “unfair” premium, the ruin probability reaches 0 for $n \to \infty$

Cf., e.g., Cummins (1991); Albrecht (1982, 1990); Smith and Kane (1994)
1. Motivation (II)

Modeling setup may be misleading from the economic point of view: Framework can give the impression that …

… risk pooling generates an added value for the policyholder

… that the premium level (or the ruin probability) plays an important role

Concept is to some extend incomplete:

If a mutual is considered, the policyholder’s owner stake must be taken into account

If a shareholder company is considered, the setup allows arbitrage opportunities

The fulfillment of A) or B) is not a necessary or sufficient condition that pooling is beneficial for a policyholder
1. Motivation

2. Introduction and research question

3. Model description

4. Numerical illustration: Case A and B

5. Conclusion
2. Introduction and research question

Under which conditions is risk pooling beneficial for policyholders in the context of mutual insurer?

Cf. Gatzert and Schmeiser (2012); Albrecht and Huggenberger (2017)

Aim of this paper:

Compare risk pooling in mutual and shareholder driven companies

Identify conditions under which a policyholder is indifferent between purchasing an insurance contract with a mutual or a shareholder insurance company

Related literature: Cf. working paper
Structure

1. Motivation

2. Introduction and research question

3. Model description

4. Numerical illustration: Case A and B

5. Conclusion
3. Model description: Pooling claims

The pool’s claims distribution consists of $n$ i.i.d. risks with distribution $X$:

$$S = \sum_{i=1}^{n} X_i$$

The policyholder $i$ receives a positive indemnity payment $I_i$ at $t = 1$ if a claim occurs.

The premium payment $\pi_i$ is fair if it equals the policyholder’s expected payoffs in $t = 1$.

We assume an incomplete market setting in which policyholders are not able to replicate future cash flows.

Policyholders value their wealth position $W$ based on exponential utility function:

$$U(W) = -e^{-a(W+K)}$$
3. Model description: Case of mutual insurer (I)

Policyholder’s wealth position at $t = 1$:

$$W_i^M = A_i (1 + r) - X_i - \pi_{i,M} (1 + r) + I_{i,M} + E_{i,M}$$

In what follows we set $r = 0$

The debtholder stake is given by

$$I_{i,M} = X_i - \frac{1}{n} max (S - \pi_M, 0)$$

and the owner’s position is

$$E_{i,M} = \frac{1}{n} max (\pi_M - S, 0)$$
3. Model description: Case of mutual insurer (II)

In any case, we have

\[
E(I_{i,M} + E_{i,M}) = E\left( X_i - \frac{1}{n} \max(S - \pi_M, 0) + \frac{1}{n} \max(\pi_M - S, 0) \right) \\
= E\left( X_i + \frac{1}{n} (\pi_M - S) \right) \\
= E\left( X_i + \frac{1}{n} (n\pi_{i,M} - S) \right) \\
= \pi_{i,M}
\]

In this sense, any premium level in the mutual company is «fair» with respect to our definition

However, the premium is in general not equal to the expected value of the claims
3. Model description: Case of mutual insurer (III)

The expected utility of policyholder $i$ does not depend on the premium level (we assume that credit risk plays no role):

For instance, the two central moments of the wealth distribution are not influenced by $\pi_i$ (cf. Gatzert and Schmeiser (2012)):

\[
E(W_i^M) = A_i - E(X_i) - \pi_{i,M} + E(I_{i,M} + E_{i,M}) = A_i - E(X_i)
\]

\[
\sigma^2(W_i^M) = \sigma^2(-X_i + I_{i,M} + E_{i,M}) = \sigma^2 \left( -X_i + X_i + \frac{1}{n}(\pi_M - S) \right)
\]

\[
= \sigma^2 \left( \frac{1}{n}(\pi_M - S) \right)
\]

\[
= \sigma^2 \left( \frac{1}{n} \sum_{i=1}^{n} X_i \right) = \frac{1}{n^2} \sum_{i=1}^{n} \sigma^2(X_i) = \frac{1}{n} \sigma^2(X_i)
\]
3. Model description: Case of a shareholder insurer (I)

Policyholder’s wealth position at $t = 1$

\[ W_{i}^{S} = A_{i}(1 + r) - X_{i} - \pi_{i,S}(1 + r) + I_{i,S} \]

Again, we set in what follows $r = 0$

The debt holder’s position is

\[ I_{i,S} = X_{i} - \frac{1}{n} \max \left( S - \pi_{S} - EC_{0}, 0 \right) \]

The payoff to the shareholders is given by

\[ E_{S} = \max \left( EC_{0} + \pi_{S} - S, 0 \right) \]
3. Model description: Case of a shareholder insurer (II)

The two central moments of the wealth distribution are given by:

\[ E(W_i^S) = A_i - E(X_i) - \pi_{i,S} + E(I_{i,S}) \]

\[ \sigma^2(W_i^S) = \sigma^2(-X_i + I_{i,S}) \]

\[ = \sigma^2(-X_i + X_i - \frac{1}{n}(max[S - \pi_S - EC_0, 0])) \]

\[ = \sigma^2 \left( \frac{1}{n} max[S - \pi_S - EC_0, 0] \right) \]

The expected wealth position equals the one in the mutual only if fair pricing takes place

The policyholder’s expected utility depends on the premium setting and the amount of equity capital of the insurer
3. Model description: Initial considerations (I)

Assumptions: (a) Fair pricing

\[ \pi_{i,M} = E[I_{i,M} + E_{i,M}] \quad \pi_{i,S} = E(I_{i,S}) \]

(b) Equal ruin probabilities \( \varepsilon \)

Because of Hence, we have

\[ \pi_{i,M} = \pi_{i,S} + EC_{0,i} \quad EC_{0,i} = EC_0/n \]

Differences in the distribution of \( W \)?

Example with the following data: Initial wealth \( A_i = 10,000 \); \( X_i \) are lognormal distributed with \( E[X_i] = 1,000 \) and \( \sigma[X_i] = 675 \); risk aversion parameter \( a = 0.75 \); constant scaling parameter \( K = -9,000 \)
### 3. Model description: Initial considerations (II)

**Table 1:** Comparison of fair premiums and policyholder’s wealth distributions at \( t = 1 \) under a mutual and a shareholder company for different safety levels \( \varepsilon \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \pi_{i,M} )</th>
<th>( \Phi(W_i^M) )</th>
<th>( \sigma(W_i^M) )</th>
<th>( \varepsilon = 10% )</th>
<th>( \pi_{i,S} )</th>
<th>( \Phi(W_i^S) )</th>
<th>( \sigma(W_i^S) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1,281.86</td>
<td>-12,815.45</td>
<td>213.45</td>
<td>296.84</td>
<td>985.02</td>
<td>-1,171.55</td>
<td>64.51</td>
</tr>
<tr>
<td>100</td>
<td>1,086.60</td>
<td>-1,282.45</td>
<td>67.50</td>
<td>90.22</td>
<td>996.38</td>
<td>-64.60</td>
<td>15.04</td>
</tr>
<tr>
<td>1,000</td>
<td>1,027.19</td>
<td>-129.14</td>
<td>21.35</td>
<td>28.27</td>
<td>998.92</td>
<td>-6.24</td>
<td>4.32</td>
</tr>
<tr>
<td>10,000</td>
<td>1,008.60</td>
<td>-13.81</td>
<td>6.75</td>
<td>8.94</td>
<td>999.66</td>
<td>-1.52</td>
<td>1.36</td>
</tr>
<tr>
<td>100,000</td>
<td>1,002.76</td>
<td>-2.28</td>
<td>2.13</td>
<td>2.86</td>
<td>999.90</td>
<td>-1.05</td>
<td>0.41</td>
</tr>
</tbody>
</table>

\( \varepsilon = 5\% \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \pi_{i,M} )</th>
<th>( \Phi(W_i^M) )</th>
<th>( \sigma(W_i^M) )</th>
<th>( \varepsilon = 5% )</th>
<th>( \pi_{i,S} )</th>
<th>( \Phi(W_i^S) )</th>
<th>( \sigma(W_i^S) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1,388.25</td>
<td>-12,815.45</td>
<td>213.45</td>
<td>395.60</td>
<td>992.64</td>
<td>-575.93</td>
<td>45.21</td>
</tr>
<tr>
<td>100</td>
<td>1,113.44</td>
<td>-1,282.45</td>
<td>67.50</td>
<td>115.13</td>
<td>998.31</td>
<td>-28.90</td>
<td>9.96</td>
</tr>
<tr>
<td>1,000</td>
<td>1,034.66</td>
<td>-129.14</td>
<td>21.35</td>
<td>35.18</td>
<td>999.48</td>
<td>-3.32</td>
<td>2.87</td>
</tr>
<tr>
<td>10,000</td>
<td>1,011.35</td>
<td>-13.81</td>
<td>6.75</td>
<td>11.49</td>
<td>999.86</td>
<td>-1.19</td>
<td>0.83</td>
</tr>
<tr>
<td>100,000</td>
<td>1,003.54</td>
<td>-2.28</td>
<td>2.13</td>
<td>3.58</td>
<td>999.95</td>
<td>-1.02</td>
<td>0.26</td>
</tr>
</tbody>
</table>

\( \varepsilon < 0.1\% \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \pi_{i,M} )</th>
<th>( \Phi(W_i^M) )</th>
<th>( \sigma(W_i^M) )</th>
<th>( \varepsilon &lt; 0.1% )</th>
<th>( \pi_{i,S} )</th>
<th>( \Phi(W_i^S) )</th>
<th>( \sigma(W_i^S) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2,405.76</td>
<td>-12,815.45</td>
<td>213.45</td>
<td>1,405.77</td>
<td>1,000.00</td>
<td>-1.04</td>
<td>0.38</td>
</tr>
<tr>
<td>100</td>
<td>1,279.04</td>
<td>-1,282.45</td>
<td>67.50</td>
<td>279.05</td>
<td>1,000.00</td>
<td>-1.01</td>
<td>0.17</td>
</tr>
<tr>
<td>1,000</td>
<td>1,081.63</td>
<td>-129.14</td>
<td>21.35</td>
<td>81.63</td>
<td>1,000.00</td>
<td>-1.00</td>
<td>0.01</td>
</tr>
<tr>
<td>10,000</td>
<td>1,023.88</td>
<td>-13.81</td>
<td>6.75</td>
<td>23.88</td>
<td>1,000.00</td>
<td>-1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>100,000</td>
<td>1,007.58</td>
<td>-2.28</td>
<td>2.13</td>
<td>7.58</td>
<td>1,000.00</td>
<td>-1.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
3. Model description: Initial considerations (III)

First results:

A risk-averse policyholder (\(a > 0\)) prefers under identical conditions in respect to fair pricing, safety level and pool size of the shareholder company over the mutual

However, it is assumed that shareholders are willing to provide the initial equity contribution in the way it is proposed in this section

If shareholders are willing to provide the equity capital proposed, depends on the valuation technique used by the insurance company’s owners

Next step:

Derive conditions under which a policyholder is indifferent between the two legal forms \((\Phi(W^M_i) = \Phi(W^S_i))\)
Structure

1. Motivation
2. Introduction and research question
3. Model description
4. Numerical illustration: Case A and B
5. Conclusion
4. Numerical illustration: Case A (I)

Case A – Fixed pool size $n$

We derive premium/equity combinations such that a risk-averse policyholder with exponential utility function is indifferent between the two legal forms.

The indifference premium is given by

$$\pi_{i,S}^* = E[I_{i,S}] + \ln \left( \frac{1 + \frac{a^2}{2} \sigma^2(W_i^M)}{1 + \frac{a^2}{2} \sigma^2(W_i^S)} \right)$$

Hence, $(\pi_{i,S}^*, EC_{0,i}^*)$ denotes a combination of premium and equity contributions such that the equation above holds.

Note that the ruin probabilities in the mutual and in the shareholder company typically differ in this case.
4. Numerical illustration: Case A (II)

(a) $n = 10$,  
$\Phi(W^M_t(n)) = -12,815.45$,  
$\Phi(W^S_t(n, \pi_i, \pi_S, EC_0)) = -12,815.45$

(b) $n = 100$,  
$\Phi(W^M_t(n)) = -1,282.44$,  
$\Phi(W^S_t(n, \pi_i, \pi_S, EC_0)) = -1,282.44$

(c) $n = 1000$,  
$\Phi(W^M_t(n)) = -129.14$,  
$\Phi(W^S_t(n, \pi_i, \pi_S, EC_0)) = -129.14$

(d) $n = 10,000$,  
$\Phi(W^M_t(n)) = -13.81$,  
$\Phi(W^S_t(n, \pi_i, \pi_S, EC_0)) = -13.81$
4. Numerical illustration: Case A (III)

<table>
<thead>
<tr>
<th>Mutual Company</th>
<th>$n$</th>
<th>$\pi_{i,M} (\varepsilon = 5%)$</th>
<th>$\pi_{i,M} (\varepsilon &lt; 0.01%)$</th>
<th>$\Phi(W_i^M)$</th>
<th>$\sigma(W_i^M)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1,388.25</td>
<td>2,405.76</td>
<td>-12,815.45</td>
<td>213.45</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1,113.44</td>
<td>1,279.04</td>
<td>-1,282.45</td>
<td>67.50</td>
<td></td>
</tr>
<tr>
<td>1,000</td>
<td>1,034.66</td>
<td>1,081.63</td>
<td>-129.14</td>
<td>21.35</td>
<td></td>
</tr>
<tr>
<td>10,000</td>
<td>1,011.35</td>
<td>1,023.88</td>
<td>-13.81</td>
<td>6.75</td>
<td></td>
</tr>
<tr>
<td>100,000</td>
<td>1,003.54</td>
<td>1,007.58</td>
<td>-2.28</td>
<td>2.13</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shareholder Company</th>
<th>$n$</th>
<th>$EC_{0,t}^*$</th>
<th>$\pi_{i,S}^*$</th>
<th>$\Phi(W_i^S)$</th>
<th>$\sigma(W_i^S)$</th>
<th>$E(I_{i,S})$</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>0.60</td>
<td>585.32</td>
<td>-12,815.45</td>
<td>215.26</td>
<td>585.34</td>
<td>99.23%</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.24</td>
<td>843.75</td>
<td>-1,282.45</td>
<td>66.99</td>
<td>843.73</td>
<td>99.39%</td>
</tr>
<tr>
<td></td>
<td>1,000</td>
<td>0.06</td>
<td>965.46</td>
<td>-129.14</td>
<td>20.38</td>
<td>965.33</td>
<td>94.95%</td>
</tr>
<tr>
<td></td>
<td>10,000</td>
<td>0.04</td>
<td>988.86</td>
<td>-13.81</td>
<td>6.49</td>
<td>988.76</td>
<td>94.96%</td>
</tr>
<tr>
<td></td>
<td>100,000</td>
<td>0.05</td>
<td>999.00</td>
<td>-2.28</td>
<td>1.56</td>
<td>998.59</td>
<td>67.58%</td>
</tr>
</tbody>
</table>

Note: Unfair premium setting according to our definition
4. Numerical illustration: Case A (IV)

\[ n = 10, \]
\[ \Phi(W_i^M(n)) = -12,815.45, \]
\[ \Phi(W_i^S(n, \pi_i, s, EC_0)) = -12,815.45 \]
4. Numerical illustration: Case B (I)

**Case B – Fixed pool size $n$ and fixed ruin probability $\varepsilon$**

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\pi_{i,M}$</th>
<th>$\Phi(W_i^M)$</th>
<th>$\sigma(W_i^M)$</th>
<th>$EC_0$</th>
<th>$\pi_{i,S}$</th>
<th>$\Phi(W_i^S)$</th>
<th>$\sigma(W_i^S)$</th>
<th>$E(I_{i,S})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1,281.86</td>
<td>-12,815.45</td>
<td>213.45</td>
<td>293.69</td>
<td>988.16</td>
<td>-12,815.45</td>
<td>64.51</td>
<td>984.97</td>
</tr>
<tr>
<td>100</td>
<td>1,086.60</td>
<td>-1,282.45</td>
<td>67.50</td>
<td>86.25</td>
<td>1,000.35</td>
<td>-1,282.45</td>
<td>15.04</td>
<td>996.36</td>
</tr>
<tr>
<td>1,000</td>
<td>1,027.19</td>
<td>-129.14</td>
<td>21.35</td>
<td>24.21</td>
<td>1,002.98</td>
<td>-129.14</td>
<td>4.32</td>
<td>998.94</td>
</tr>
<tr>
<td>10,000</td>
<td>1,008.60</td>
<td>-13.81</td>
<td>6.75</td>
<td>6.00</td>
<td>1,002.60</td>
<td>-13.81</td>
<td>1.36</td>
<td>999.66</td>
</tr>
<tr>
<td>100,000</td>
<td>1,002.76</td>
<td>-2.28</td>
<td>2.13</td>
<td>1.82</td>
<td>1,000.94</td>
<td>-2.28</td>
<td>0.41</td>
<td>999.90</td>
</tr>
</tbody>
</table>

$p = 10\%$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\pi_{i,M}$</th>
<th>$\Phi(W_i^M)$</th>
<th>$\sigma(W_i^M)$</th>
<th>$EC_0$</th>
<th>$\pi_{i,S}$</th>
<th>$\Phi(W_i^S)$</th>
<th>$\sigma(W_i^S)$</th>
<th>$E(I_{i,S})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1,388.25</td>
<td>-12,815.45</td>
<td>213.45</td>
<td>391.49</td>
<td>996.75</td>
<td>-12,815.45</td>
<td>45.21</td>
<td>992.62</td>
</tr>
<tr>
<td>100</td>
<td>1,113.44</td>
<td>-1,282.45</td>
<td>67.50</td>
<td>110.08</td>
<td>1,003.35</td>
<td>-1,282.45</td>
<td>9.96</td>
<td>998.30</td>
</tr>
<tr>
<td>1,000</td>
<td>1,034.66</td>
<td>-129.14</td>
<td>21.35</td>
<td>30.29</td>
<td>1,004.37</td>
<td>-129.14</td>
<td>2.87</td>
<td>999.49</td>
</tr>
<tr>
<td>10,000</td>
<td>1,011.35</td>
<td>-13.81</td>
<td>6.75</td>
<td>8.23</td>
<td>1,003.12</td>
<td>-13.81</td>
<td>0.83</td>
<td>999.86</td>
</tr>
<tr>
<td>100,000</td>
<td>1,003.54</td>
<td>-2.28</td>
<td>2.13</td>
<td>2.51</td>
<td>1,001.03</td>
<td>-2.28</td>
<td>0.26</td>
<td>999.96</td>
</tr>
</tbody>
</table>
4. Numerical illustration: Case B (II)

<table>
<thead>
<tr>
<th>Mutual Company</th>
<th>Shareholder Company</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>$\pi_{t,M}$</td>
</tr>
<tr>
<td>10</td>
<td>2,405.76</td>
</tr>
<tr>
<td>100</td>
<td>1,279.04</td>
</tr>
<tr>
<td>1,000</td>
<td>1,081.63</td>
</tr>
<tr>
<td>10,000</td>
<td>1,023.88</td>
</tr>
<tr>
<td>100,000</td>
<td>1,007.58</td>
</tr>
</tbody>
</table>

$\varepsilon < 0.1\%$

Premiums must be higher in the mutual company

For finite $n$ (and given a mutual as an alternative), policyholders accept unfair premiums in a shareholder insurance company
1. Motivation

2. Introduction and research question

3. Model description

4. Numerical illustration: Case A and B

5. Conclusion
5. Conclusion

The classical risk pooling framework must be specified in several ways in order to explain the rationality of pooling claims from the policyholder’s perspective – and the business model of insurance companies per se.

In this paper, we compare the merits of pooling claims for mutual and stock insurance companies.

At first glance and under the assumptions taken (in particular fair pricing), shareholder companies seem to offer higher utility levels to their policyholders than mutual insurers.

However, we must assume that in competitive markets, parameter conditions in respect to premium and equity contributions take place that lead to (approximately) the same utility levels for both legal forms.

If utility levels are the same and \( n \) and \( \varepsilon \) are fixed, the premium level in the shareholder company is below the mutual, but exceeds the fair price.