Institute of Insurance Economics

University of St.Gallen

Heterogeneous Premiums for Homogeneous Risks? Asset Liability Management under Default Probability and Price-Demand Functions

Florian Klein and Hato Schmeiser

Outline

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1. Relevance of the Topic

How to improve the performance of insurers?

- In times of low interest rates and possible "bubbles" in various asset classes, improving the insurer's performance is challenging
- Various researchers focus on optimal investment decisions (e. g., Braun et al. 2014; 2017, Eckert and Gatzert 2017)
- We set the underwriting sector (premiums – claims) as the origin of acquiring and improving the insurer's profitability
- Thereby, the insurer's profitability is measured by net present value calculus

Outline

2. Basic Idea

Default probability and insurance premium
(Wakker et al. 1997, Zimmer et al. 2009, 2016)

Asset Liability model

Price-demand functions and premiums

Framework

Individuals overweight very unlikely events
(Tversky and Kahneman, 1979)

Bounded rationality (Simon, 1955)

General assumptions:
a) Policyholders cannot replicate future cash-flows and are risk-averse
b) Insurance companies have access to frictionless, complete capital markets, and can replicate future cash-flows

Linear price-demand function
(Gatzert et al. 2012,
Einav and Finkelstein 2011)

Convex price-demand function
(Zimmer et al. 2016)

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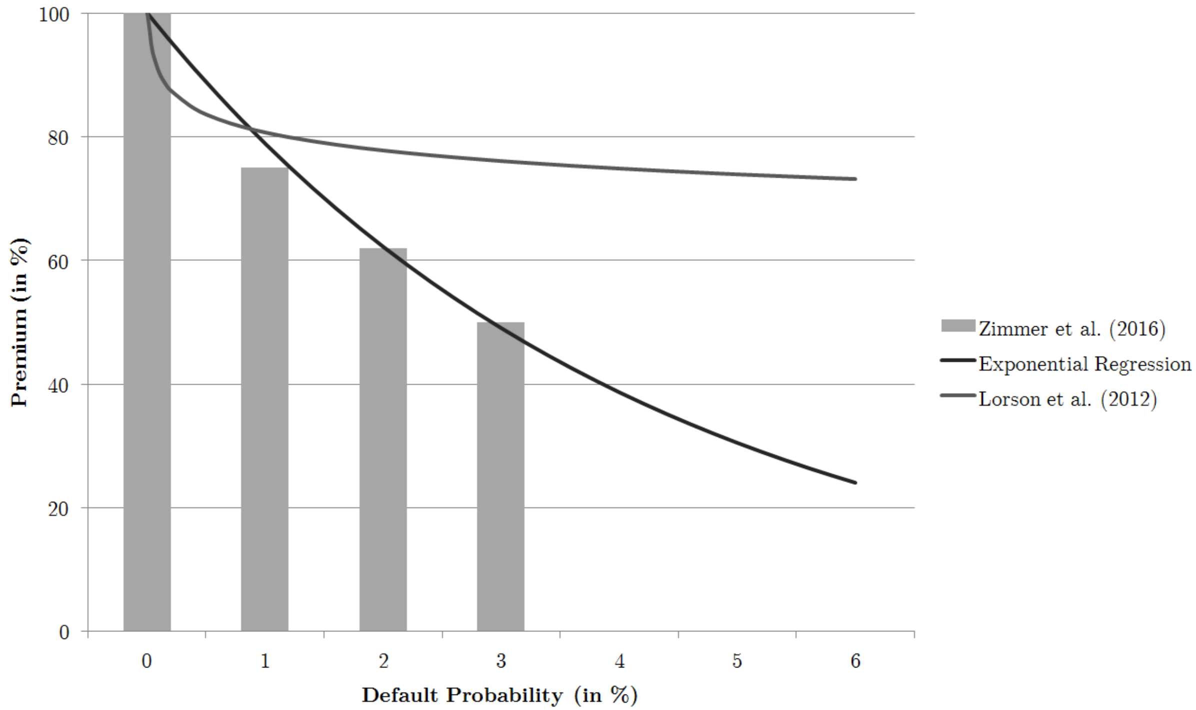
1	Relevance of the Topic
2	Basic Idea
3	**Relation between Default Probability, Price-Demand Function, and Insurance Premiums**
4	Asset Liability Model Framework
5	Numerical Example
6	Conclusion

3. Relation between Default Probability, Price-Demand Function, and Insurance Premiums

- Default probability and premium



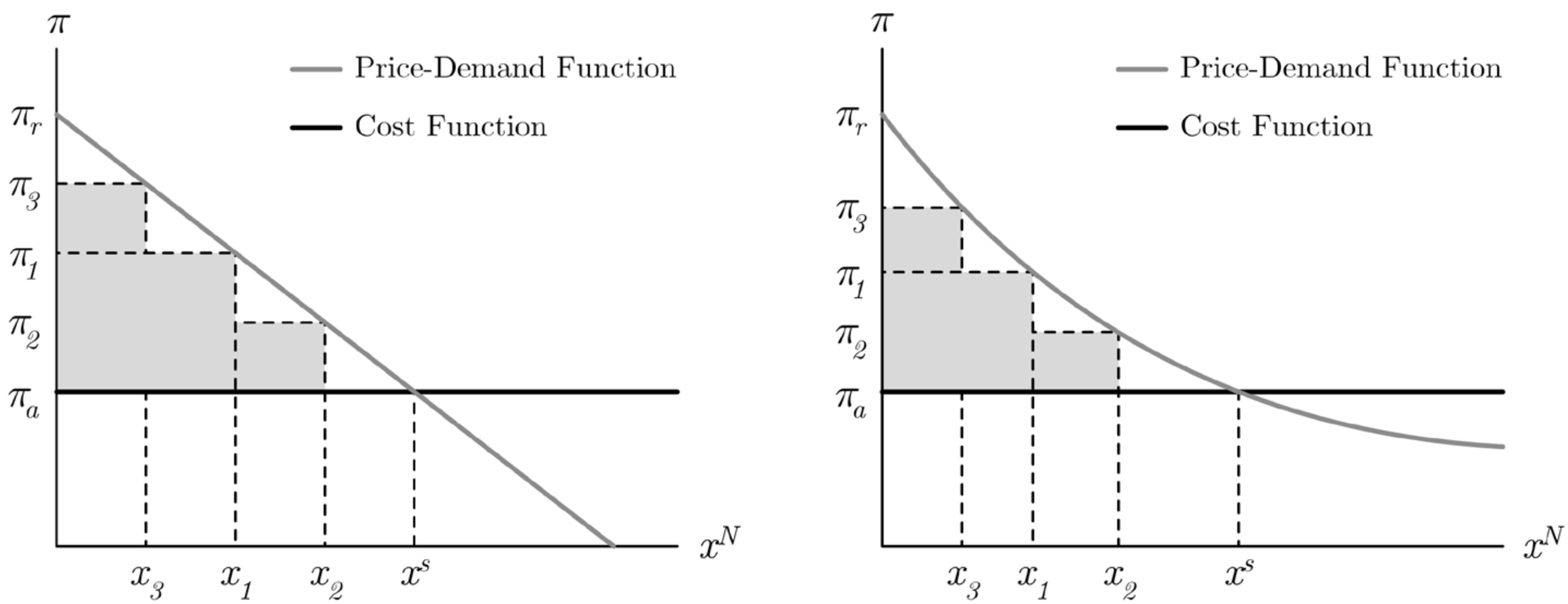
- Linear price-demand function: $\pi_{mean}^{PDF}(DP^A, x^N) = (\pi_r - \theta \cdot x^N) \cdot \exp\left(-23.75 \cdot DP^A\right)$

- Convex price-demand function: $\pi_{mean}^{PDF,con}(DP^A, x^N) = \dfrac{\pi_r}{\eta \cdot x^N + 1} \cdot \exp\left(-23.75 \cdot DP^A\right)$

3. Relation between Default Probability, Price-Demand Function, and Insurance Premiums

- Heterogeneous premiums for homogeneous risks

- Idea: Adapt the premium to the willingness to pay instead the willingness to pay to the premium (which is only downwards possible)



3. Relation between Default Probability, Price-Demand Function, and Insurance Premiums

- **Argumentation:**

- No perfect competition in the insurance market (otherwise no premium higher the equilibrium premium can exist)

- Imperfect Information:

Customer ⟷ Market: Market access or intransparent markets
Customer ⟷ Insurer:

- Various risk attitutes of individuals (e.g., Tversky and Wakker, 1995; Ellsberg, 1961)
- Moral hazard (Holmström, 1979) and adverse selection (Rothschild and Stiglitz, 1997)
⟶ **heterogeneous willingness to pay** (empirical validity see Zimmer et al. 2016)

- **Heterogeneous premiums are supported by heterogeneous willingness to pay, customers' limitations, and the acceptance of satisfactory results**

Outline

4. Asset Liability Model

- One period, non-life model

$$EC_1(DP^A, x^N) = max(A_1(DP^A, x^N) - L_1(DP^A, x^N), 0)$$

where

$$A_0(DP^A, x^N) = EC_0 + \pi^{PDF}_{mean}(DP^A, x^N) \cdot x^N$$

$$E(L_1(DP^A, x^N)) = E(C_1(DP^A, x^N)) = E(c_1(DP^A)) \cdot x^N$$

$$E(C_1(DP^A, x^N)) = E(C_1^0(DP^A, x^N)) + \xi$$

$$\underset{DP^A \in (0,1], x^N \in \mathbb{N}}{\arg\max} \quad NPV(EC_1(DP^A, x^N) - EC_0)$$

4. Asset Liability Model

- Based on two geometric Brownian motions

$$A_1(DP^A, x^N) = A_0(DP^A, x^N) \cdot exp[r - \sigma_A^2/2 + \sigma_A(W_{A_1}^Q - W_{A_0}^Q)]$$
$$L_1(DP^A, x^N) = L_0(DP^A, x^N) \cdot exp[r - \sigma_L^2/2 + \sigma_L(W_{L_1}^Q - W_{L_0}^Q)]$$

- We use the Margrabe-Fischer option pricing formula to determine

$$\begin{aligned} PV(EC_1(DP^A, x^N)) &= E^Q[exp(-r)\, max(A_1(DP^A, x^N) - L_1(DP^A, x^N), 0)] \\ &= A_0(DP^A, x^N) \cdot \Phi(d_1) - L_0(DP^A, x^N) \cdot \Phi(d_2) \end{aligned}$$

where

$$d_1 = \frac{ln(\frac{A_0(DP^A, x^N)}{L_0(DP^A, x^N)}) + \frac{\sigma^2}{2}}{\sigma}; d_2 = d_1 - \sigma; \sigma = \sqrt{\sigma_A^2 + \sigma_L^2 - 2\rho \cdot \sigma_A \cdot \sigma_L}$$

4. Asset Liability Model

- Constraints:

$$P(A_1(DP^A, x^N) < L_1(DP^A, x^N)) \leq DP^A$$

with uncorrelated assets and liabilities, we reach

$$\Phi(\frac{ln(\frac{L_0(DP^A,x^N)}{A_0(DP^A,x^N)}) - \sigma_L^2/2 + \sigma_A^2/2}{\sqrt{\sigma_A^2 + \sigma_L^2}}) \leq DP^A$$

- Premium levels restricted to 3:

- Different premium levels lead to classification costs (see, e.g., Gatzert et al. 2012, optimal risk classification)

- Reputational risk may occur if customer recognizes heterogeneous premiums

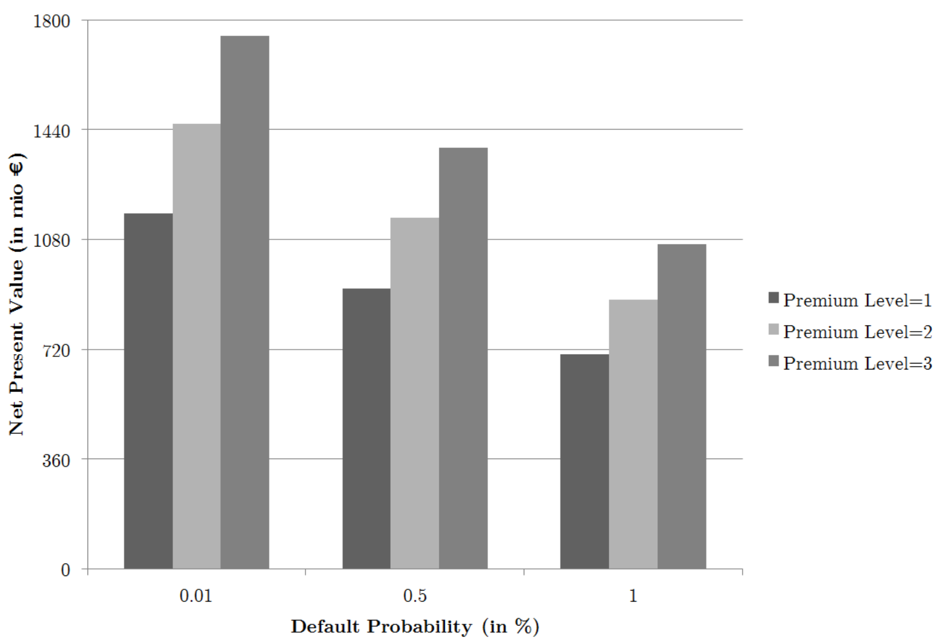
Outline

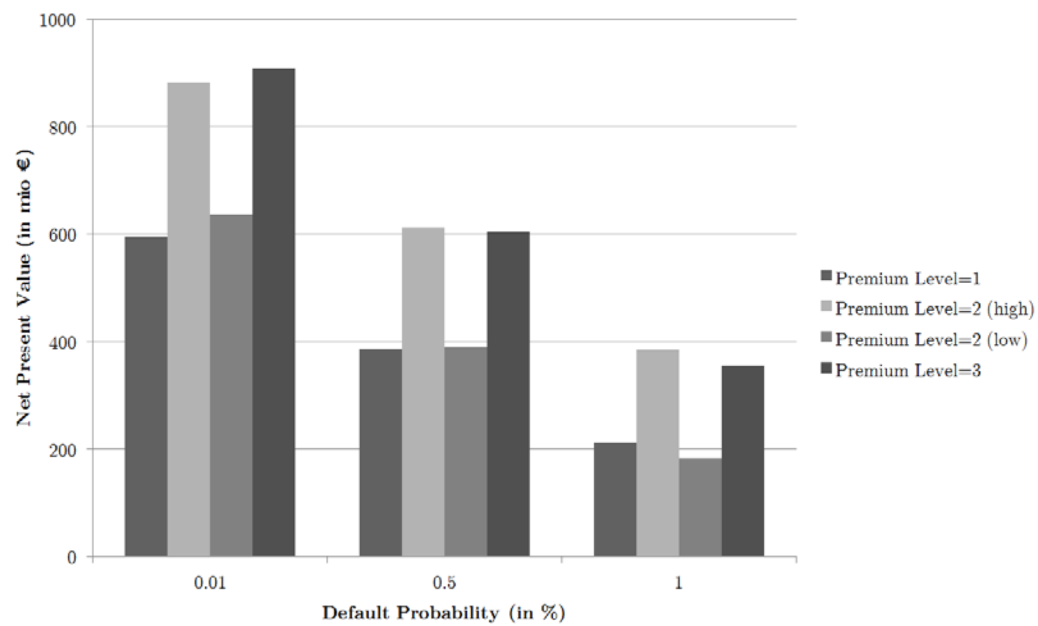
5. Numerical Example

- Linear price-demand function

- Perfect expectations (no cost shift)

Cost shift: 50% increase of the expected costs



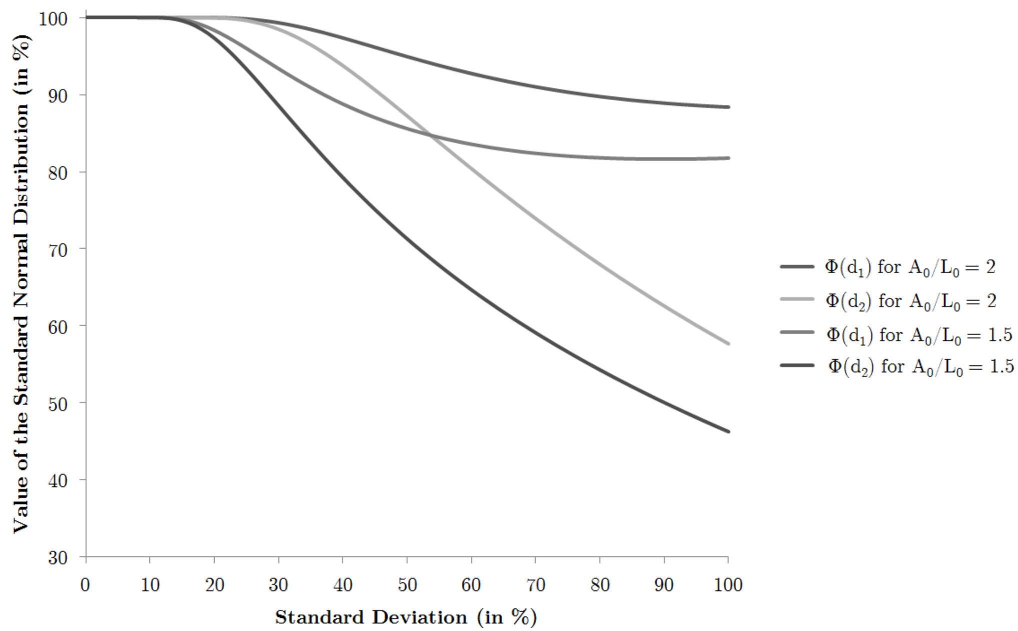


5. Numerical Example

- For 0.01 (0.5) percent default probability, a lower premium adaptation increases the net present value although premium < expected costs: Why?

$$PV(EC_1(DP^A, x^N)) = E^Q[exp(-r)\, max(A_1(DP^A, x^N) - L_1(DP^A, x^N), 0)]$$
$$= A_0(DP^A, x^N) \cdot \Phi(d_1) - L_0(DP^A, x^N) \cdot \Phi(d_2)$$

The gap of the standard normal distribution functions increases for a lower premium adaptation



5. Numerical Example

- Optimal demand varies for net present value maximization between underwriting (premiums minus costs) and overall perspective (asset liability model)
- Optimal demand variation: For the linear price demand function by 5.37 percent
- Internal Solvency Constraint:

Table 5: Perfect Expectations: Reported and Actual Default Probability in Percent

π^{level}	1	2(high)	2(low)	3	1	2(high)	2(low)	3	1	2(high)	2(low)	3
DP^A	0.01	0.01	0.01	0.01	0.5	0.5	0.5	0.5	1	1	1	1
DP^{lin}	0.15	0.08	0.61	0.39	0.19	0.11	0.78	0.53	0.21	0.13	0.92	0.65
					✓	✓			✓	✓	✓	✓

Table 6: Cost Shift: Reported and Actual Default Probability in Percent

π^{level}	1	2(high)	2(low)	3	1	2(high)	2(low)	3	1	2(high)	2(low)	3
DP^A	0.01	0.01	0.01	0.01	0.5	0.5	0.5	0.5	1	1	1	1
DP^{lin}	2.47	1.54	6.71	4.92	2.91	1.95	7.89	6.06	3.21	2.30	8.84	6.96

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6. Conclusion

- Default probability (if transparent) substantially reduces the willingness to pay, incentive to reduce default risk or sell additional insurance contracts which pay in the case when the first contract does not pay
- Heterogeneous premiums can improve the net present value, a lower premium adaptation is only beneficial if premium > costs (if premium < costs, results are not unique)
- Heterogeneous premiums are possible because of a heterogeneous willingness to pay and if the market is not fully competitive
- Solvency constraints do not or only delayingly protect for dramatic changes
- Optimal price setting varies between the underwriting and the overall (i.e., Asset / Liability) perspective