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THE MERITS OF POOLING CLAIMS: MUTUAL VS. STOCK INSURERS

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Abstract

This paper aims to identify combinations of premiums, equity capital contributions, ruin probabilities, and pool sizes, under which a risk-averse policyholder, with an exponential utility function, is indifferent between purchasing insurance coverage with a mutual or a shareholder insurance company. In a mutual company, and in contrast to the shareholder insurer, the amount of premiums charged do not influence the policyholder's utility level. For the shareholder company, we analyze combinations of premiums and equity within a fixed pool size, such that a policyholder is indifferent to either of the two legal forms in focus. In addition, we include the restriction of a fixed ruin probability in the mutual and shareholder insurance company. A numerical illustration shows that given a fixed ruin probability, a determined pool size and fair premiums (i.e., the premium equals the expected indemnity payments), policyholders in the shareholder company experience higher utility levels than policyholders in a mutual insurance company. Hence, a risk-averse policyholder accepts an unfair premium in the shareholder company to obtain the same utility level as provided by the mutual company.

1 Introduction

The term "risk pooling" in insurance is commonly used to refer to the spreading of risk among a large number of participants in a portfolio to reduce the impact of losses on the individual's wealth position (Smith & Kane, 1994; Albrecht, 1984; Albrecht & Huggenberger, 2017). In exchange for this risk transfer, individuals pay an upfront insurance premium that is typically given by the expected value of the claim plus a safety loading. In the case of identical and independently distributed claims and a fixed ruin probability, the safety loading per risk tends to zero if the number of risks in the pool goes to infinity. This behavior is caused by the underlying risk structure which is, in this case, of pure unsystematic nature.

Risk pooling and the resulting merits for policyholders are typically focused on the context of mutual insurance companies[1]. Risk sharing in a mutual includes the splitting of assets leftover whenever the aggregate premiums exceed the claims in the pool at the end of the period. Individuals insured by a shareholder company only face

[1]Cf. Gatzert & Schmeiser (2012) and the literature cited in this paper.

the default risk of the insurer. Consequently, the number of participants in the pool, as well as the debt and equity capital provided by the policyholders and shareholders, determines the safety level of the insurer.

The literature on the subject of pooling insurance risks has mainly focused on diversification effects regarding different loss distributions, dependency structures, pool sizes, risk measures and rate making schemes (cf., e.g., Albrecht & Huggenberger (2017); Cummins (1991); Powers (2006))[2]. Gatzert & Schmeiser (2012) study the merits of pooling claims, from a policyholder's point of view, in the case of a mutual insurer. If a policyholder with μ/σ-preferences cannot replicate future cash-flows via assets traded on the capital market, an increase in the pool size has a positive effect on his/her utility level. In this way, identical and independently distributed risks are assumed and both policyholder stakes in the mutual insurance company (debt and owner's position) are evaluated. Moreover, the level of the initial premium contribution provided by the policyholders do *ceteris paribus* not influence the utility level of the participants of the pool if counterparty risk within the pool members does not exist. Albrecht & Huggenberger (2017) extend these findings for a sequence of exchangeable risks and prove that the utility of the policyholder strictly increases with the size of the risk pool for the various preferences obtained (e.g., expected utility, Choquet expected utility, distorting risk measures). The model setup provided by the authors is very general. Hence, no specific distributional assumptions are needed for individual risks and stochastic dependencies among the participants in the pool.

Braun et al. (2015) analyze the relationship between stock and mutual insurance premiums based on a contingent claims framework. In general and in a fully competitive market, mutual insurance companies must charge higher premiums compared to shareholder driven companies. The authors confront their theoretical findings with the results from an empirical sample retrieved from the German motor insurance market. Thereby, no statistically significant differences in the premium levels of mutual and shareholder companies were found. The authors discuss their reasoning to suspect that considerable wealth transfer exists between policyholders and shareholders. Their focus is the fair pricing of the insurance policies and only assume the policyholders have higher willingness to participate in the pool as long as their utility is higher than the no insurance case. However, Braun et al. (2015) overlook the actual risk valuation that policyholder may perform when comparing mutual and stock insurers from the utility theory point of view.

The primary purpose of this paper is to identify combinations of premiums, required equity contributions, default risk, and pool sizes (i.e., number of risks in the

[2] An extensive literature review in the matter of risk pooling is presented in Gatzert & Schmeiser (2012) pp. 186-188 and Albrecht & Huggenberger (2017) pp. 180-181.

portfolio) under which the policyholders, using an exponential utility function, are indifferent between purchasing an insurance contract with a mutual or a shareholder insurance company. In competitive markets, the two legal forms should provide (at least) similar utility levels to their customers, in order to explain their existence in the long run. In other words, policyholders always have the option to set up a mutual insurer in order to diversify their risks and increase their utility position. Rational policyholders only consider insurance with a shareholder driven company, if their utility position equals the benchmark case of the mutual. In this paper, we derive conditions that lead to an indifference position from the viewpoint of the policyholders.

The initial setting in the first part of the paper analyses the policyholder's wealth distribution in a mutual and shareholder insurance company, when *ceteris paribus* conditions are presumed to take place. In respect to identical conditions, we refer to the claims distribution, pool size, insurer's ruin probability and fair premiums[3]. The variance of the policyholder's wealth illustrates substantial discrepancies under the two legal forms. In particular, policyholders insured under the mutual model have an owners' share, and hence, a claim on the remaining assets whenever the insurer is not in a state of default. Therefore, their wealth's variance is not only affected by the default risk (as it is in the case of a shareholder company), but also by the upward return opportunity.

Under the conditions mentioned and assuming an exponential utility function, the policyholders in the shareholder company experience a higher utility level than policyholders in the mutual company. In addition, we illustrate that, in contrast to a shareholder company, the amount of premiums charged by a mutual company have no influence on the policyholder's utility (as long as credit risk of the policyholders does not exist). Moreover, any premium level in the mutual is "fair" with respect to our definition (cf. Footnote No.3).

In the second part of the paper, we analyze three insurance pool scenarios under which policyholders are indifferent between being insured in a mutual or a shareholder insurance company. The first scenario involves investigating the pool characteristics, such that policyholders face the same ruin probability in both legal forms. Whenever the ruin probability is fixed and fair premiums are charged, the policyholder's utility level is mainly driven by the variance of the future wealth distribution. In the case of a mutual insurer, the pool size needs to be substantially larger, compared to the pool in a shareholder company, to lead to the same ruin probabilities and utility levels.

[3]We understand fair insurance premium as a premium that equals the expected payoff to the policyholder. In the case of the mutual company, the expected payoff includes the expected indemnity payment as well as the payments for the policyholder's stake as an owner. In the shareholder insurance company a fair premium equals the expected payoff which is given by the expected indemnity payments.

The second scenario assumes that the pool size is fixed for both companies. For the shareholder company, we look for premium and equity contributions, such that policyholders would be indifferent to either of the two legal forms. In general, the two insurance companies in this scenario are not subject to the same ruin probability. An increase in the insurance premium leads to a higher equity contribution, and therefore, a lower ruin probability for the shareholder company. The pool size plays a major role in determining the set of suitable premium-equity combinations. More specifically, a larger number of risks in the pool reduces the amount of equity contributions per policyholder. The indifference premium interval (i.e., the absolute difference between the upper and lower bound of the indifference premium) becomes smaller whenever the pool size increases. Moreover, the indifference premium reaches the expected value of the policyholder's claim for a large number of participants within the pool. For a large number of risks in the pool, changes in the equity amount per policyholder do not have a severe influence on the indifference premium, but have a large impact on the insurer's ruin probability.

The third scenario focuses on a policyholder being indifferent between purchasing insurance within a mutual and a shareholder company, as the two legal forms offer the same ruin probability for a fixed pool size. In this case, the indifference premium of the shareholder company exceeds the policyholder's expected claim (i.e., the premium is not fair because it does not equal the expected indemnity payments to the policyholder). Hence, policyholders are willing to accept an unfair premium in the shareholder company to reach the same utility and safety level that the mutual insurer provides.

The rest of this paper is structured as follows. Section 2 establishes general assumptions and a modeling framework concerning collective risk pooling in a mutual and in a shareholder insurance company. Section 3 presents the differences between the policyholder's wealth position in a mutual and a stock company. An analysis of the conditions under which the pooling of claims leads to identical policyholder's utility in a mutual and a shareholder insurance company is conducted in Section 4. A final comment on the impact of the policyholder risk aversion on the indifference premium is also included in Section 4.4. Section 5 concludes the paper.

2 Pooling Claims

Our first case considers that n risks are pooled in a portfolio within a specified reporting period $t = 0, 1$. The risks X_i with $i = 1, ..., n$ are independent and identically distributed, with a finite and positive variance $\sigma(X_i) > 0$. The stochastic total claim S for the portfolio at time $t = 1$ is given by:

$$S = \sum_{i=1}^{n} X_i \tag{1}$$

A policyholder pays a premium π_i at $t = 0$ to for the risk transfer of X_i. The policyholder receives a positive indemnity payment I_i at $t = 1$ if a claim occurs. We define π_i as a "*fair premium*" if it equals the policyholder's expected payoff at $t = 1$. The expected payoff includes the expected indemnity (claim payment in the mutual and shareholder company) as well as any agreed policyholder participation in the insurance company profits (case of mutual insurer).

We assume an incomplete market setting in which policyholders are not able to replicate future cash flows with the given market instruments traded on the capital markets. Hence, a valuation based on policyholder preferences is required. We assume that policyholders judge their decision concerning insurance using an exponential utility function for their wealth W. In formal terms, we have:

$$U(W) = -e^{-a(W+K)} \tag{2}$$

where $a > 0$ corresponds to the risk aversion coefficient and K is a scaling constant[4]. The second-order approximation (mean-variance) for the expected utility $EU[\cdot]$ is given by:

$$\Phi(W) = E[U(W)] \approx -e^{-a(\mu(W)+K)}\left[1 + \frac{a^2}{2}\sigma^2(W)\right] \tag{3}$$

where $\mu(W) = E[W]$ and $\sigma^2(W) = E[(W - \mu(W))^2]$. The second order Taylor approximation[5] used for $U(W)$ is described in the Appendix.

3 Policyholders Perspective

3.1 Case of a Mutual Insurer

The value of the pooling, from the policyholder perspective, depends on the underlying assumptions concerning the diversification opportunities. We now assume a mutual insurance company denoted by M. In what follows, the policyholders are owners and debt holders of the insurance firm. The policyholder's wealth W_i^M at time $t = 1$,

[4]The main purpose of the scaling parameter is to keep the utility function different than zero with large wealth values.

[5]Note that if W is normally distributed with $\mathcal{N}(\mu, \sigma^2)$, the exact value of the expected utility $E[U(W)] = -M_W(-a) = e^{-a\mu+\frac{1}{2}\sigma^2 a^2}$ and the utility level is completely and precisely defined by the behavior of $\mu - \frac{a}{2}\sigma^2$. However, we use the general second-order approximation, which allows for the calculation of the expected utility for more general random variables. In particular, in our setting, the account for default risk leads to payoff distributions that are not normal in shareholder insurance companies, even for large n.

assuming that he/she buys insurance coverage for the risk X_i is given by[6]

$$W_i^M = A_i(1+r) - X_i - \pi_{i,M}(1+r) + I_{i,M} + E_{i,M} \tag{4}$$

where A_i represents the initial wealth of the policyholder at $t = 0$ that accumulates to $t = 1$ at a risk-free rate of return r. $\pi_{i,M}$ is the premium charged by the mutual company and $I_{i,M}$ corresponds to the indemnity payment for the loss X_i. Finally, $E_{i,M}$ stands for the policyholder's owner stake in the mutual insurance company (i.e., the remaining assets after all claims are paid). Hereafter, we assume, for the sake of simplicity, that the risk-free rate of return is equal to zero (i.e., $r = 0$).

Given the limited liability of the participants of the pool in the mutual company, the insurer would declare bankruptcy if the assets at $t = 1$ would not be enough to pay the accumulated claims S. The policyholder's indemnity payment $I_{i,M}$ at $t = 1$ would be reduced in the case of bankruptcy. Thus, it is given by

$$I_{i,M} = X_i - \frac{1}{n} max(S - \pi_M, 0) \tag{5}$$

where $\pi_M = n \cdot \pi_{i,M}$. Once the mutual insurer has fully paid the indemnity payments, the remaining assets would be distributed to the pool members. Thus, $E_{i,M}$ is given by

$$E_{i,M} = \frac{1}{n} max(\pi_M - S, 0) \tag{6}$$

Note that, in all cases, the following equivalence holds:

$$\begin{aligned} E(I_{i,M} + E_{i,M}) &= E\left(X_i - \frac{1}{n} max(S - \pi_M, 0) + \frac{1}{n} max(\pi_M - S, 0)\right) \\ &= E\left(X_i + \frac{1}{n}(\pi_M - S)\right) \\ &= E\left(X_i + \frac{1}{n}(n\pi_{i,M} - S)\right) \\ &= \pi_{i,M} \end{aligned} \tag{7}$$

In this sense, the premium always equals the policyholder's expected payoff. Hence, any premium level in the mutual company is "fair" with respect to our definition (cf. Section 2). However, the premium does in general not equal the expected value of the claim (i.e., $E[I_{i,M} + E_{i,M}] \neq E[X_i]$).

Since Equation 7 holds in all cases for the mutual insurance, we assume that - without loss of generality - individual premium[7] $\pi_{i,M}$ is given by

[6] For the case of the mutual insurer, we rely on the findings presented in Gatzert & Schmeiser (2012).

[7] Cf. Equation 7 where $\pi_{i,M}$ satisfies

$$\pi_{i,M} = E(X_i) + E\left(\frac{1}{n}(\pi_M - S)\right)$$

$$\pi_{i,M} = E(X_i) + c \tag{8}$$

where $c \in (-E(X_i), \infty)$. In particular, $\pi_{i,M}$ could be set such that it satisfy the following equation:

$$P(S > n \cdot \pi_{i,M}) = \varepsilon \tag{9}$$

where ε denotes the company safety level. Hence, the safety loading c would be dependant of the safety level ε and the pool size n, i.e., $c = c(n, \varepsilon)$.

Policyholder's utility

The policyholder's expected utility at $t = 1$ in the mutual insurance is given by:

$$\Phi(W_i^M) = -e^{-a(\mu(W_i^M)+K)} \left[1 + \frac{a^2}{2}\sigma^2(W_i^M)\right] \tag{10}$$

where the expected value $\mu(W_i^M)$ and variance $\sigma^2(W_i^M)$ of the policyholder's wealth at $t = 1$ corresponds to:

$$\mu(W_i^M) \doteq E(W_i^M) = A_i - E(X_i) - \pi_{i,M} + E(I_{i,M} + E_{i,M}) = A_i - E(X_i) \tag{11}$$

and

$$\begin{aligned}
\sigma^2(W_i^M) &= \sigma^2(-X_i + I_{i,M} + E_{i,M}) = \sigma^2\left(-X_i + X_i + \frac{1}{n}(\pi_M - S)\right) \\
&= \sigma^2\left(\frac{1}{n}(\pi_M - S)\right) = \sigma^2\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) \\
&= \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2(X_i) = \frac{1}{n}\sigma^2(X_i)
\end{aligned} \tag{12}$$

As shown in Gatzert & Schmeiser (2012), Equations (11) and (12) do not depend on $\pi_{i,M}$. More generally, the policyholders wealth distribution, and hence, the resulting utility, is independent of the premium level if the credit risk is omitted from the analysis[8].

[8] In practice, the credit risk plays a major role in the case of low premium payments at $t = 0$. Thereby, payments between policyholders without a claim to policyholders with a claim must take place at $t = 1$ to ensure the merits of the pooling effect modelled in Equation (10) to (12). We assume, in what follows, that such payments can be made (and hence, no credit risk takes place) for all policyholders. Thus, for large n, each policyholder − with or without a claim − pays approximately $E[X_i]$.

3.2 Case of the Shareholder Insurance Company

In the shareholder company denoted by S, in what follows, the owners of the company and the policyholders are two, entirely or partially, separated groups. The policyholder's wealth W_i^S at $t = 1$ and the indemnity benefit $I_{i,S}$, given the shareholders limited liability, result from:

$$W_i^S = A_i(1+r) - X_i - \pi_{i,S}(1+r) + I_{i,S} \tag{13}$$

and

$$I_{i,S} = X_i - \frac{1}{n} max\left(S - \pi_S - EC_0, 0\right). \tag{14}$$

As such, $\pi_{i,S}$ corresponds to the premium charged by the insurer to each policyholder. Hence, we have $\pi_S = n\pi_{i,S}$ and EC_0 stands for the equity capital contribution paid by the shareholders. The shareholders' claim (E_S) on the remaining assets, after the policyholder's claim, is entirely paid out and is given by:

$$E_S = max(EC_0 + \pi_S - S, 0) \tag{15}$$

Policyholder's utility

The policyholder's expected utility at $t = 1$ in the shareholder company is:

$$\Phi(W_i^S) = -e^{-a(\mu(W_i^S)+K)}\left[1 + \frac{a^2}{2}\sigma^2(W_i^S)\right] \tag{16}$$

The expected value $\mu(W_i^S)$ and the variance $\sigma^2(W_i^S)$ of the policyholder's wealth corresponds to:

$$\mu(W_i^S) \doteq E(W_i^S) = A_i - E(X_i) - \pi_{i,S} + E(I_{i,S}) \tag{17}$$

and

$$\begin{aligned}\sigma^2(W_i^S) &= \sigma^2(-X_i + I_{i,S}) \\ &= \sigma^2(-X_i + X_i - \frac{1}{n}(max[S - \pi_S - EC_0, 0]) \\ &= \sigma^2\left(\frac{1}{n}max[S - \pi_S - EC_0, 0]\right)\end{aligned} \tag{18}$$

Note that the premium $\pi_{i,S}$ can explicitly be set such that it equals the expected indemnity payment $E(I_{i,S})$. In this case, we have $E[W_i^S] = E[W_i^M]$. Such a setting corresponds to our definition of a fair premium. However, in general, the equality between the premium $\pi_{i,S}$ and the expected policyholder's payoffs $E(I_{i,S})$ do not hold (i.e., $\pi_{i,S} \neq E(I_{i,S})$). Finally, the comparison of the expected utility in both business models illustrates that the policyholder's utility in the shareholder company directly depends on the premium π^s and the equity contribution EC_0. This stands in contrast to the mutual company, in which the utility is independent of the premium amount paid up-front (if the policyholders' credit risk does not play a role).

3.3 Initial Considerations

We assume that the insurer charges fair premiums to policyholders. More precisely, we set the initial insurance premium in the way that it equals the policyholder's expected payoff. Thus $\pi_{i,M} = E[I_{i,M} + E_{i,M}]$ and $\pi_{i,S} = E(I_{i,S})$ are the premiums charged to each pool member in the mutual and in the shareholder company, respectively. The premium $\pi_{i,M}$ for the mutual and the combination ($\pi_{i,S}$,EC_0) for the shareholder company are specified such that the ruin probability does not exceed a predefined safety level ε in either of the companies. Thus, the following equivalence holds:

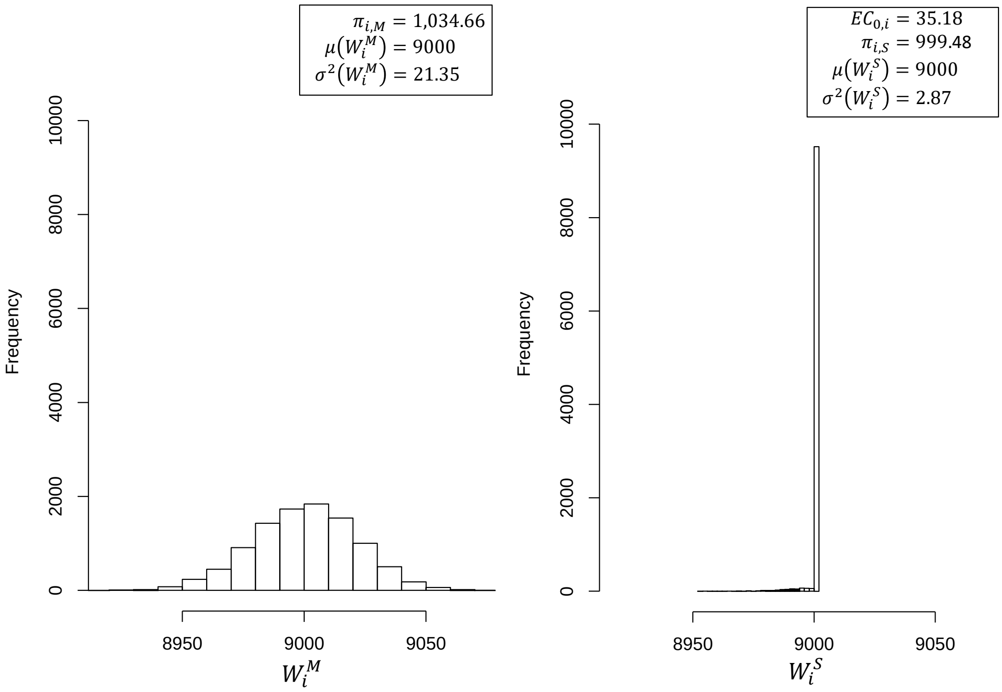
$$Pr\left[S > \sum_{i=1}^{n} \pi_{i,M}\right] = \varepsilon = Pr\left[S > \sum_{i=1}^{n} \pi_{i,S} + EC_0\right] \tag{19}$$

Consequently, we have:

$$\pi_{i,M} = \pi_{i,S} + EC_{0,i} \tag{20}$$

where $EC_{0,i} = EC_0/n$ denotes the shareholder equity contribution per policyholder that is required in the shareholder company. In our numerical illustration, we assume that the policyholder has an initial wealth of A_i = 10,000 and the risks X_i, $i = 1, ..., n$ are Lognormally distributed with an expected value of $E[X_i]$ = 1,000 and a standard deviation $\sigma(X_i)$ = 675. Additionally, we set the policyholder's specific risk aversion parameter to a = 0.75 and the constant scaling parameter to K = -9,000 (i.e., $U(W) = -e^{-0.75(W-9{,}000)}$).

Figure 1 illustrates the differences in the distribution of the policyholder's final wealth under the two business models, assuming that fair premiums are charged to the policyholders, and the ruin probability is set at 5%. The main difference in the distributions corresponds to the variance of the policyholder's wealth. Indeed, the variance of policyholder's wealth with insurance is smaller than the variance of the individual with no insurance. More precisely, we have $\sigma(A_i - X_i) > \sigma(W_i^M)$ and $\sigma(W_i^M) > \sigma(W_i^S)$ for $n > 1$ (cf. Gatzert & Schmeiser, 2012, p. 185). On the one hand, in a mutual company, the policyholder would share the volatility of the risk with the pool (cf. Figure 1a). The reduction in the variance of the policyholder's wealth would depend on the pool size n: $\sigma(W_i^M) = \sigma(X_i)/\sqrt{n}$. The portion of risk retained by the policyholder corresponds to the default risk and the policyholder's participation in the assets' surplus after all claims are paid.



(a) Mutual company **(b)** Shareholder company

Figure 1: Comparison of the distribution of the policyholder's wealth at $t = 1$ for the case of the mutual company $W_{i,M}$ and shareholder company $W_{i,S}$. $n = 1,000$, $E(X_i) = 1,000$, $\sigma(X_i) = 675$, $A_i = 10,000$ and $\varepsilon = 5\%$.

On the other hand, if the policyholder is insured with a shareholder insurance company, he/she transfers the risk solely to the shareholders, as long as the insurer is solvent (cf. Figure 1b). Hence, any participation in the assets' surplus would be distributed to the shareholders, instead of being shared with the policyholders. The shareholders must buffer most of the volatility of the claims with their equity capital. At the same time, the shareholders' limited liability would charge policyholders with default costs. Hence, in this case, the variance of the policyholder's wealth $\sigma^2(W_i^S)$ results from the policyholder's deficit, in case the insurer declares bankruptcy.

In summary, we see that the policyholders in the mutual company face an upside risk (whenever the stochastic wealth in $t = 1$ exceeds $E(W_i^M) = A_i - E(X_i)$) and a downside risk. The premium paid to the mutual company, and hence, the resulting default risk, does not influence the policyholder's utility level. Both points do not hold true for the shareholder company. In particular, the policyholder only faces the downside risk. More precisely, the stochastic wealth in $t = 1$ is below $E(W_i^S) = A_i - E(X_i)$ if the shareholder company is in default. The distribution of the downside risk differs widely between the two organizational forms.

Table 1 illustrates the policyholder's utility in the mutual and shareholder

Table 1: Comparison of fair premiums and policyholder's wealth distributions at $t = 1$ under a mutual and a shareholder company for different safety levels ε.

	Mutual Company			Shareholder Company			
n	$\pi_{i,M}$	$\Phi(W_i^M)$	$\sigma(W_i^M)$	$EC_{0,i}$	$\pi_{i,S}$	$\Phi(W_i^S)$	$\sigma(W_i^S)$
				$\varepsilon = 5\%$			
10	1,388.25	-12,815.45	213.45	395.60	992.64	-575.93	45.21
100	1,113.44	-1,282.45	67.50	115.13	998.31	-28.90	9.96
1,000	1,034.66	-129.14	21.35	35.18	999.48	-3.32	2.87
10,000	1,011.35	-13.81	6.75	11.49	999.86	-1.19	0.83
100,000	1,003.54	-2.28	2.13	3.58	999.95	-1.02	0.26
				$\varepsilon = 1\%$			
10	1,633.48	-12,815.45	213.45	634.72	998.76	-101.37	18.89
100	1,168.40	-1,282.45	67.50	168.66	999.73	-5.01	3.77
1,000	1,050.44	-129.14	21.35	50.51	999.92	-1.30	1.03
10,000	1,016.04	-13.81	6.75	16.06	999.98	-1.02	0.29
100,000	1,005.03	-2.28	2.13	5.04	999.99	-1.00	0.09
				$\varepsilon = 0.5\%$			
10	1,695.04	-12,815.45	213.45	695.81	999.23	-67.52	15.38
100	1,185.02	-1,282.45	67.50	185.16	999.86	-3.16	2.77
1,000	1,055.28	-129.14	21.35	55.32	999.96	-1.15	0.73
10,000	1,017.92	-13.81	6.75	17.93	999.99	-1.01	0.17
100,000	1,005.51	-2.28	2.13	5.52	1,000.00	-1.00	0.06
				$\varepsilon = 0.01\%$			
10	2,405.76	-12,815.45	213.45	1,405.77	1,000.00	-1.04	0.38
100	1,279.04	-1,282.45	67.50	279.05	1,000.00	-1.01	0.17
1,000	1,081.63	-129.14	21.35	81.63	1,000.00	-1.00	0.01
10,000	1,023.88	-13.81	6.75	23.88	1,000.00	-1.00	0.00
100,000	1,007.58	-2.28	2.13	7.58	1,000.00	-1.00	0.00

companies when both companies set fair premiums and offer the same safety level. In the case that the available assets (premiums plus equity contributions) are not enough to cover the insurance claims, the policyholders will see their indemnity payments reduced (i.e., the incurred claims cannot be fully reimbursed).

For a fixed ruin probability ε, the variance of the policyholder's wealth decreases as the pool size n increases. Hence, the owner stake, per policyholder, has almost no value for large n. Consequently, the fair premiums in the mutual and in the shareholder company merge. For the shareholder insurance company, a lower ruin probability implies a decrease in the variance of the policyholder's wealth and, ceteris paribus, an increase in the fair premiums will take place. For a fixed ruin probability ε, we have: $\pi_{i,S} = E[I_{i,S}] \leq E[X_i]$ with $\lim_{n\to\infty} E[I_{i,S}] = E[X_i]$.

If the premiums in the shareholder company are set such that $\pi_{i,S} = E[I_{i,M}]$ and the safety level ε is fixed, the corresponding safety loading $c(n,\varepsilon)$ and the equity contribution $EC_0(n,\varepsilon) = n \cdot EC_{0,i}(n,\varepsilon)$ are fully determined. Moreover, the safety loading in the mutual company and the amount of equity contribution per policyholder, $EC_{0,i}(n,\varepsilon)$, in the shareholder company decrease and tend to zero as the pool size increases. In formal terms, we have $c(n,\varepsilon) < c(m,\varepsilon)$ and $EC_{0,i}(n,\varepsilon) < EC_{0,i}(m,\varepsilon)$ for $n > m$. Nonetheless, larger pool sizes in aggregated terms would always require higher amounts of equity contributions in absolute terms (i.e.,$EC_0(n,\varepsilon) > EC_0(m,\varepsilon)$ for $n > m$). In fact, we have $EC_{0,i}(n,\varepsilon) \geq c(n,\varepsilon)$[9], given that equity contributions and safety loadings are set such that policyholders face, at most in either of the companies, a default risk smaller than or equal to ε. Finally, higher safety levels would require larger safety loadings, as well as equity contributions (i.e., $c(n,\varepsilon) > c(n,\zeta)$ and $EC_0(n,\varepsilon) > EC_0(n,\zeta)$ for $\varepsilon < \zeta$).

If fair premiums are charged in the mutual and shareholder company, we have $E[W_i^M] \doteq E[W_i^S]$ (cf. Equations (11) and (17)). Hence, policyholder's utility levels are, in general, driven by the wealth distribution and the variance of the policyholder's wealth. The variance of the policyholder's wealth in the mutual company is dependent on the pool size, but independent of the insurance premium and the ruin probability. On the contrary, the variance of the policyholder's wealth in the shareholder company is dependent on the premium, equity contribution, pool size, and ruin probability.

The policyholders' utility is, in all cases, higher in the shareholder company than the mutual company when fair premiums are set and a positive risk aversion coefficient is fixed for the exponential utility function. However, the utility levels in the mutual and shareholder company are comparable when the size of the pool is sufficiently large, such that $\sigma[W_i^M]$ and $\sigma[W_i^S]$ are approximatively zero. In this case, the underlying unsystematic risk is fully diversified.

It can be concluded that, in most cases, especially for the exponential utility function, the policyholders would prefer the shareholder company to the mutual company if similar pools are compared under identical conditions (i.e., fair premiums, identical safety levels and the same positive risk aversion coefficient for all participants in the pool). However, in this scenario, it is assumed that shareholders are willing to

[9] In formal terms, we have (cf. Equation (20))

$$\begin{aligned}
\pi_{i,S} &\leq E[X_i] \\
\pi_{i,S} + c(n) &\leq E[X_i] + c(n) = \pi_{i,S} + EC_{0,i} \\
\pi_{i,S} + c(n) &\leq \pi_{i,S} + EC_{0,i} \\
c(n) &\leq EC_{0,i}
\end{aligned}$$

provide the initial equity contribution in the way that is proposed in this section. The question then becomes if shareholders are willing to provide the necessary equity capital or not, which depends on the valuation techniques used by the owners of the insurance company. Hence, if the shareholders achieve a risk-adequate return on their initial contribution out of the scope of our model setting.

4 Policyholders Equity-Premium Indifference Curve (EC-P Curve)

In this section, we seek conditions under which a policyholder would be indifferent between joining a mutual insurer or a shareholder company. In a competitive market, we expect premium settings that lead to similar utility levels in order that both firms can exist in the long run. A policyholder is indifferent between the two business models if the perceived utility is the same in either of the two companies:

$$\Phi(W_i^M) = \Phi(W_i^s)$$
$$-e^{-a(\mu(W_i^M)+K)}\left[1+\frac{a^2}{2}\sigma^2(W_i^M)\right] = -e^{-a(\mu(W_i^S)+K)}\left[1+\frac{a^2}{2}\sigma^2(W_i^S)\right] \tag{21}$$

For our numerical example, we set $a = 0.75$ and $K =$ -9,000, as in Section 3.3.

4.1 Case A - Fixed ruin probability ε

In this section, we analyze the scenario in which the risk aversion parameter $a > 0$ and the safety level ε would be the same for the mutual and the shareholder company. That being said, the composition of the pools in each of the companies may differ.

If fair premiums are to be set by both companies, the difference in the utility levels would be mainly determined by the variance of the policyholder's wealth at $t = 1$ (cf. Section 3.1 and Section 3.2). Hence, if a fair premium, equity contributions, and pool size are given for the shareholder company, then the adequate pool size in the mutual company can be calculated by:

$$\Phi(W_i^M) = \Phi(W_i^S)$$
$$-e^{-a(\mu(W_i^M)+K)}\left[1+\frac{a^2}{2}\sigma^2(W_i^M)\right] = -e^{-a(\mu(W_i^S)+K)}\left[1+\frac{a^2}{2}\sigma^2(W_i^S)\right]$$
$$1+\frac{a^2}{2}\frac{\sigma^2(X_i)}{n_M} = 1+\frac{a^2}{2}\sigma^2(W_i^S)$$
$$\frac{\sigma^2(X_i)}{n_M} = \sigma^2(W_i^S(n_s))$$
$$n_M = \frac{\sigma^2(X_i)}{\sigma^2(W_i^S(n_s))} \tag{22}$$

where n_M and n_s are the pool sizes in the mutual and shareholder company, respectively[10]. On the side of the mutual company, the variance of the policyholder's wealth is independent of the safety level. On the contrary, Table 1 in Section 3.3 illustrates that the safety level plays a major role in reducing the variance of the policyholder's wealth at $t = 1$ in the shareholder company.

Table 2: Case A. Comparison of premiums and equity contributions, such that policyholders are indifferent between the mutual insurer and the shareholder company when the ruin probability is set to ε.

Mutual Company				Shareholder Company				
n	$\pi_{i,M}$	$\Phi(W_i^M)$	$\sigma(W_i^M)$	n	EC_0	$\pi_{i,S}$	$\Phi(W_i^s)$	$\sigma(W_i^S)$
				$\varepsilon = 5\%$				
223	1,074.35	-575.64	45.20	10	395.60	992.64	-575.93	45.21
4,593	1,016.38	-28.90	9.96	100	115.13	998.31	-28.90	9.96
55,138	1,004.73	-3.32	2.87	1,000	35.18	999.48	-3.32	2.87
658,635	1,001.37	-1.19	0.83	10,000	11.49	999.86	-1.19	0.83
6,688,899	1,000.43	-1.02	0.26	100,000	3.58	999.95	-1.02	0.26
				$\varepsilon = 1\%$				
1,277	1,043.94	-101.35	18.89	10	634.72	998.76	-101.37	18.89
31,988	1,008.78	-5.01	3.77	100	168.66	999.73	-5.01	3.77
431,000	1,002.39	-1.30	1.03	1,000	50.51	999.92	-1.30	1.03
5,334,510	1,000.68	-1.02	0.29	10,000	16.06	999.98	-1.02	0.29
58,947,736	1,000.20	-1.00	0.09	100,000	5.04	999.99	-1.00	0.09
				$\varepsilon = 0.5\%$				
1,926	1,039.62	-67.53	15.38	10	695.81	999.23	-67.52	15.38
59,405	1,007.13	-3.16	2.77	100	185.16	999.86	-3.16	2.77
847,946	1,001.89	-1.15	0.73	1,000	55.32	999.96	-1.15	0.73
15,456,510	1,000.44	-1.01	0.17	10,000	17.93	999.99	-1.01	0.17
136,623,936	1,000.15	-1.00	0.06	100,000	5.52	1,000.00	-1.00	0.06
				$\varepsilon = 0.01\%$				
3,213,647	1,001.40	-1.04	0.38	10	1,405.77	1,000.00	-1.04	0.38
15,781,766	1,000.63	-1.01	0.17	100	279.05	1,000.00	-1.01	0.17
8,126,105,048	1,000.03	-1.00	0.01	1,000	81.63	1,000.00	-1.00	0.01
27,313,479,858	1,000.02	-1.00	0.00	10,000	23.88	1,000.00	-1.00	0.00
92,325,155,417	1,000.01	-1.00	0.00	100,000	7.58	1,000.00	-1.00	0.00

Table 2 presents the size of the adequate insurance pool and fair premium, such that policyholders would individually assess the insurance products from each of the companies equally. If fair premiums are set, then the same utility level would imply a similar variance of the policyholder's wealth under both business models. It can be seen that if the mutual company seeks to offer the same utility level as a shareholder company, significant effort should be made to capture enough participants. The number of additional participants in the pool in the mutual company would be dependent on the desired safety level. Higher safety levels would require larger pools in

[10]Note that we assumed that the expected utility is approximated by the second-order Taylor polynomial and policyholder pays fair premium in both companies, i.e., $\mu(W_i^M) = \mu(W_i^S)$

the mutual companies (i.e., $n_M >> n_S$) to reach the same variance of the policyholder's wealth in the shareholder company.

In the case of a very low ruin probability (e.g., $\varepsilon < 0.01\%$), the variance of the policyholder's wealth in the shareholder company could be reduced to (almost) zero with a finite number of participants in the pool, as well as a finite amount of equity contributions. However, in the case of the mutual company, the size of the pool would need to tend to infinity if the goal was to reach a zero variance of the policyholder's wealth. The variance of W_i^M depends only on the pool size, and hence, only reaches the level of zero as n_M grows to infinity (i.e., $\sigma^2(W_i^M) = \sigma^2(X_i)/n_M$).

4.2 Case B - *Fixed pool size* n

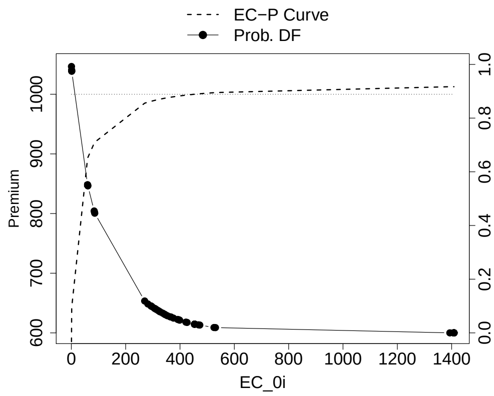
Let $n > 1$ be the finite size of the pool. In this case, the two different companies do not have the same ruin probability. In what follows, we look for the premium/equity combinations, such that policyholders are indifferent to either of the two legal forms. In formal terms, the interrelationship between the premium in a mutual company and the premium in a shareholder company is:

$$\pi^*_{i,S} = E[I_{i,S}] + ln\left(\frac{1 + \frac{a^2}{2}\sigma^2(W_i^M)}{1 + \frac{a^2}{2}\sigma^2(W_i^S)}\right) \tag{23}$$

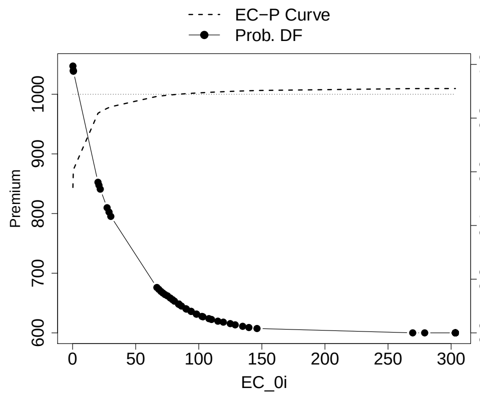
The term *"Indifference premium"* is used to refer to $\pi^*_{i,S}$. Thus, $(\pi^*_{i,S}, EC^*_{0,i})$ denotes a combination of premium and equity contributions, such that Equation (23) holds.

Figure 2 illustrates feasible combinations of premium and equity contributions per policyholder $(\pi^*_{i,S}(n), EC^*_{0,i}(n))$, that −, if adopted by the shareholder company, −, result in the policyholder being indifferent between the two organizational forms. Given the pool size, and after setting the premium, the required equity contribution $EC_0(n) = n \cdot EC_{0,i}(n)$ is entirely determined. If the insurer increases the premium, the shareholders' equity contributions must also be increased to preserve the utility of the policyholder at the same level as in the case of the mutual insurer for a given pool size n.

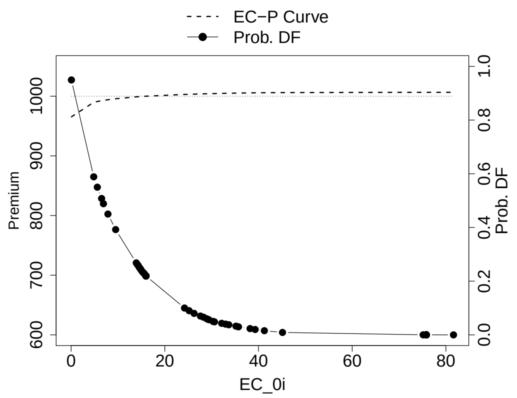
If the pool size increases, the amount of the equity contribution per policyholder decreases, even though the total amount of the equity contribution must increase. In formal terms, we have $EC_{0,i}(n) < EC_{0,i}(m)$ and $EC_0(n) > EC_0(m)$ for $n > m$ (cf. Section 3.3). Clearly, an increase in the premiums and equity contributions reduces *ceteris paribus* the ruin probability of the insurer. Hence, an upper bound for the premium and equity combinations are provided by setting the ruin probability at the lowest possible level.



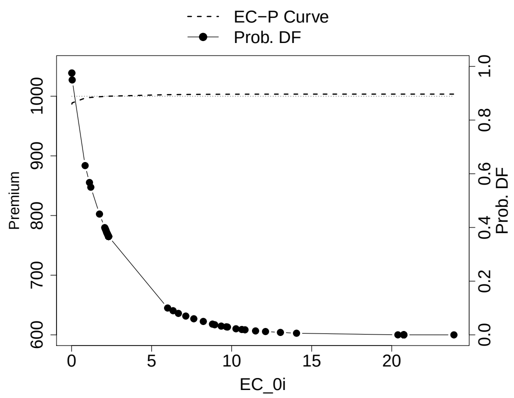
(a) $n = 10$,
$\Phi(W_i^M(n)) = -12,815.45$,
$\Phi(W_i^S(n, \pi_{i,S}, EC_0)) = -12,815.45$



(b) $n = 100$,
$\Phi(W_i^M(n)) = -1,282.44$,
$\Phi(W_i^S(n, \pi_{i,S}, EC_0)) = -1,282.44$



(c) $n = 1000$,
$\Phi(W_i^M(n)) = -129.14$,
$\Phi(W_i^S(n, \pi_{i,S}, EC_0)) = -129.14$



(d) $n = 10,000$,
$\Phi(W_i^M(n)) = -13.81$,
$\Phi(W_i^S(n, \pi_{i,S}, EC_0)) = -13.81$

Figure 2: The EC-P indifference curve and the resulting ruin probabilities for the shareholder company. Combinations of premiums and equity contributions are chosen in such a way that the policyholder is indifferent between purchasing an insurance contract with the shareholder or the mutual company.

The graphical comparison of the curves shows that larger pools constrain the shareholder company, with respect to the premium level required to reach the same policyholder's utility level, as it is offered by the mutual insurer. For instance, if the premium is set to 750 and $n = 10$, it is possible to calculate an adequate positive equity contribution per policyholder, $(EC_{0,i}(10))$, such that the policyholder perceives the same utility as if they purchase insurance from the mutual company. However, if the pool size is $n = 100$, 1,000 or 10,000, and the premium is set at 750, an equity contribution that leads to equal utility levels for both legal forms does not exist. In formal terms, we have $(850, EC_{0,i}(n)) \notin \{(\pi^*_{i,S}, EC^*_{0,i}(n)) | \Phi(W^S_i(n)) = \Phi(W^M_i(n))\}$, $n = 100$, 1,000 or 10,000 for all $EC_{0,i} \geq 0$.

The EC-P curves $(\pi^*_{i,S}, EC^*_{0,i})$ (cf. the dotted curve in Figure 2), also illustrate an increase in the sensitivity of the equity contributions and ruin probability to changes in the premium level $\pi^*_{i,S}$ whenever the pool size varies. More precisely, indifference premiums show a lower sensitivity to changes in the equity contribution for larger pools. For example, if $n = 10{,}000$, policyholders would not experience substantial variations in the premium as a consequence of the shareholders choice for any equity contribution in the interval $EC^*_{0,i} \in (0, 20.75)$.

Table 3 presents three specific combinations of premium and equity contributions in the shareholder company, such that the policyholder's utility is the same under both business models. First, the combinations $(\pi^*_{i,S}, EC^*_{0,i})$ refer to the minimum equity contribution and minimum indifference premium for five distinct pool sizes. It can be observed that the "lower bound" for the indifference premium increases if the pool size increases too. However, low insurance premiums and equity contributions lead −, even for large n − to very high ruin probabilities in the shareholder company. For the "lower bound" case, the indifference premium for large n will converge to 999 and the ruin probability to 67.58%.

The second set of combinations in Table 3 corresponds to a premium set to be the expected value of the claim (i.e., $E[X_i]$). In this case, we noticed a substantial decrease in the variance of the policyholder's wealth, compared to the "lower bound" situation. Moreover, according to our definition of a fair premium, this case illustrates a premium setting which is unfair, since we have $\pi_{i,S} = E[X_i] > E[I_{i,S}]$ whenever $\varepsilon > 0$. If fair premiums $\pi_{i,S} = E[I_{i,S}]$ are set, discrepancies between the policyholder's utility in the mutual and the shareholder company, for a given pool size and a given equity contribution, will occur. They arise from differences regarding the variance of the policyholder's wealth (cf. Equation (14) to (20)).

Finally, the last case shows an upper bound of the indifference premium with

Table 3: Case B. Premium and equity combinations in the shareholder company, such that the policyholder is indifferent between the mutual insurer and the shareholder company.

	Mutual Company					
n	$\pi_{i,M}$ ($\varepsilon = 5\%$)	$\pi_{i,M}$ ($\varepsilon < 0.01\%$)	$\Phi(W_i^M)$	$\sigma(W_i^M)$		
10	1,388.25	2,405.76	-12,815.45	213.45		
100	1,113.44	1,279.04	-1,282.45	67.50		
1,000	1,034.66	1,081.63	-129.14	21.35		
10,000	1,011.35	1,023.88	-13.81	6.75		
100,000	1,003.54	1,007.58	-2.28	2.13		
	Shareholder Company					
n	$EC_{0,i}^*$	$\pi_{i,S}^*$	$\Phi(W_i^S)$	$\sigma(W_i^S)$	$E(I_{i,S})$	ε
	Lower bound $\pi_{i,S}^*$					
10	0.60	585.32	-12,815.45	215.26	585.34	99.23%
100	0.24	843.75	-1,282.45	66.99	843.73	99.39%
1,000	0.06	965.46	-129.14	20.38	965.33	94.95%
10,000	0.04	988.86	-13.81	6.49	988.76	94.96%
100,000	0.05	999.00	-2.28	1.56	998.59	67.58%
	$\pi_{i,S}^* = E[X_i]$					
10	453.70	1,000.00	-12,815.45	35.97	995.26	3.25%
100	84.13	1,000.00	-1,282.45	15.58	996.11	10.72%
1,000	15.97	1,000.00	-129.14	7.26	997.20	21.94%
10,000	2.31	1,000.00	-13.81	3.19	998.30	36.65%
100,000	0.35	1,000.00	-2.28	1.13	999.31	43.47%
	Upper Bound $\pi_{i,S}^*$					
10	1,408.44	1,012.61	-12,815.45	0.02	1,000.00	0.01%
100	303.39	1,009.54	-1,282.45	0.04	1,000.00	0.01%
1,000	75.82	1,006.48	-129.14	0.00	1,000.00	0.01%
10,000	20.75	1,003.50	-13.81	0.00	1,000.00	0.01%
100,000	7.58	1,001.10	-2.28	-	1,000.00	0.00%

respect to the pool size n. For $n = 100$, the premium must be increased by around 9.5% (from 1,000 to 1,009.54) to reduce the ruin probability from around 10.72% to approximately zero. However, for a pool size of n =100,000, the premium should be increased by only 1.1% to have an improvement of 43.5% in the ruin probability. The particular equity contributions, according to the safety level, are analyzed in detail in Section 4.3.

4.3 Case C - Fixed pool size n and ruin probability ε

In this section, we analyze the conditions under which the policyholders are indifferent between the mutual company and the shareholder company when both business models seek to offer the same safety levels, given identical pool sizes.

Table 4: Case C. Premium and equity contribution in the shareholder company, such that the policyholder is indifferent between the mutual insurer and the shareholder company when the ruin probability ε is fixed.

	Mutual Company			Shareholder Company				
n	$\pi_{i,M}$	$\Phi(W_i^M)$	$\sigma(W_i^M)$	EC_0	$\pi_{i,S}$	$\Phi(W_i^S)$	$\sigma(W_i^S)$	$E(I_{i,S})$
				$\varepsilon = 5\%$				
10	1,388.25	-12,815.45	213.45	391.49	996.75	-12,815.45	45.21	992.62
100	1,113.44	-1,282.45	67.50	110.08	1,003.35	-1,282.45	9.96	998.30
1,000	1,034.66	-129.14	21.35	30.29	1,004.37	-129.14	2.87	999.49
10,000	1,011.35	-13.81	6.75	8.23	1,003.12	-13.81	0.83	999.86
100,000	1,003.54	-2.28	2.13	2.51	1,001.03	-2.28	0.26	999.96
				$\varepsilon = 1\%$				
10	1,633.48	-12,815.45	213.45	628.27	1,005.21	-12,815.45	18.89	998.75
100	1,168.40	-1,282.45	67.50	161.27	1,007.13	-1,282.45	3.77	999.73
1,000	1,050.44	-129.14	21.35	44.38	1,006.06	-129.14	1.03	999.93
10,000	1,016.04	-13.81	6.75	12.59	1,003.45	-13.81	0.29	999.98
100,000	1,005.03	-2.28	2.13	3.94	1,001.09	-2.28	0.09	999.99
				$\varepsilon = 0.5\%$				
10	1,695.04	-12,815.45	213.45	688.82	1,006.22	-12,815.45	15.38	999.22
100	1,185.02	-1,282.45	67.50	177.15	1,007.86	-1,282.45	2.77	999.86
1,000	1,055.28	-129.14	21.35	49.03	1,006.26	-129.14	0.73	999.96
10,000	1,017.92	-13.81	6.75	14.44	1,003.48	-13.81	0.17	999.99
100,000	1,005.51	-2.28	2.13	4.42	1,001.10	-2.28	0.06	1,000.00
				$\varepsilon = 0.01\%$				
10	2,405.76	-12,815.45	213.45	1,393.16	1,012.60	-12,815.45	0.17	1,000.00
100	1,279.04	-1,282.45	67.50	269.56	1,009.49	-1,282.45	0.38	1,000.00
1,000	1,081.63	-129.14	21.35	75.15	1,006.48	-129.14	0.01	1,000.00
10,000	1,023.88	-13.81	6.75	20.38	1,003.50	-13.81	0.00	1,000.00
100,000	1,007.58	-2.28	2.13	6.48	1,001.10	-2.28	0.00	1,000.00

Table 4 presents a comparison between the premium in the mutual company and the indifference premium $\pi_{i,S}^*$, as well as equity contributions that must be set in the shareholder company, such that both business models offer the same safety and utility levels. It is worth noting that, in the case of small pools and equal safety levels, the shareholders must pay equity contributions per policyholder that are higher than the safety loading charged to policyholders in the mutual company. However, when the size of the pool increases, the adequate equity contribution per policyholder $EC_{0,i}$ decreases for the shareholder. Thereby, $EC_{o,i}$ is smaller than the safety loading that would be charged by the mutual company. In this case, the indifference premium would be increased, resulting in $\pi_{i,S}^* > E[X_i]$. Hence, in all cases, the policyholders would pay premiums that are higher than the expected indemnity payment $E(I_{i,S})$.

The variance of the policyholder's wealth in the shareholder company turns out to be identical in the case of indifference premiums and fair premiums, whenever the safety level is fixed (cf. Table 1 in Section 3.3). Hence, the decrease in the utility levels for the policyholder in the shareholder company arises from the unfair premium, i.e, $\pi_{i,S} \neq E[I_{i,S}]$. In all cases, the policyholder should always pay lower insurance premiums in the shareholder company than in the mutual company if the same safety conditions are offered by both legal forms. This result is not surprising, taking into account that policyholders possess an owner's stake if they purchase insurance from a mutual insurance company. Indeed, expected payoffs are generally higher in the mutual than in the shareholder company, and hence, higher premiums should be charged.

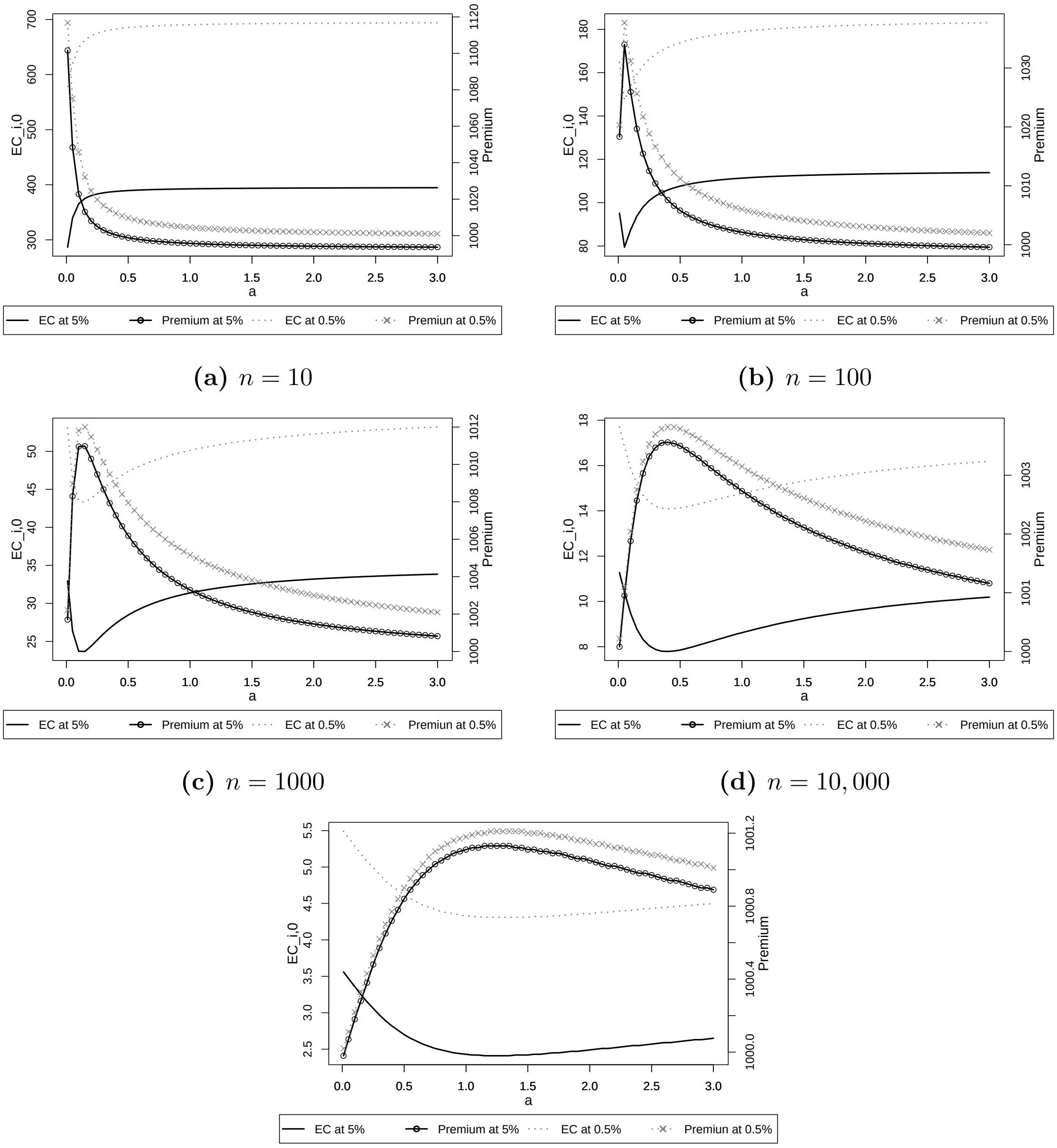
4.4 Final comments on the policyholder's risk aversion coefficient

This section studies the role of the risk aversion coefficient in determining the policyholder's willingness to pay for insurance in the shareholder company when the pool size n and ruin probability ε are fixed.

Figure 3 illustrates the indifference premium and equity contributions in the shareholder company dependent on the policyholder's risk aversion a for different safety levels and pool sizes. It can be observed that two policyholders with different risk-aversion coefficient would have distinct indifference premium. For a given safety level, it can be calculated a maximum indifference premium value $\pi^*_{i,S}(a)$ and the corresponding minimum equity contribution $EC_{i,0}$ that the insurance company would be allowed to charge such that policyholders are indifferent between the mutual and the shareholder company. Indeed, it exists a risk-aversion coefficient a' such that $\pi^*_{i,S}(a') \geq \pi^*_{i,S}(a)$ for all $a > 0$.

The result shows that if two policyholders have different risk aversion coefficient such that $a_i > a_j \geq a'$ then $\pi^*_{i,S}(a_i) < \pi^*_{i,S}(a_j)$ (cf. Appendix B for $a = 0.25, 0.75$ and 1.25). The policyholder's wealth distribution in the shareholder company is fully determined by the safety level and the pool size[11]. Hence, the policyholder with the higher risk aversion coefficient prefers to pay less when the expected indemnity payments $E[I_{i,S}]$ is given. If the insurer is unable to provide to policyholder any improvement in the risk level, e.g., lower ruin probability ε, the policyholder will be less willing to accept unfair insurance premiums. Therefore, the indifference premium $\pi^*_{i,S}$ tend to the fair premium and shareholders should increase their equity contributions to preserve the fixed safety level ε.

[11]For a fixed safety level ε and pool size n, the expected indemnity $E(I_{i,S}) = E(X_i) - E(max(S - \pi_S - EC_0)/n)$ and the variance of policyholder's wealth $\sigma(W_i^S)$ are completely determined and constant for any combination of premium π_S and equity contribution EC_0 such that $Pr[S > \pi_S + EC_0] = \varepsilon$ holds.



(a) $n = 10$ **(b)** $n = 100$ **(c)** $n = 1000$ **(d)** $n = 10,000$ **(e)** $n = 100,000$

Figure 3: Equity contribution and indifferent premium curves for the shareholder company as dependent variables of the risk aversion level for ruin probability $\epsilon = 5\%$ and 0.5%. The equity contribution and indifference curve are such that policyholder is indifferent between purchasing and insurance coverage with the shareholder or the mutual company.

5 Summary

In this paper, we analyze and compare the merits of pooling claims in mutual and shareholder insurance companies. In particular, we seek conditions of premiums, safety levels, pool sizes, and equity contributions with which a policyholder would be indifferent between being insured in a mutual or a shareholder company.

We initially studied the differences in the policyholder's wealth distribution when he/she participates in the mutual or the shareholder company under identical pool conditions. By identical conditions, we refer to fair premiums, identical pool sizes (measured by the number of participants) and equal ruin probabilities. In the case of the mutual company, the distribution of the policyholder's wealth is only determined by the pool size. Hence, neither the safety level, nor the premium charged to the policyholder, influence the policyholder's wealth distribution whenever the credit risk, on the side of the policyholders, is not taken into account. On the contrary, the policyholder's wealth in the shareholder company would not only be driven by the pool size, but also by the chosen safety and premium levels.

If fair premiums are applied and the safety level is fixed, the required equity contributions in a shareholder company are entirely defined. The comparison of the policyholder's wealth position illustrates that, in general, policyholders in the mutual company face a higher risk (measured, e.g., by the standard deviation of the wealth position) than policyholders in the shareholder company. When a shareholder company insures policyholders, the owners buffer the volatility of the claims and policyholders only sustain the default risk of the insurer.

However, out of these findings, we do not conclude that shareholder companies would bring higher utility levels to their policyholders than mutual insurers. Rather, we must assume that, in competitive markets, parameter conditions take place which lead to (approximately) the same utility levels for the policyholders, in order to explain the long-term existence of the two legal forms. Subsequently, we explore and analyze the requirements of the pool conditions under which policyholders would be indifferent between being insured by a mutual insurer or a shareholder company.

In the first scenario, we study the pool characteristics, such that policyholders face the same risk level − measured by the insurer's ruin probability − in the mutual and the shareholder company. We show that if the safety level is fixed and fair premiums are charged, the policyholder's utility level is entirely determined by the variance of the policyholder's wealth. In the case of a mutual insurer, the pool size needs to be substantially higher than in the shareholder company to lead to the same risk and utility level.

For the second case, we assumed that the pool size is fixed for both companies. For the shareholder company, we look for premium and equity contributions, such that policyholders would be indifferent to either of the companies. In this scenario, the two legal forms are generally not subject to the same ruin probability. The numerical results illustrate that indifferent premiums and equity contributions are positively related. In this sense, an increase in the insurance premium leads to a higher equity contribution, and consequently, a higher safety level for the shareholder company.

The pool size plays a major role in determining the set of suitable premium-equity combinations. On the one hand, larger pools reduce the amount of equity contributions that must be provided by the shareholders per policyholder participating in the pool. The indifference premium interval gets smaller whenever the pool size increases (i.e., the absolute difference between the upper and lower bound of the indifference premium). On the other hand, the indifference premium reaches the expected value of the policyholder's claim when n tends to infinity. For large n, changes in the equity amount per policyholder do not have a serious influence on the indifference premium, but have a large impact on the insurer's safety level.

Founded on the results of the second case, we analyze a third scenario based on the conditions under which the policyholders are indifferent between a mutual and a shareholder company. These two legal forms offer the same safety level within a given pool size. In this case, the indifference premium is higher than the expected payments to the policyholder and, in general, exceed the expected claim. The policyholder is therefore willing to accept an unfair premium in the shareholder driven company in order to obtain the same utility and safety level as is provided by the mutual company.

The last section examines the impact of policyholder's risk aversion coefficient in the indifference premium fixed. More risk-averse policyholders are less willing to accept unfair contracts whenever the insurer does not provide any improvement in the risk from the policyholder perspective.

Finally, our analysis focuses on the risk premium charged to policyholders. Future studies in the context of "merits of pooling claims" may also consider the fact that these two organizational forms may lead to differences in management expenses and agency costs.

Appendix

A. Taylor Approximation for the Expected Utility

Following Jondeau & Rockinger (2006), the infinite-order Taylor series expansion of the utility function can be written as:

$$U(W) = \sum_{k=0}^{\infty} \frac{U^{(k)}(\overline{W})(W - \overline{W})^k}{k!}$$

where $\overline{W} = E[W]$. The expected utility is given by:

$$E[U(W)] = E\left[\sum_{k=0}^{\infty} \frac{U^{(k)}(\overline{W})(W - \overline{W})^k}{k!}\right] = \sum_{k=0}^{\infty} \frac{U^{(k)}(\overline{W})}{k!} E[(W - \overline{W})^k]$$

In particular, if $U(W)$ is defined by:

$$U(W) = -exp(-aW)$$

where a measures the investor's risk aversion, then the approximation for the expected utility is given by

$$E[U(W)] = -exp(-a\overline{W}) \left(1 + \sum_{k=2}^{\infty} \frac{(-1)^{(k)} a^k}{k!} E[(W - \overline{W})^k]\right)$$

Tsiang (1972) concludes that the exponential utility can be approximated by the second-degree Taylor Series.

B. Detail values for different risk aversion coefficients

Table 5: Indifference premium and equity contributions such that the policyholder is indifferent between the mutual and the shareholder company for different risk aversion coefficients.

	Mutual[a,b]		$a = 0.25$					$a = 0.75$					$a = 1.25$				
ϵ	$\pi_{i,M}$	$\sigma(W_i^M)$	EC_0	$\pi_{i,S}$	$\Phi(W_i^S)$	$\sigma(W_i^S)$	$E(I_{i,S})$	EC_0	$\pi_{i,S}$	$\Phi(W_i^S)$	$\sigma(W_i^S)$	$E(I_{i,S})$	EC_0	$\pi_{i,S}$	$\Phi(W_i^S)$	$\sigma(W_i^S)$	$E(I_{i,S})$
								$n = 10$									
10.00%	1,281.86	213.45	287.34	994.52	-1,424.83	64.51	984.97	293.69	988.16	-12,815.45	64.51	984.97	294.97	986.89	-35,596.70	64.51	984.97
5.00%	1,388.25	213.45	383.27	1,004.97	-1,424.83	45.21	992.62	391.49	996.75	-12,815.45	45.21	992.62	393.15	995.10	-35,596.70	45.21	992.62
1.00%	1,633.48	213.45	615.67	1,017.81	-1,424.83	18.89	998.75	628.27	1,005.21	-12,815.45	18.89	998.75	630.85	1,002.63	-35,596.70	18.89	998.75
0.50%	1,695.04	213.45	675.28	1,019.76	-1,424.83	15.38	999.22	688.82	1,006.22	-12,815.45	15.38	999.22	691.61	1,003.43	-35,596.70	15.38	999.22
0.10%	1,897.90	213.45	873.58	1,024.32	-1,424.83	8.21	999.74	893.78	1,008.46	-12,815.45	8.11	999.83	895.49	1,005.02	-35,596.70	8.15	999.81
0.01%	1,530.96	213.45	1,390.48	1,029.05	-1,424.83	0.03	1,000.00	1,406.81	1,012.61	-12,815.45	0.03	1,000.00	1,410.65	1,008.38	-35,596.70	0.04	1,000.00
								$n = 100$									
10.00%	1,086.60	67.50	78.73	1,007.88	-143.38	15.04	996.36	86.25	1,000.35	-1,282.45	15.04	996.36	87.84	998.76	-3,560.57	15.04	996.36
5.00%	1,113.44	67.50	100.92	1,012.52	-143.38	9.96	998.30	110.08	1,003.35	-1,282.45	9.96	998.30	112.09	1,001.35	-3,560.57	9.96	998.30
1.00%	1,168.40	67.50	150.27	1,018.12	-143.38	3.77	999.73	161.27	1,007.13	-1,282.45	3.77	999.73	164.12	1,004.28	-3,560.57	3.77	999.73
0.50%	1,185.02	67.50	166.16	1,018.86	-143.38	2.77	999.86	177.15	1,007.86	-1,282.45	2.77	999.86	180.18	1,004.84	-3,560.57	2.77	999.86
0.10%	1,234.37	67.50	214.65	1,019.72	-143.38	1.01	999.98	225.18	1,009.19	-1,282.45	1.01	999.98	228.31	1,006.05	-3,560.57	1.01	999.98
0.01%	1,135.96	67.50	288.72	1,019.86	-143.38	0.08	1,000.00	298.10	1,009.53	-1,282.45	0.09	999.99	300.83	1,006.53	-3,560.57	0.09	999.99
								$n = 1,000$									
10.00%	1,027.19	21.35	19.19	1,008.00	-15.24	4.32	998.94	24.21	1,002.98	-129.14	4.32	998.94	25.74	1,001.45	-356.96	4.32	998.94
5.00%	1,034.66	21.35	25.19	1,009.47	-15.24	2.87	999.49	30.29	1,004.37	-129.14	2.87	999.49	32.08	1,002.59	-356.96	2.87	999.49
1.00%	1,050.44	21.35	39.74	1,010.69	-15.24	1.03	999.93	44.38	1,006.06	-129.14	1.03	999.93	46.29	1,004.15	-356.96	1.03	999.93
0.50%	1,055.28	21.35	44.49	1,010.79	-15.24	0.73	999.96	49.03	1,006.26	-129.14	0.73	999.96	50.90	1,004.38	-356.96	0.73	999.96
0.10%	1,066.70	21.35	55.82	1,010.88	-15.24	0.30	999.99	60.26	1,006.44	-129.14	0.30	999.99	62.06	1,004.64	-356.96	0.30	999.99
0.01%	1,035.57	21.35	71.34	1,010.90	-15.24	0.00	1,000.00	75.75	1,006.48	-129.14	0.00	1,000.00	77.53	1,004.70	-356.96	0.00	1,000.00

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Table 5 – continued from previous page

	Mutual[a,b]			$a = 0.25$					$a = 0.75$					$a = 1.25$			
ϵ	$\pi_{i,M}$	$\sigma(W_i^M)$	EC_0	$\pi_{i,S}$	$\Phi(W_i^S)$	$\sigma(W_i^S)$	$E(I_{i,S})$	EC_0	$\pi_{i,S}$	$\Phi(W_i^S)$	$\sigma(W_i^S)$	$E(I_{i,S})$	EC_0	$\pi_{i,S}$	$\Phi(W_i^S)$	$\sigma(W_i^S)$	$E(I_{i,S})$
								$n = 10,000$									
10.00%	1,008.60	6.75	5.62	1,002.98	-2.42	1.36	999.66	6.00	1,002.60	-13.81	1.36	999.66	6.78	1,001.82	-36.60	1.36	999.66
5.00%	1,011.35	6.75	8.04	1,003.32	-2.42	0.83	999.86	8.23	1,003.12	-13.81	0.83	999.86	8.96	1,002.39	-36.60	0.83	999.86
1.00%	1,016.04	6.75	12.53	1,003.51	-2.42	0.29	999.98	12.59	1,003.45	-13.81	0.29	999.98	13.23	1,002.81	-36.60	0.29	999.98
0.50%	1,017.92	6.75	14.39	1,003.53	-2.42	0.17	999.99	14.44	1,003.48	-13.81	0.17	999.99	15.07	1,002.85	-36.60	0.17	999.99
0.10%	1,020.59	6.75	17.05	1,003.54	-2.42	0.07	1,000.00	17.09	1,003.50	-13.81	0.07	1,000.00	17.71	1,002.87	-36.60	0.07	1,000.00
0.01%	1,011.79	6.75	20.67	1,003.54	-2.42	0.00	1,000.00	20.70	1,003.50	-13.81	0.00	1,000.00	21.33	1,002.88	-36.60	0.00	1,000.00
								$n = 100,000$									
10.00%	1,002.76	2.13	2.35	1,000.41	-1.14	0.41	999.90	1.82	1,000.94	-2.28	0.41	999.90	1.74	1,001.01	-4.56	0.41	999.90
5.00%	1,003.54	2.13	3.06	1,000.48	-1.14	0.26	999.96	2.51	1,001.03	-2.28	0.26	999.96	2.41	1,001.13	-4.56	0.26	999.96
1.00%	1,005.03	2.13	4.51	1,000.52	-1.14	0.09	999.99	3.94	1,001.09	-2.28	0.09	999.99	3.83	1,001.20	-4.56	0.09	999.99
0.50%	1,005.51	2.13	4.99	1,000.53	-1.14	0.06	1,000.00	4.42	1,001.10	-2.28	0.06	1,000.00	4.31	1,001.21	-4.56	0.06	1,000.00
0.10%	1,006.56	2.13	6.03	1,000.53	-1.14	0.02	1,000.00	5.46	1,001.10	-2.28	0.02	1,000.00	5.34	1,001.21	-4.56	0.02	1,000.00
0.01%	1,003.43	2.13	7.23	1,000.53	-1.14	0.00	1,000.00	6.65	1,001.10	-2.28	0.00	1,000.00	6.55	1,001.21	-4.56	0.00	1,000.00

[a] Recall $\Phi(W_i^M)$ is independent of the premium level but only affected by the variance of policyholder's final wealth in the mutual company.

[b] $\Phi(W_i^M) = \Phi(W_i^S)$ since the premiums presented for the shareholder company are indifference premiums.

References

Albrecht, P. (1984). Ausgleich im Kollektiv und Prämienprinzipien. *Zeitschrift für die gesamte Versicherungswissenschaft*, *73*(1/2), 167–180.

Albrecht, P., & Huggenberger, M. (2017). The Fundamental Theorem of Mutual Insurance. *Insurance: Mathematics and Economics*, *75*, 180–188.

Braun, A., Schmeiser, H., & Rymaszewski, P. (2015). Stock vs. Mutual Insurers: Who Should and Who Does Charge More? *European Journal of Operational Research*, *242*(3), 875–889.

Cummins, I. D. (1991). Statistical and Financial Models of Insurance Pricing and the Insurance Firm. *Journal of Risk and Insurance*, *58*(2), 261–302.

Gatzert, N., & Schmeiser, H. (2012). The Merits of Pooling Revisited. *Journal of Risk Finance*, *13*(3), 184–198.

Jondeau, E., & Rockinger, M. (2006). Optimal Portfolio Allocation under Higher Moments. *European Financial Management*, *12*(1), 29–55.

Powers, M. R. (2006). An Insurance Paradox. *The Journal of Risk Finance*, *7*(2), 113–116.

Smith, M. L., & Kane, S. A. (1994). The Law of Large Numbers and the Strength of Insurance. In *Insurance, Risk Management, and Public Policy*, (pp. 1–27). Springer Netherlands.

Tsiang, S. C. (1972). The Rationale of the Mean-Standard Deviation Analysis, Skewness Preference and the Demand for Money. *The American Economic Review*, *62*(3), 354–371.