Valuation of Participating Life Insurance Portfolios Under Stochastic Interest Rate Assumptions: Is Fair Pricing Possible?

Abstract

Previous research has attempted to answer the question about the existence of fair prices for the portfolios of insurance policies when the default risk is taken into account. However, the complexity of the interactions among generations leads, in some cases, to oversimplifying or neglecting the key portfolio dynamics. These dynamics directly affect the policyholders and shareholders risk positions. We focus on voluntary savings schemes in which participants are risk-averse and make decisions based on a ? ? risk preference. This study aims to assess the impact of additional informational assumptions on fair pricing and the willingness to pay for insurance (WTP) of new generations that join an insurance portfolio. The insurance portfolio is modelled under various scenarios of stochastic interest rates. We search for adequate risk management strategies: investment strategies, equity contributions and minimum interest rate guarantees. In this way, all generations joining the portfolio will pay fair premiums under the assumptions that reflect the real market dynamics.

Keywords: Participating life insurance, portfolio of policies, fair pricing

1 INTRODUCTION

Pension plans and insurance contracts with minimum guarantees, policyholder participation, and several embedded options are popular insurance products used to establish old age provisions in most European countries. In general, participants in a pension system expect to pay fair contributions, meaning they pay adequate contributions during the accumulation period and receive the corresponding appropriate benefit during retirement. The questions surrounding fairly priced products have appealed to regulators and policyholders concerns. Starting with the seminal paper by Grosen & Jørgensen (2002), the pricing of individual customers has been widely researched. There is a clear idea about how (in financial terms) fair conditions for policyholders can be obtained, both with and without default risk.

However, pension funds and insurance companies typically pool their insurance policies in portfolios and have many contracts with different inception and maturity dates. The study of insurance premiums, in the context of insurance portfolios, has been limited. Despite this, policyholders need to be seen and treated as part of an insurance portfolio which may have redistribution issues among various generations. This is also referred to as inter-generational subsidies. These inter-generational subsidies are primarily due to the uncertainty in the long-term returns and demographic changes.

Orozco-Garcia & Schmeiser (2017) present insight into the possibility of charging fair prices for insurance contracts when multiple generations are pooled. The authors investigate whether an insurer is likely to charge fair premiums, and simultaneously, ensure identical levels of default risk to all generations joining the portfolio with homogeneous contracts and under mandatory pension schemes. They conclude that by adopting precise investment strategies and equity contributions, it is possible to simultaneously avoid intergenerational subsidies and provide the same default risk, from an ex-ante perspective. Nevertheless, in general, it is not possible to integrate fair prices and identical default risk levels for all generations joining the portfolio of insurance policies, even under naive assumptions.

The simplicity of the main hypotheses (e.g., deterministic riskless returns), may underestimate or neglect portfolio dynamics that usually constrain the managers of the insurance company. Hieber et al. (2016) incorporated stochastic interest rates in the context of insurance portfolios. The authors illustrate the existence of parameter combinations (e.g., equity and minimum rate guaranteed), which allows individual insurance contracts with a guaranteed annual return (cliquet-style) to be incorporated into an existing portfolio of insurance policies with no wealth transfer between the two groups. Even though it can seemingly be proven that a fair price for the new contract exists, the specific parameter combination can only be accessed numerically, when the portfolio is composed of more than one generation. Their analysis does not extend to possible cross-subsidizations between sequences of generations. In addition, the authors concentrate on single premiums, rather than periodical premiums, which replicate a more realistic cash-flow of premiums in a pension scheme.

In mandatory schemes, portfolio participants may be unable to take any measures to avoid intergenerational subsidies (Orozco-Garcia & Schmeiser, 2017). In contrast, in non-compulsory schemes, policyholders would have the choice of joining or not joining the insurance portfolio. Indeed, policyholders exhibit different levels of risk aversion and propensities to plan and save. More specifically, younger generations joining the insurance portfolio possess more information than older generations, and hence, might evaluate the insurance coverage differently. Thus, insurers should take into account the policyholder perspective when designing the insurance features and fix the prices.

The dynamics of interest rates are essential in determining the price of insurance contracts, as well as adequate investment strategies, for the insurer to meet customer obligations. On the one hand, current down-trend interest rates have put insurers under pressure. They seem to have difficulties fulfilling prior promises. Furthermore, constraints, such as the maximum probability of default or minimum share in riskless investments, may require the implementation of sophisticated risk management strategies to cope with the dynamics of the day-to-day operations of real insurance companies. Under these circumstances, policyholders need to invest additional effort when making decisions about old-age provisions.

Under the assumption of a competitive insurance market with stochastic interest rates and a non-compulsory pension system (similar to current voluntary pension plans), customers and insurers make decisions under volatile interest rate environments. As a result, insurers must adapt product prices to reflect prevalent economic scenarios and meet policyholders expectations. New policyholders must also make decisions by taking into account the up-to-date financial market information. Hence, new generations may value the available insurance opportunities differently than previous generations of participants in the insurance portfolio.

The primary aim of this paper is to investigate the impact of updated financial assumptions on the fair pricing of a portfolio of voluntary insurance policies. We analyze: (a) to what extent the fair pricing of a new participating generation, as well as the entire portfolio of insurance, is viable, and (b) the policyholder's willingness to join the insurance portfolio, assuming that the new generation consists of risk-averse policyholders. In the context of stochastic interest rates, insurers require dynamic risk management strategies to achieve fairness on the proposed conditions to the generations that are expected to participate in the insurance portfolio. We compare the new generation's willingness to join the insurance portfolio when fair conditions are offered versus unfair individual premiums, but fair in the portfolio sense. Particular focus will be placed on the current financial environment of low, or even negative, interest rates.

We recursively simulate a portfolio of insurance policies under a non-compulsory scheme. The policyholders could join the insurance portfolio at different points in time and have different maturity dates. The policyholders pay annual premiums (similar to the annual contributions that a policyholder may pay during his/her working lifetime). The annual premiums are continuously collected by the insurance company at the beginning of each year. In the first stage, we investigate the impact of changing informational assumptions on the pricing of an insurance portfolio consisting of two generations with different issuance dates. The results illustrate that fair insurance portfolios could only exist if the insurer adopts dynamic risk management measures. Changes in the interest rate environment would affect the adequate risk management measures and the policyholder's willingness to participate in the insurance portfolio. Policyholders willingness to pay would be driven by the expected returns, which depends on the financial market conditions and the future expectations.

Finally, the analysis concentrates on adequate risk management strategies to be adopted for the case $(n+1)$ generations in the insurance portfolio. The increase in the portfolio size, together with the stochastic interest rates, complicates the task of determining the adequate investment strategy and contract parameters to offer fair contracts to the new generations that are expected to be part of the insurance portfolio. However, it is possible to find strategies (e.g., intergenerational subsidies) that are negligible for each generation. We compare the changes in the management strategies and the new policyholder's willingness to participate in insurance when the same policy conditions are offered to all participants, versus a portfolio that intends to provide fair conditions between the outstanding portfolio and the joining generation. We find that the policyholder that values the portfolio, based on the returns, would prefer the insurance portfolio with higher returns, disregarding the wealth transfer when no extreme drop in the interest rate is expected.

The remainder of this paper is structured as follows. Section 2 introduces the asset and liability models. Section 2.4 describes the indirect method used for the measurement of the policyholders willingness to participate in the portfolio. Section 3 presents the numerical implementation of the pricing framework for the case of an insurance portfolio that is assembled recursively. Section 3.3 presents the analysis of a portfolio with one and two generations and guides the readers, with respect to the interpretations of the concepts presented in this paper, applied to simple portfolio composition. Section 3.4 presents the recursive development of an insurance portfolio when more than two generations participate in the insurance portfolio. Finally, Section 4 summarizes the findings of the paper.

2 MODEL DESCRIPTION

Our research focuses on participating endowment contracts , similar to those offered by insurance companies in many European countries. Our analysis concentrates on the savings component and does not consider any premiums or benefits related to the term life insurance coverage or the associated transaction costs. In addition, we assume that premiums are paid annually. The surrender option will not be used by the policyholder. We consider an insurance company with two types of stakeholders: policyholders and shareholders.

The following framework for a portfolio of policies is an extension of the accounting model described by Orozco-Garcia & Schmeiser (2017). We assume that the outstanding portfolio of insurance contracts consists of m generations that joined the insurance portfolio at some time prior to $t = 0$ and have a maturity date $\mathcal{T}_i > 0$ for $i = 1, ..., m$. Let $\Omega_0 = \{1, 2..., m\}$ be the subset of indices of the policies in force at $t = 0$ and $AV_{i,0}$ be the corresponding account values. The current value of assets A_0 correspond to the current equity value EC_0, plus the value of the accrued liability V_0 (i.e., $A_0 = EC_0 + V_0$) given by

$$V_0 = \sum_{i \in \Omega_0} AV_{i,0}$$

Let $\tau_1, \tau_2, ..., \tau_m \leq \tau_{m+1}$ denote the sequence of times at which the insurance company issued the insurance contracts. Consequently, we assumed that one new generation may join the insurance portfolio at $\tau_{m+1} = 0$. The policyholders sign a contract with a duration of $T_1, T_2, ..., T_{m+1}$ years and pay annual premiums $P_1, P_2, ..., P_{m+1}$. Let $\mathcal{T}_{max}$ denote the last maturity payment or portfolio expiration (i.e., $\mathcal{T}_{max} = max\{\mathcal{T}_j = \tau_j + T_j, j = 1, 2, ..., m+1\}$) where $\mathcal{T}_j = \tau_j + T_j$ correspond to the time to maturity of generation j. Each policyholder receives a minimum annual interest rate guarantee, g_j. The policyholders participate in the insurer's investment returns that exceed the guaranteed interest rate at rate α.The shareholders pay equity contributions, EC_{m+1}, at the issuance date of the insurance contract. The capital contributions represents in our model, any risk management measure (e.g., reinsurance, hedging) that the insurance company could adapt to control the company's default put option value. We allow the equity capital to be adjusted at discrete points in time. New premium payments and equity contributions are deposited into the shared asset account, A_t, and invested in a preferred investment strategy for the entire portfolio of insurance policies.

2.1 *The Financial Market*

Our model assumes a complete, perfect, frictionless and competitive market. Let $\mathbf{W(t)} = [\mathbf{W(t)}, \mathbf{W_r(t)}]^T$, $t > 0$,denote a standard two-dimensional Wiener process defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathbb{P}$ denotes the real-world probability measure. The two-dimensional Wiener process satisfies $\mathrm{d}W_S(t)\mathrm{d}W_r(t) = \rho \mathrm{d}t$ with a constant correlation coefficient $\rho \in [-1, 1]$. We assume that the dynamics of the short-term rate are given by:

$$\mathrm{d}r(t) = \kappa(\theta - r(t))\mathrm{d}t - \sigma_r \sqrt{k_1 r(t) + k_2}\mathrm{d}W_r(t) \tag{1}$$

where κ, θ and σ_r are positive real constants. Note that the dynamics focus on two models for the financial market: the Vasicek (resp. Cox-Ingersoll-Ross CIR) dynamics when k_1 (resp. k_2) is equal to zero. Equation 1 produces short rates, which are always positive for $k_1 = 1$ if the parameters fulfil the condition $\kappa\theta > \sigma_r^2/2$. In addition, assuming the absence of arbitrage and a market price $\lambda(t, r)$ of the interest rate risk of the special form $\lambda(t, r) = \lambda_r \sqrt{k_1 r(t) + k_2}$ with a parameter $\lambda_r \in \mathbb{R}$, the short interest rate under the risk neutral measure $\mathbb{Q}$ follows the same process as in Equation 1, but with the parameters $\hat{\kappa} = \kappa - \sigma_r \lambda_r k_1$ and $\hat{\theta} = \frac{\kappa\theta - \sigma_r \lambda_r k_2}{\hat{\kappa}}$.

We assumed that the market is composed of three financial asset categories. The first category is an instantaneously riskless money market asset (cash) $M(t)$, whose price is evolving according to:

$$\frac{\mathrm{d}M(t)}{M(t)} = r(t)\mathrm{d}t, \quad M(0) = 1 \tag{2}$$

The second category is a variable return asset (i.e., a stock or a basket of stocks), whose price is denoted by $S(t)$. The dynamics of $S(t)$ are:

$$\frac{\mathrm{d}S(t)}{S(t)} = r(t)\mathrm{d}t + \sigma_1 \left(\mathrm{d}W_S(t) + \lambda_S \mathrm{d}t\right) + \sigma_2 \sqrt{k_1 r(t) + k_2} \left(\mathrm{d}W_r(t) + \lambda_r \sqrt{k_1 r(t) + k_2}\mathrm{d}t\right) \tag{3}$$

where the stock volatility equals $\sigma_S^2 \equiv \left(\sigma_1^2 + \sigma_2^2(k_1 r_0 + k_2)\right) \geq 0$, $\rho_{S,r} \equiv \sigma_2 \sigma_S^{-1} \sqrt{k_1 r_0 + k_2}$ and λ_S denotes the constant market price of the risk of the stock.

The third asset category includes fixed interest assets (i.e., bonds). The price dynamics of a zero coupon bond, with a maturity in T years, $b(t,T)$, under the objective probability measure $\mathbb{P}$, are:

$$\frac{\mathrm{d}b(t,T)}{b(t,T)} = r(t)\mathrm{d}t + \sigma_T(T-t) \left(\mathrm{d}W_r(t) + \lambda_r \sqrt{k_1 r(t) + k_2}\mathrm{d}t\right) \tag{4}$$

with

$$\sigma_T(T-t) = h(T-t)\sigma_r \sqrt{k_1 r(t) + k_2}$$

and the terminal condition $b(T,T) = 1$. When $k_1 = 1$:

$$h(s) = \frac{2\left(e^{ms} - 1\right)}{m - (\kappa - \sigma_r \lambda_r) + e^{ms}(m + \kappa - \sigma_r \lambda_r)}$$

$$m = \sqrt{(\kappa - \sigma_r \lambda_r)^2 + 2\sigma_r^2}$$

and for $k_2 = 1$ we have:

$$h(s) = \frac{1 - e^{-\kappa(T-t)}}{\kappa}$$

2.2 Management Model

Asset Allocation

The value of the asset account, A_t, is affected by the in-flow of the periodical premiums paid by the policyholders, equity contributions, equity withdrawals, maturity payments, and gains or losses from the investments. The company holds a fraction γ_S of its invested assets in a stock index and a fraction γ_B in bonds,while the remaining capital is deposited in an instantaneously riskless money market fund (e.g., bank account) (BajeuxBesnainou et al., 2003; Gao, 2008; Gerstner et al., 2008; Zaglauer & Bauer, 2008). The bonds fraction γ_B consists of a portfolio of zero coupon bonds with different maturities that equal the policies maturities and the expiration date of the portfolio (i.e., represents the risk-free asset). The number of stock and j-bond units (i.e., bond with maturity data $\mathcal{T}_j$ held at t) are denoted by $\eta_{\mathbf{S}}(t)$ and $\eta_{\mathbf{B}}(j,t)$, respectively.We assume that there are no transaction costs, and thereby, the insurer rebalances its assets at the beginning of each period. The value of the asset account, at the end of period $t$before any benefit (out-flow) or premium (in-flow) payment, is:

$$A_t^+ = [1-\gamma_S-\gamma_B]A_t^- exp\left(\int_{t-1}^{t} r_s ds\right)+\eta_{\mathbf{S}}(t-1)S(t)+\sum_{j\in\Omega_{t-1}} \eta_{\mathbf{B}}(j,t-1)b(t,\mathcal{T}_j)+\eta_{\mathbf{B}}(max,t-1)b(t,\mathcal{T}_{max}) \quad (5)$$

where Ω_{t-1} corresponds to the subset of indices of the policies in force at the end of period t. Hence, the value of the asset account at the beginning of the period $t+1$ (A_{t+1}^-) is defined as:

$$A_{t+1}^- = A_t^+ - L_{k,t}\mathbf{1}_{t=\mathcal{T}_k} + \sum_{j\in\Omega_t} P_j\mathbf{1}_{t<\mathcal{T}_j} + EC_{m+1}\mathbf{1}_{t=\tau_{m+1}} \quad (6)$$

where $\Omega_t = \Omega_{t-1} \cup \{m+1|t=\tau_{m+1}\} \setminus \{k|t=\mathcal{T}_k\}$ is the updated subset of indices of the policies in force at the beginning of period $t+1$, P_j is the premium paid by the i-th generation and $L_{k,t}$ is the maturity payment to the k-th generation. The re-allocation of assets have the following dynamics. The new number of units of stocks held is:

$$\eta_{\mathbf{S}}(t) = \frac{A_{t+1}^- \cdot \gamma_S}{S(t)} \quad (7)$$

The number reallocated of units $\eta_B(j,t)$ of the in-force bonds (i.e., $j \in \Omega_{t-1}\setminus\{k|t=\tau_k+T_k\}$), corresponds to:

$$\eta_{\mathbf{B}}(j,t) = \frac{\left[A_t^+ - max\{L_{k,t}[1-\gamma_B]+\eta_B(k,t-1)b(t,\mathcal{T}_k), L_{k,t}\}\mathbf{1}_{t=\mathcal{T}_k}\right]\cdot\gamma_B\cdot w_{j,t}}{b(t,\mathcal{T}_j)} \quad (8)$$

where

$$w_{j,t} = \frac{\eta_B(j,t-1)b(t,\mathcal{T}_j)}{\sum_{j\in\Omega_{t-1}\setminus\{k|t=\tau_k+T_k\}}\eta_B(j,t)b(t,\mathcal{T}_j)} \quad (9)$$

The units of the new bonds are:

$$\eta_{\mathbf{B}}(m+1,t) = \frac{(P_{m+1}+EC_{m+1})\mathbf{1}_{t=\tau_{m+1}}\cdot\gamma_B}{b(t,\mathcal{T}_{m+1})} \quad (10)$$

Finally, the available assets from bond maturities are reinvested also in bonds. The number of units reinvested in bonds (e.i., bond with maturity date $\mathcal{T}_{max}$)

$$\eta_{\mathbf{B}}(max,t) = \frac{A_{t+1}^-\cdot\gamma_B - \sum_{j\in\Omega_t}\eta_{\mathbf{B}}(j,t)b(t,\mathcal{T}_j)}{b(t,\mathcal{T}_{max})} \quad (11)$$

Liability model

The j-th generation's account $AV_{j,t}$ is established when the first periodical premium, P_j, is paid at $t=\tau_j$. In subsequent years while the contract is in force (i.e., $t=\tau_j+1,...,\mathcal{T}_j$), the j-th generation's account $AV_{j,t}$ earns an annual interest rate $R_{j,t}$ and accumulates the periodical premium collected. The interest rate earned by the policyholders is the greater value between the guaranteed interest rate, g_j, and the fraction α of the annual return on the assets account, R_P. The annual participation, in turn, becomes part of the guarantee after being credited to the policyholder's account (cliquet-style guarantee). Thus, the value of the

j-th generation's account, $AV_{j,t}$, is:

$$AV_{j,t} = \begin{cases} AV_{j,t-1} \cdot max\left(g_j, \alpha \cdot R_{P,t}\right) + P_j & \text{if } \tau_j < t \leq \mathcal{T}_j \\ 0 & \text{elsewhere} \end{cases} \tag{12}$$

where $R_{P,t} = \left[A_t^+/A_t^- - 1\right]$ and $AV_{j,\tau_j} = P_j$ is the initial value of the policyholder's account that equals the initial premium paid by the new j-th generation at the issuance date τ_j.

The accumulated insurer's liability, V_t at time t, is:

$$V_t = \sum_{j \in \Omega_t} AV_{j,t}$$

We assume the insurer's solvency position is monitored annually. The insurer is permitted to continue its activity (i.e., to issue new contracts and make payments at maturity dates), as long as the total assets value at the end of period t, A_t^+, is greater than the accumulated liability, V_t. In formal terms:

$$A_t^+ > V_t = \sum_{j \in \Omega_t} AV_{j,t} \tag{13}$$

If the available assets, A_t^+, at the end of the period, t, do not satisfy the condition in (13), the insurance company declares bankruptcy, the insurer is not allowed to issue new contracts, and all policyholders must be paid out (shareholders do not receive any payments in this case). The bankruptcy corresponds to the application of simple accounting rules in which assets value are smaller than liabilities values. The model could also include solvency regulations, in which the insurance company must declare bankruptcy earlier (e.g., if the assets are smaller than a multiple $\theta$of the liabilities $A_t^+ < \theta V_t$ for some $\theta \geq 1$, Orozco-Garcia & Schmeiser (2017)). The time to default is defined as:

$$\xi = \begin{cases} inf\left\{t \in \{1, ..., \mathcal{T}_{max}\} | V_t > A_t^+\right\} & \text{if } \exists t \leq \mathcal{T}_{max} \quad V_t > A_t^+ \\ \infty & \text{if } \forall t \leq \mathcal{T}_{max} \quad V_t \leq A_t^+ \end{cases} \tag{14}$$

with V_t and A_t^+ calculated under the empirical measure $\mathbb{P}$. The cost of insurer insolvency, D_t, corresponds to the difference between the accumulated liabilities and the available assets at A_t^+ (Doherty & Garven, 1986; Butsic, 1994). This cost is known as the default put option, defined by:

$$D_t = \left(V_t - A_t^+\right)^+$$

where $(\cdot)^+$ stands for $max(\cdot, 0)$. Given the limited liability of shareholders, the cost of the insurer's insolvency will be borne by the in-force generations (i.e., those policyholders whose contracts remain in force at the moment bankruptcy is declared). By considering the insolvency risk, the policyholders would experience a reduction in their individual account value at the moment of bankruptcy. The proportional cost of the insurer insolvency, $D_{j,t}$, undertaken by the j-th generation is:

$$D_{j,t} = \left(AV_{j,t} - \beta_{j,t} \cdot A_t^+\right)^+ \quad \text{for} \quad j \in \Omega_t \tag{15}$$

and:

$$\beta_{j,t} = \frac{P_{j,t}}{\sum_{j \in \Omega_t} P_{j,t}} \quad \text{for} \quad j \in \Omega_t \tag{16}$$

where $\beta_{j,t}$ is the share of the insurer's accumulated liability belonging to the i-th generation, whose contract remains in force at t. This share also corresponds to the proportion of assets A_t^+ that the j-th generation may claim in the case of bankruptcy (Ibragimov et al., 2010). The "conditional" pay-off of the j-th generation, $L_{j,t}$ when the company has operated for at least $t > 0$ years, equals the account value at time $AV_{j,t}$ minus the proportional default cost, $D_{j,t}$, charged to the i-th generation in the case of bankruptcy. In summary, $L_{j,t}$ is:

$$L_{j,t}|\xi > \tau_j = \begin{cases} AV_{j,t} - D_{j,t} & \text{if } V_t \geq A_t^+ \text{ and } \tau_j < t < \mathcal{T}_j \\ AV_{j,t} & \text{if } V_t < A_t^+ \text{ and } t = \mathcal{T}_j \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

where the cost of default, $D_{j,t}$, assumed by the j-th generation in force at time t, is also defined by Equation (15). Finally, the shareholders stake, E_t, at time t corresponds to the remaining assets after the accumulated liabilities have been subtracted. Consequently, E_t is:

$$E_t = (A_t^+ - V_t)^+ \tag{18}$$

2.3 *Shareholders' valuation technique*

The assumption of a competitive market signifies that *fair prices*for the contracts would be settled. Fair prices imply that, under the shareholders point of view, both policyholders and shareholders receive risk-adequate returns on their premiums and equity contributions. Let $\Pi_{j,t}^P$denote the contract market value of the j-th generation at $t \geq 0$.This corresponds to the sum of the conditional expected values for the customer payoff, assuming the insurance company does not go bankrupt before issuing the j-th contract. $L_{j,h}$ is defined in Equation (17):

$$\Pi_{j,t}^P = \sum_{h=t+1}^{\mathcal{T}_j} E^{\mathbb{Q}} \left[exp\left(-\int_t^h r(s)ds \right) L_{j,h}|\xi > \tau_j \right] \qquad \forall j = 1, ..., m+1 \tag{19}$$

An insurance contract is fair if the present value of the future pay-off is equal to the present value of the premiums paid by the policyholder. The conditional present value of the insurance premiums paid by the j-th generation between $t \geq \tau_i$ and the maturity date $\mathcal{T}_j$ is denoted by $\pi_{j,t}$ and given by:

$$\pi_{j,t} = \sum_{h=t \vee \tau_j}^{\mathcal{T}_j} E^{\mathbb{Q}} \left[exp\left(-\int_t^h r(s)ds \right) P_j|\xi > h \right] \tag{20}$$

where $a \vee b := max\{a, b\}$. Let $NPV_{j,t}$ be the net present value of the j-th generation under the risk-neutral measure, $\mathbb{Q}$, at the issuance date, τ_j, where:

$$NPV_{j,t} = \begin{cases} \Pi^P_{j,t} - \pi_{j,t} & \text{if } t = \tau_j \\ \Pi^P_{j,t} - [AV_{j,t} + \pi_{j,t}] & \text{if } t > \tau_j \end{cases} \tag{21}$$

The market value of the insurance portfolio Π^P_0, under the risk-neutral measure, $\mathbb{Q}$ at $t_0 = 0$,is:

$$\Pi^P_0 = \sum_j \Pi^P_{j,0} \tag{22}$$

We define the net present value of the portfolio (NPV_P) as the sum of the individual generation's net present values at t=0, where:

$$NPV_P = \sum_j NPV_{j,0} \tag{23}$$

We say that a portfolio of insurance policies is fairly priced, overall, if the portfolio's net present value NPV_P is equal to zero. Notably, the portfolio's net present value NPV_P might be zero, even though the individual net present values are not zero for all generations. Thus, it is possible that some customers will not pay fair premiums, although the insurer's overall portfolio is fairly priced.

The value of the shareholders' stakes, $\left(\Pi^E_0\right)$, and the net present value for shareholders, NPV_{EC}, under the risk-neutral measure, respectively, are:

$$\Pi^E_0 = \sum_{t=1}^{\mathcal{T}_{max}} E^{\mathbb{Q}} \left[exp\left(-\int_0^t r(s)ds \right) E_t \mathbf{1}_{t=min\{\xi, \mathcal{T}_{max}\}} \right] \tag{24}$$

and

$$NPV_{EC} = \Pi^E_0 - EC_0 - \sum_{j>1} E^{\mathbb{Q}} \left[exp\left(-\int_0^{\tau_j} r(s)ds \right) EC_{j-1} | \xi > \tau_j \right] \tag{25}$$

where E_t is derived from Equation (18).

2.4 Policyholder's point of view

We assume that the policyholders save a fixed annual amount W_0 for retirement during T_j years. Customers may either buy insurance coverage (I) or directly invest their wealth in the capital markets (NI). There is no possibility of a partial investment in the insurance portfolio. Hence, the customer invests all of his/her wealth W_0 either in insurance or alternative investments. In the first case, the customer joins the insurance portfolio and purchases a participating life insurance policy with a cliquet-style guarantee and annual premium $P = W_0$. However, the shareholders possess limited liability, and hence, the policyholders are exposed to a default risk. In the case of default at $t = \xi$, the policyholder would immediately invest the default payout $L_{j,\xi}$ and future contributions W_0 in the bank account until the date $\mathcal{T}_j$ is reached (i.e., the initial maturity date that can be interpreted as the retirement date). The policyholder's accumulated wealth $W^{(I)}_{\mathcal{T}_j}$ at the maturity date $\mathcal{T}_j$ is:

$$W^{(I)}_{\mathcal{T}_j} = \begin{cases} L_{j,\mathcal{T}_j} & \text{if } \xi > \mathcal{T}_j \\ L_{j,\xi} exp\left(\int_{\xi}^{\mathcal{T}_j} r_s ds\right) + \sum_{t=\xi}^{\mathcal{T}_j - 1} W_0 exp\left(\int_{t}^{\mathcal{T}_j} r_s ds\right) & \text{if } \tau_j < \xi < \mathcal{T}_j \end{cases} \tag{26}$$

By assuming that payouts and future premiums, in the case of default, are invested in the bank account (cash) until the maturity date, we intend to avoid additional complexity with respect to time dependence preferences.

In the second case, the customer directly invests in the financial market in a mix of cash-stocks. A fixed portion γ_{PH} of his/her income W_0 is invested in stocks and the remaining portion is deposited in a bank account (cash). Hence, the customer directly faces a financial risk. During T_j years, the customer would continue with -exactly the same investment strategy. More specifically, until their retirement date $\mathcal{T}_j$, he/she would buy assets, but not make any withdrawals. The policyholder's accumulated wealth $W^{(NI)}_{\mathcal{T}}$ after T_j years under the alternative strategy (i.e., not insurance NI) is:

$$W^{(NI)}_{\mathcal{T}_j} = \sum_{t=\tau_j}^{\mathcal{T}_j - 1} W_0 \tilde{Z}_{t,\mathcal{T}_j}$$

where $\tilde{Z}_{t,T}$ corresponds to the stochastic return of an alternative investment portfolio $(1-\gamma_{PH}, \gamma_{PH})$ (cash-stocks respectively) between t and $\mathcal{T}_j$.

Policyholder payouts in participating life insurance products, as in the one described in this paper (cliquet-style with periodical premiums), are calculated as a function of the premiums paid. Consequently, the assessment of the policyholders willingness to pay becomes a complex task if we intend to measure it as a function of the insurance premium. Hence, we intend to indirectly measure the policyholders' willingness to join the insurance portfolio. We assume that each policyholder assigns a utility score to competing investment portfolios based on the final wealth expected value and standard deviation (Saha, 1997). The policyholders' preference function is:

$$\Phi(W) = E[W] - Var[W]^{a/2} \tag{27}$$

where $a > 0$ corresponds to the risk aversion coefficient and W corresponds to the customer's wealth at the retirement date $\mathcal{T}_i$. Let $\mathcal{Q}_a$ be the set of all possible alternative investment strategies that lead to a greater utility than the insurance policy for a policyholder with a risk aversion coefficient a. In formal terms:

$$\mathcal{Q}_a = \left\{\gamma_{PH} \in [0,1] : \Phi_a(W^{(NI)}(\gamma_{PH})) > \Phi_a(W^{(I)})\right\}$$

We define the willingness to pay WTP_a of a policyholder with a risk aversion coefficient a as:

$$WTP_a := \begin{cases} 1 & \text{if } \mathcal{Q}_a = \emptyset \\ 1 - (Q_a - q_a) & \text{Otherwise} \end{cases} \tag{28}$$

where $Q_a = \sup \mathcal{Q}_a$ and $q_a = \inf \mathcal{Q}_a$ the corresponding supremum and infimum of $\mathcal{Q}_a$, respectively. If $\mathcal{Q}_a$ is an empty set, the policyholder would always decide to join the insurance portfolio. In particular, let a' be:

$$a' = inf\{a : WTP = 1\}$$

The value of $WTP = 1$indicates that the policyholder's perceived utility, when buying the insurance policy, would always be higher than any other available alternative investment in the market. In contrast, if $WTP_a < 1$ (i.e., $\mathcal{Q}_a$ is not an empty set, such that Q_a and q_a exist),then the customer could find alternative investments to insurance that provide him/her with a higher utility. Pure (fully) rational policyholders would choose the investment strategy in $\mathcal{Q}_a$ that maximizes his/her utility, and hence, only choose to join the insurance portfolio when WTP_a is equal to one. However, in reality, policyholders may not be pure (fully) rational customers, though $WTP_a < 1$ becomes an important indicator of the policyholders' driver decisions, but not an absolute determinant of it. $WTP_{j,a} < WTP_{i,a}$suggests that the jth generation has a larger set of alternative investment strategies that provide a higher utility than insurance, and hence, lower willingness to join the insurance policy than the i-th generation.

3 NUMERICAL ILLUSTRATION

3.1 Contract parameters

We consider the savings premium of a participating life insurance policy and do not take into account the surrender or pay-up options or death benefits. Our numerical analysis assumes that the insurance portfolio is built recursively and only one generation at a time would join the insurance portfolio. The insurer provides insurance policies with a term to maturity of $T = 20$ years, a fixed annual premium equal to 100 C.U., and a fixed participation rate of $\alpha = 0.9$, when the asset returns exceed the minimum interest rate guarantee. The guarantee level g_j for the new generations is fixed for the entire insurance contract, according to the prevailing financial market conditions and the insurance portfolio composition at the issuance date. Therefore, we assume that once the insurance contract is issued, the guarantee level cannot be modified. The insurer's risk management tools are equity contributions and investment strategies.

3.2 Financial market parameters and the regulatory environment

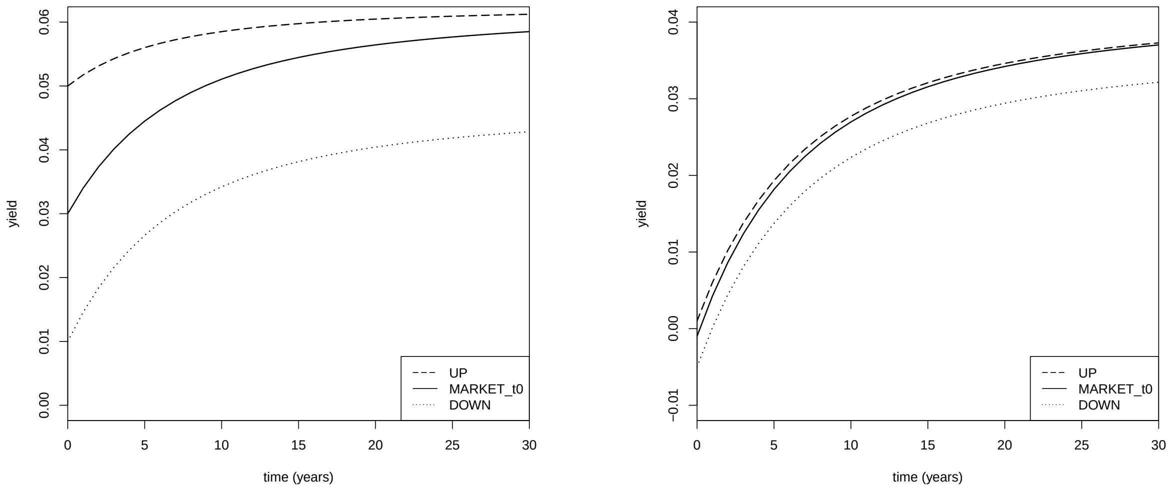
Recently , interest rates have reached and remained negative in many developed countries (e.g., Germany, Sweden, Finland, Switzerland). It is important to incorporate this particularity in our numerical analysis. The investment portfolio is simulated by assuming that the interest rate follows the Vasicek model dynamics (i.e, $k_1 = 0$ and $k_2 = 1$ in Equation 1).We analyze the fair pricing of individual policyholders and insurance portfolios under two main interest rate scenarios: positive and negative interest rates. Vasicek's parameters model the interest rate evolution in two main interest rate scenarios that are summarized in Table 1.

Table 1: Main Scenarios. Vasicek Model Parameters

parameter	Vasicek Positive	Vasicek Negative
k_1	0	0
k_2	1	1
$r(0)$	**3.000%**	**-0.100%**
κ	0.246	0.246
θ	**4.480%**	**2.480%**
σ_r	1.890%	1.890%
σ_S	20.000%	20.000%
λ_S	0.428	0.428
λ_r	-0.272	-0.272
ρ	-0.100	-0.100

Note that the positive and negative scenarios differ in the initial or current interest rate level $r(0)$and the long-term parameter θ. As generations do not join the insurance portfolio at the same point in time, there is additional financial information that should be considered when pricing new insurance contracts. Consequently, the analysis of the potential changes in the fair pricing of the insurance policies of different generations under different informational assumptions becomes essential. We assume that the new generations use the most updated market information to make decisions concerning their long-term investments. Therefore, the main scenarios, described in Table 1, have two sub-scenarios that intend to replicate the new information scenarios the insurer and policyholders would possess at the time of the pricing of the insurance contract of the entering generation.

Figure 1illustrates the different shapes that are assumed in the sub-scenarios. The up-trend and down-trend scenarios intend to reproduce the changes in the financial information that are available at the date of issuance. This information is essential to take into account the pricing of the insurance portfolio. The main variation in the interest rate hypotheses, to price any new generation that joins the insurance portfolio, corresponds to the variation in the long-term interest rate θ and the changes in the initial interest rate r_0 at the issuance date. The up-trend scenario would refer to information in which the current hypotheses of the interest rates would illustrate improvements or optimistic increases in the interest rates. In the case of negative interest rates, the upward trend scenario would correspond to an improvement in the interest rate level, but would remain a low long-term parameter. The downward trend scenario corresponds to the case in which the interest rates are expected to drop.



(a) Positive Interest Rate Sub-Scenarios

(b) Negative Interest Rate Sub-Scenarios

Figure 1: Interest rate sub-scenarios for additional financial information. Mean value curves.

3.3 The case of two policyholders

In this section, the analysis focuses on the simplest interaction among generations. Only two generations compose the insurance portfolio. As we focus on voluntary saving plans, at the moment of issuing a new generation, the insurer possesses information about the in-force composition of the insurance portfolio. However, there is no certainty concerning the portfolio size. The behavior of the future generations is unknown (i.e., future generations join or do not join the insurance portfolio) and is assumed to depend on the financial conditions at the issuance of the insurance contract. The conditions differ for each generation in the portfolio. Consequently, the insurer would take into account the possible intergenerational interactions with the in-force portfolio (i.e., generations which are already part of the insurance portfolio) and neglect the interactions with future generations, as is suggested in Hieber et al. (2016).

The insurer maintains an unchanged investment strategy γ as long as no new customer joins the insurance portfolio. Thus, the risk management strategies may be adjusted if new policyholders decide to take part in the insurance portfolio. We assume that the investment strategy γ, as well as any required equity contributions **EC**, are adjusted at the issuance of any new insurance policy. The in-force insurance portfolio would be evaluated at the market prices and the new generation would merge with the in-force portfolio by considering the latest financial information available: the interest rate environment.

The insurance portfolio, consisting of two generations, would then be established at two different times $t = 0$ and $t = 1$. At $t = 0$, the first generation would be priced individually. At $t = 1$, the second generation is priced assuming that it joins a portfolio of insurance. There is no certainty about participation in the portfolio. Consequently, the size of the insurance portfolio would depend on the new policyholders' willingness to join, or not to join, the insurance portfolio. We assume that the new intergenerational interactions that are created in the portfolio do not affect the willingness to persist in the insurance portfolio of older generations. This assumption may not hold, in reality, and may not be rational, in the cases in which surrendering the insurance contract is allowed. However, this case is out of the scope of this paper.

3.3.1 *Insurance Portfolio at* $\tau_1 = 0$ *with* $n = 1$

In this section, we include the analysis of fair pricing and the willingness to join the insurance portfolio for a single generation (i.e., the first generation that joins the portfolio at $t = 0$) (Grosen & Jørgensen, 2002; Braun et al., 2015). We intend to set the initial conditions of the insurance portfolio. The differences in the initial portfolio are principally driven by the interest rate levels, as well as the regulatory environment under which the insurance company operates.

Table 2shows the specific risk management strategies, such that the generation at $t = 0$is fairly priced, and the generation's willingness to pay for the participating insurance portfolio is for different guarantee levels. Various aspects can be analyzed from the results. The insurer should make decisions concerning the adequate risk management measures to be adopted to avoid a wealth transfer between policyholders and shareholders. It can be observed that lower guarantee levels would allow the insurer to take riskier investment strategies. Consequently, the policyholders have a higher expected wealth value, but also a higher wealth variance. The insurer could adopt similar investment strategies in the positive and negative scenarios of the interest rate by adjusting the guarantee levels. Concerning the equity contributions, stable values can be observed among the different guarantee values, except for $g = 0\%$, in the negative Scenario IR.

The results illustrate that, under a scenario of low-interest rates, positive guarantees are unlike to be offered. Accordingly, the insurer is required to invest most of the assets in safe investments (i.e., 2.5% in risky assets), and shareholders must double the equity contributions.

Shareholders and managers decisions drive variations in the policyholders' willingness to buy the insurance policy. Since we assume that policyholders judge their investments by assessing their expected final wealth and the implied risk (i.e., $\mu - \sigma$ preferences), then the asset return would play an important role in the policyholder's decision to take part of the insurance portfolio.

Risk-neutral customers (i.e., risk aversion coefficient $a = 0$), only have a concern about the expected value of their wealth and neglect any variance. For this reason, risk neutral customers would judge that investing directly in the capital markets is more attractive than sharing the returns with the insurer (the participation rate α is 0.9 smaller than where 10%of the profit is taken by the insurer). For example, if the guarantee level is set to $g = 1.25\%$ in a positive interest rate environment, risk neutral customers perceive a higher utility by investing at least 28%of their wealth in stocks. The set of alternative investments that

Table 2: Fair Pricing and Willingness to Pay of Individual Customers

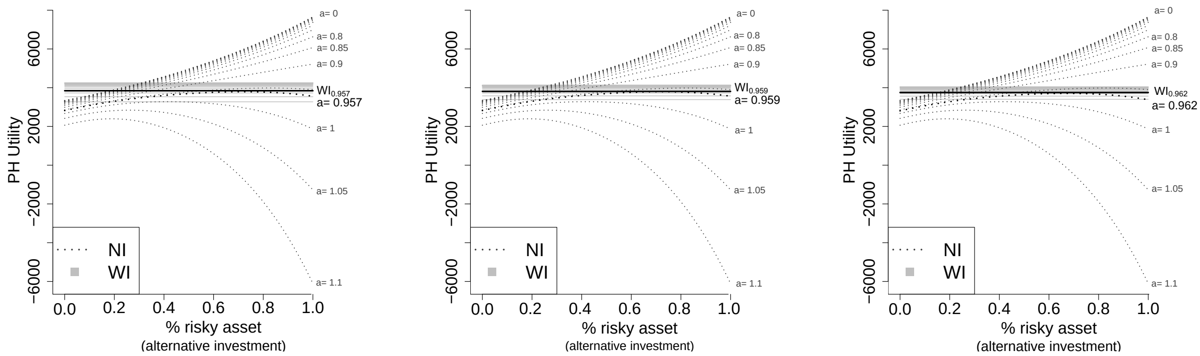
	Positive Scenario IR			Negative Scenario IR		
	$g = 0.625\%$	$g = 1.25\%$	$g = 1.875\%$	$g = -1.25\%$	$g = -0.625\%$	$g = 0.00\%$
RM Parameters[a]						
γ_S	21.300%	16.666%	12.958%	13.897%	7.674%	2.485%
γ_B	70.000%	72.380%	71.042%	70.971%	73.338%	70.686%
EC	128.000	123.494	134.404	138.814	138.064	285.173
PD^b	0.157%	0.167%	0.117%	0.060%	0.089%	0.007%
Portfolio NPVs[c]						
NPV_E	0.000	0.001	0.000	0.000	-0.001	-0.383
NPV_P	0.000	-0.001	0.000	0.000	0.001	0.383
Individual NPVs[c]						
NPV_1	0.000	-0.001	0.000	0.000	0.001	0.383
ΔNPV_1	0.000%	0.000%	0.000%	0.000%	0.000%	0.025%
$E[W^{(I)}]$	4249.423	4130.531	4041.443	3243.416	3084.701	2969.104
$\sigma_{W^{(I)}}$	522.088	408.707	353.912	297.376	217.379	224.285
Individual WTP[d]						
a'	0.957	0.959	0.962	0.995	1.002	1.014
WTP_0	0.313	0.278	0.250	0.220	0.170	0.133
$\mathcal{Q}_0$	(0.3125,1)	(0.2775,1)	(0.25,1)	(0.22,1)	(0.17,1)	(0.1325,1)
$WTP_{0.75}$	0.335	0.303	0.275	0.238	0.190	0.150
$\mathcal{Q}_{0.75}$	(0.335,1)	(0.3025,1)	(0.275,1)	(0.2375,1)	(0.19,1)	(0.15,1)
WTP_1	1.000	1.000	1.000	1.000	0.865	0.585
$\mathcal{Q}_1$	(0,0)	(0,0)	(0,0)	(0,0)	(0.47,0.605)	(0.3175,0.7325)
$WTP_{1.25}$	1.000	1.000	1.000	1.000	1.000	1.000
$\mathcal{Q}_{1.25}$	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)

[a] *RM:* Risk Management.
[b] PD: Probability of default. $Pr[\xi < \mathcal{T}_{max}]$.
[c] NPV: Net Present Value.
[d] WTP: Willingness to pay.

provide a higher expected utility than the insurance contract corresponds to $\mathcal{Q}_a = (0.28, 1]]$. In this case, the customer maximizes his/her utility by investing 100% of his/her wealth in stocks. However, as the policyholder becomes more risk averse, the size of the set of alternative investments $\mathcal{Q}_a$ is reduced. Hence, the policyholders' willingness to pay for insurance tends to 1 (i.e., the interval size of the alternative investment strategy narrows).

We assume that the customers are purely rational, which implies that a customer would always reject an insurance contract if his/her $\mathcal{Q}_a$is a non-empty set (i.e., $WTP_a < 1$). Nevertheless, if the customers are not purely rational, $WTP < 1$would provide an indicator of a policyholder's set of alternatives that he/she may consider when making an investment decision. In reality, there are external factors, besides the policyholder's final wealth distribution, might influence the policyholder's final decision about signing or not signing an insurance contract. The WTP, in this case, could be interpreted as a percentage of the cases in which the insurance policy would be preferred over the alternative investments.

Figure 2 and Figure 3 graphically illustrate the policyholder expected utility for the insurance contract for di?erent interest rate guarantees, as compared to the policyholder's expected utility with alternative



(a) $g = 0.625\%$ **(b)** $g = 1.25\%$ **(c)** $g = 1.875\%$

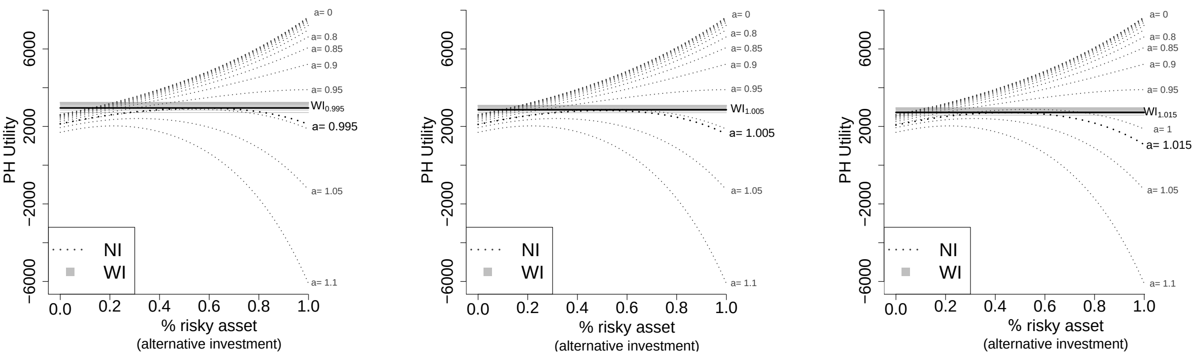
Figure 2: Positive Scenario IR. The policyholder's expected utility with and without insurance for different risk-aversion coefficients and different guarantee levels. The gray area corresponds to the policyholder's utility level with insurance (I) when $a \in [0, 1.1]$. The dotted lines correspond to the policyholder's utility level without insurance (NI), which is dependent on the share of the investment in the risky asset. The bold lines correspond to the policyholder's utility level with insurance at a'.

strategies (i.e.,NI no insurance), for different risk aversion coefficients. The gray area represents the policyholder's utility level with insurance(I) for $a \in [0, 1.1]$. The dotted lines describe the policyholder's utility level without insurance (NI) dependent on the share of the investment in the risky asset. The intersection between the utility curves with and without insurance, at a certain risk-aversion level a, are the corresponding bounds of the interval $\mathcal{Q}_a$ presented in Table 2.

The bold lines corresponds to $\Phi_{a'}(W^{(I)}(g))$ such that $a' = inf\{a : WTP_a = 1\}$. In other words, the bold lines correspond to the maximum utility level at which policyholder would prefer always to buy insurance. Any policyholder with risk-aversion coefficient greater than a' would always decide to join the insurance portfolio. While customers with risk-aversion coefficient smaller than $a < a'$, would have a non-empty set of alternative investments $\mathcal{Q}_a$ and hence maximize their utility without an insurance contract.

Figures 2a, 2b and 2cshow that, for the scenario of positive interest rates, the three different guarantees lead to similar expected utility levels. However, the a' aversion coefficient is slightly different. Indeed, if the guarantee level increases, then a'also increases. Hence, lower guarantees would capture more customers. If the guarantee is $g = 1.25\%$, any customer with a risk-aversion coefficient greater than $a = 0.96$ would join the insurance portfolio. That being said, if a higher guarantee is offered $g = 1.875\%$, then only customers with a risk-aversion coefficient greater than $a = 0.965$,would always join the insurance portfolio.

In the scenario of negative interest rates (Figures 3a, 3b and 3c), only very risk-averse policyholders would join the insurance portfolio (i.e., $a' > 1$). Higher guarantees would require more conservative investment strategies (i.e., lower share of risky assets) (Table 2), and hence, lower policyholder returns. Therefore, in the scenario of low-interest rates, policyholders would value alternative investments as more attractive than



(a) $g = -1.25\%$ **(b)** $g = -0.625\%$ **(c)** $g = 0.00\%$

Figure 3: Negative Scenario IR.The policyholder's expected utility with and without insurance for different risk-aversion coefficients and different guarantee levels. The gray area corresponds to the policyholder's utility level with insurance (I) when $a \in [0, 1.1]$. The dotted lines correspond to the policyholder's utility level without insurance (NI), which is dependent on the share of the investment in the risky asset. The bold lines correspond to the policyholder's utility level with insurance at a'.

investing in the insurance contract.

In general, there is an inverse relationship between the guarantee level and the policyholder's WTP (Braun et al., 2015). Higher guarantees would require safer investment strategies (i.e., lower share of risky assets). Hence, lower returns on the asset investments decrease the policyholders' WTP.

3.3.2 *Insurance Portfolio of Subsequent generations at* $\tau_2 = 0$ *,* $n = 2$

In this section, two subsequent generations would compose the insurance portfolio. In non-compulsory schemes, policyholders are not obliged to join the insurance portfolio. Therefore, we analyze the insurance portfolio when only one additional generation is potentially entering the insurance portfolio. No further information is known a priori regarding the future generations that might sign a contract. The assumption of future generations, in the case of non-mandatory schemes, may bias the view of intergenerational wealth transfers. We intend to avoid mispricing the insurance contract at the issuance caused by taking into account interactions with generations for which there is still high uncertainty about their final behavior (i.e., may or may not join the insurance portfolio).

We assume that the insurance company is solvent at $\tau_2 = 0$ and the in-force portfolio involves the generation that joined the insurance portfolio $\tau_1 = -1$. Additionally, at the issuance date, the new customers would consider the latest financial information when deciding whether or not to join the insurance portfolio. The most recent financial information in this setting corresponds to changes in interest rate scenarios. The upward and downward sub-scenarios of the interest rate are described in Section 3.2. The insurer could consider offering different guarantee levels accordingly. We seek investment strategies and equity contributions such that, in all cases, the overall insurance portfolio is fairly priced. Therefore, a wealth transfer between generations may occur, but no wealth transfer occurs between the shareholders and the policyholders.

Table 3: Positive Upward Sub-Scenario Fair Pricing and Willingness to Pay of Individual Customers.

$r(0) = 5.00\%$ and $\theta = 4.48\%$

		Same Guarantee		Fair Guarantee					
Initial Portfolio									
	EC_{ini}	130.891		130.891		130.891		130.891	
	V_0	106.267		106.267		106.267		106.267	
RM Parameters[a]									
	γ_S	17.777%		17.162%		15.263%		13.113%	
	γ_B	74.636%		75.558%		80.336%		84.996%	
	EC	112.105		109.742		153.115		155.198	
	PD^b	0.315%		0.218%		0.129%		0.091%	
Portfolio NPVs[c]									
	NPV_E	0.000		0.001		0.001		-0.008	
	NPV_P	0.000		-0.001		-0.001		0.008	
Individual NPVs[c]									
		PH_1	PH_2	PH_1	PH_2	PH_1	PH_2	PH_1	PH_2
	T_i	19	20	19	20	19	20	19	20
	g_i	1.250%	1.250%	1.250%	1.401%	1.250%	1.419%	1.250%	1.438%
	$V_{i,0}$	106.267	0.000	106.267	0.000	106.267	0.000	106.267	0.000
	NPV_i	2.105	-2.105	0.017	-0.017	0.040	-0.040	0.049	-0.042
	ΔNPV_i	0.169%	-0.181%	0.001%	-0.001%	0.003%	-0.003%	0.004%	-0.003%
	$E[W^{(I)}]$	-	4239.014	-	4233.269	-	4226.528	-	4189.994
	$\sigma_{W^{(I)}}$	-	446.608	-	428.873		405.409	-	363.853
Individual WTP[d]									
	a'	-	0.956	-	0.955	-	0.954	-	0.955
	$WTP_{0,i}$	-	0.288	-	0.285	-	0.283	-	0.273
	$\mathcal{Q}_{0,i}$	-	(0.2875,1)	-	(0.285,1)	-	(0.2825,1)	-	(0.2725,1)
	$WTP_{0.75,i}$	-	0.313	-	0.313	-	0.310	-	0.300
	$\mathcal{Q}_{0.75,i}$	-	(0.3125,1)	-	(0.3125,1)	-	(0.31,1)	-	(0.3,1)
	$WTP_{1,i}$	-	1.000	-	1.000	-	1.000	-	1.000
	$\mathcal{Q}_{1,i}$	-	(0,0)	-	(0,0)	-	(0,0)	-	(0,0)
	$WTP_{1.25,i}$	-	1.000	-	1.000	-	1.000	-	1.000
	$\mathcal{Q}_{1.25,i}$	-	(0,0)	-	(0,0)	-	(0,0)	-	(0,0)

[a] *RM:* Risk Management.
[b] PD: Probability of default. $Pr[\xi < \mathcal{T}_{max}]$.
[c] NPV: Net Present Value.
[d] WTP: Willingness to pay.

Table 4: Positive Downward Sub-Scenario Fair Pricing and Willingness to Pay of Individual Customers.

$r(0) = 1.00\%$ and $\theta = 2.98\%$

	Same Guarantee		Fair Guarantee					
Initial Portfolio								
EC_{ini}	130.891		130.891		130.891		130.891	
V_0	106.267		106.267		106.267		106.267	
RM Parameters[a]								
γ_S	1.112%		1.304%		1.155%		1.176%	
γ_B	70.000%		70.005%		75.002%		80.015%	
EC	2048.000		1644.039		1755.800		1522.990	
PD^b	0.000%		0.000%		0.000%		0.000%	
Portfolio NPVs[c]								
NPV_E	-26.205		-0.071		-0.230		-0.152	
NPV_P	26.205		0.071		0.230		0.152	
Individual NPVs[c]								
	PH_1	PH_2	PH_1	PH_2	PH_1	PH_2	PH_1	PH_2
T_i	19	20	19	20	19	20	19	20
g_i	1.250%	1.250%	1.250%	0.370%	1.250%	-0.556%	1.250%	-1.875%
$V_{i,0}$	106.267	0.000	106.267	0.000	106.267	0.000	106.267	0.000
NPV_i	18.272	7.933	18.586	-18.515	30.986	-30.757	44.466	-44.314
ΔNPV_i	1.199%	0.542%	1.219%	-1.264%	2.033%	-2.100%	2.917%	-3.025%
$E[W^{(I)}]$	-	3138.274	-	3073.065	-	3050.669	-	3032.556
$\sigma_{W^{(I)}}$	-	217.063	-	225.909	-	219.729	-	208.338
Individual WTP[d]								
a'	-	1.005	-	1.013	-	1.015	-	1.016
$WTP_{0,i}$	-	0.133	-	0.110	-	0.100	-	0.095
$\mathcal{Q}_{0,i}$	-	(0.1325,1)	-	(0.11,1)	-	(0.1,1)	-	(0.095,1)
$WTP_{0.75,i}$	-	0.153	-	0.130	-	0.123	-	0.115
$\mathcal{Q}_{0.75,i}$	-	(0.1525,1)	-	(0.13,1)	-	(0.1225,1)	-	(0.115,1)
$WTP_{1,i}$	-	0.753	-	0.608	-	0.583	-	0.573
$\mathcal{Q}_{1,i}$	-	(0.38,0.6275)	-	(0.3,0.6925)	-	(0.2875,0.705)	-	(0.2825,0.71)
$WTP_{1.25,i}$	-	1.000	-	1.000	-	1.000	-	1.000
$\mathcal{Q}_{1.25,i}$	-	(0,0)	-	(0,0)	-	(0,0)	-	(0,0)

[a] *RM:* Risk Management.
[b] PD: Probability of default. $Pr[\xi < \mathcal{T}_{max}]$.
[c] NPV: Net Present Value.
[d] WTP: Willingness to pay.

Table 3 and 4 present the adequate risk management measures to fairly price an insurance portfolio and the willingness to join the insurance portfolio of a new generation when different guarantee levels are proposed under different informational assumptions: positive upward and downward scenarios of interest rates, respectively. The adequate risk management measures correspond to the optimal combinations of investment strategies and equity combinations, such that the wealth transfer among generations is minimized for the corresponding portfolio setting (i.e., proposed guarantee levels, maturities and annual premiums). We assume that the insurance policy conditions of the in-force portfolio are set at the issuance date and cannot be modified. The WTP for the in-force portfolio is not reported in Table 3 and 4, given that we assumed that the contract does not have any surrender options. Hence, policyholders that are already part of the insurance portfolio would not be the subject of any decision.

The results illustrate that, in general, the insurance company should adopt dynamic investment strategies to ensure the fair pricing of the insurance portfolio when additional financial information is available. More precisely, the interest rate environment drives the suitable investment strategy for fair pricing. If new financial information reveals that interest rates have an upward trend (Table 3) and the insurer offers the same guarantee level as the first generation guarantee (i.e., $g = 1.25\%$), the insurance company must adapt its investment strategy. The insurer is allowed to increase the share of the risky asset $\gamma_S$to ensure the fair pricing of the insurance portfolio. However, the fair price of the entire portfolio does not imply that each generation would also be fairly priced (Orozco-Garcia & Schmeiser, 2017).

In the case in which the same guarantee level is provided to all participants in the portfolio, we find that the joining generation is over-priced (i.e., $NPV_2 < 0$), although the insurance portfolio is fairly priced. Here, the insurer may decide to adjust the guarantee level to offer a fair insurance contract to the new generation (Hieber et al., 2016). Table 3 illustrates the different guarantee levels that could be offered by the insurer in an upward interest rate scenario, such that the joining generation pays fair premiums at issuance. This fair guarantee is not unique. However, it does depend on the investment strategy. For instance, if the bond investment share is held at 75%, the guarantee level should be adjusted to 1.401% to furnish fair contracts. The insurer can also provide higher guarantees, but this would imply that the risky investment share should be reduced (Table 2).

The insurer should also consider the policyholders $WTP$when deciding upon the optimal guarantee level offered to the joining generation. The policyholder's WTP − also a' − would be similar if the insurer provided the same guarantee to all generations in the portfolio, or a fair guarantee to the joining generation. In general, any policyholder with a risk-aversion coefficient that is greater than 0.955 would join the insurance portfolio, regardless of the portfolio fairness, if insurance offers only one possibility. However, the customer would perceive the highest expected utility when the guarantee level is set at $g_2 = 1.419\%$ and lowest expected utility would be perceived when $g_2 = 1.25\%$.

In contrast to the scenario of positive interest rates, a downward movement in the interest rate curve would restrict the managers of the insurance company with respect to the guarantee level to offer to the new generation. Indeed, if the interest rate sinks, then the fair insurance portfolio might not always exist (Table 4). In this case, a low share of investment in the risky asset, together with extremely high equity contributions, might still generate wealth transfers between shareholders and policyholders whenever the insurer persists in offering the same guarantee to all generations in the insurance portfolio. More precisely, from the shareholder's point of view, providing the same guarantee level to the new generation is unreasonable. Indeed, the plausible guarantee level to be offered to the second generation should be lower than the guarantees in the in-force portfolio to at least the fair price of the insurance portfolio, as a whole. This guarantee is lower than the first generation guarantee $g_2 = 0.37\% < g_1 = 1.25.\%$. Regardless, only a fair portfolio can be offered (i.e., no wealth transfer between the policyholders and the shareholders).

In this case, a wealth transfer among generations is unavoidable. The new generation would reduce its WTP_2, compared to the first generation WTP_1 at τ_1 and a guarantee level $g_1 = 1.25\%$. Only customers with a risk-aversion coefficient greater than $a' > 1.013$ would join the insurance portfolio (i.e., very risk-averse). The insurer would be obliged to adopt a very conservative investment strategy (i.e., only a 1% investment in the risky asset), and consequently, alternative investments would provide the policyholder with much higher returns.

Tables 5 and 6 present the adequate risk management measures necessary to fairly price an insurance portfolio and the willingness of a new generation to join the insurance portfolio when different guarantee levels are offered under various informational assumptions in an environment of low-interest rates. An environment of negative interest rates would restrict the insurer concerning the guarantee levels that could be provided to new customers. In this case, we assume that the insurer has already offered a low negative guarantee level -0.625% to the first generation that joined the insurance portfolio at ?0. As was discussed in the case of positive interest rates with an upward trend (i.e., increasing interest rates), offering the same guarantee would be possible, but wealth transfers among generations would take place (i.e., only a fair price for the entire portfolio exists, while the second generation would be over-priced).

The insurer might also decide to offer fair guarantees by increasing the guarantee level g_2. The insurer could provide different fair guarantees by adjusting the investment strategy. However, higher guarantees would reduce the risky share of investments, and hence, the policyholders WTP_2 could be affected. In this case, the highest WTP_2 for all risk aversion coefficients can be observed for a guaranteed level of $g_2 = -0.372\%$. In particular, the policyholders that join the insurance portfolio are very risk-averse customers with $a' > 1$. In the scenario of negative interest rates, the sensitivity of the policyholder WTP_2changes with the guarantee level. Small changes in the guarantee level would decrease the WTP would remain almost constant whenever the interest rates have an upward trend in a scenario of positive interest rates (Table 3).

Table 5: Negative Upward Sub-Scenario Fair Pricing and Willingness to Pay of Individual Customers.

$r(0) = 1.00\%$ and $\theta = 2.48\%$

	Same Guarantee		Fair Guarantee					
Initial Portfolio								
EC_{ini}	141.050		141.050		141.050		141.050	
V_0	103.072		103.072		103.072		103.072	
RM Parameters[a]								
γ_S	10.778%		9.488%		6.884%		2.233%	
γ_B	70.029%		71.040%		75.257%		80.245%	
EC	33.720		105.451		125.862		834.667	
PD[b]	0.366%		0.142%		0.082%		0.001%	
Portfolio NPVs[c]								
NPV_E	0.001		0.000		0.001		0.001	
NPV_P	-0.001		0.000		-0.001		-0.001	
Individual NPVs[c]								
	PH_1	PH_2	PH_1	PH_2	PH_1	PH_2	PH_1	PH_2
T_i	19	20	19	20	19	20	19	20
g_i	-0.625%	-0.625%	-0.625%	-0.372%	-0.625%	-0.353%	-0.625%	-0.334%
$V_{i,0}$	103.072	0.000	103.072	0.000	103.072	0.000	103.072	0.000
NPV_i	3.250	-3.252	0.012	-0.013	0.018	-0.019	0.001	-0.002
ΔNPV_i	0.219%	-0.229%	0.001%	-0.001%	0.001%	-0.001%	0.000%	0.000%
$E[W^{(I)}]$	-	3111.006	-	3144.563	-	3096.121	-	3015.994
$\sigma_{W^{(I)}}$	-	363.180	-	244.520	-	226.278	-	234.273
Individual WTP[d]								
a'	-	1.018	-	1	-	1.003	-	1.012
$WTP_{0,i}$	-	0.168	-	0.180	-	0.165	-	0.138
$\mathcal{Q}_{0,i}$	-	(0.1675,1)	-	(0.18,1)	-	(0.165,1)	-	(0.1375,1)
$WTP_{0.75,i}$	-	0.180	-	0.200	-	0.185	-	0.155
$\mathcal{Q}_{0.75,i}$	-	(0.18,1)	-	(0.2,1)	-	(0.185,1)	-	(0.155,1)
$WTP_{1,i}$	-	0.570	-	1.000	-	0.825	-	0.625
$\mathcal{Q}_{1,i}$	-	(0.3025,0.7325)	-	(0,0)	-	(0.4425,0.6175)	-	(0.3325,0.7075)
$WTP_{1.25,i}$	-	1.000	-	1.000	-	1.000	-	1.000
$\mathcal{Q}_{1.25,i}$	-	(0,0)	-	(0,0)	-	(0,0)	-	(0,0)

[a] *RM:* Risk Management.
[b] PD: Probability of default. $Pr[\xi < \mathcal{T}_{max}]$.
[c] NPV: Net Present Value.
[d] WTP: Willingness to pay.

Table 6: Negative Downward Sub-Scenario Fair Pricing and Willingness to Pay of Individual Customers.

$r(0) = -0.5\%$ and $\theta = 1.98\%$								
	Same Guarantee		Fair Guarantee					
Initial Portfolio								
EC_{ini}	141.050		141.050		141.050		141.050	
V_0	103.072		103.072		103.072		103.072	
RM Parameters[a]								
γ_S	1.112%		1.112%		4.261%		5.868%	
γ_B	70.000%		70.000%		75.342%		80.127%	
EC	1433.600		1410.201		1299.484		6.373	
PD^b	0.000%		0.000%		0.003%		0.479%	
Portfolio NPVs[c]								
NPV_E	-1.700		-0.457		0.004		0.000	
NPV_P	1.700		0.457		-0.004		0.000	
Individual NPVs[c]								
	PH_1	PH_2	PH_1	PH_2	PH_1	PH_2	PH_1	PH_2
T_i	19	20	19	20	19	20	19	20
g_i	-0.625%	-0.625%	-0.625%	-0.672%	-0.625%	-2.109%	-0.625%	-3.164%
$V_{i,0}$	103.072	0.000	103.072	0.000	103.072	0.000	103.072	0.000
NPV_i	5.140	-3.440	5.140	-4.683	22.656	-22.660	32.535	-32.537
ΔNPV_i	0.309%	-0.213%	0.309%	-0.290%	1.364%	-1.403%	1.959%	-2.014%
$E[W^{(I)}]$	-	2748.300	-	2745.702	-	2795.299	-	2762.649
$\sigma_{W^{(I)}}$	-	200.837	-	201.179	-	195.396	-	182.230
Individual WTP[c]								
a'	-	1.027	-	1.028	-	1.021	-	1.023
$WTP_{0,i}$	-	0.105	-	0.103	-	0.120	-	0.110
$\mathcal{Q}_{0,i}$	-	(0.105,1)	-	(0.1025,1)	-	(0.12,1)	-	(0.11,1)
$WTP_{0.75,i}$	-	0.123	-	0.123	-	0.140	-	0.130
$\mathcal{Q}_{0.75,i}$	-	(0.1225,1)	-	(0.1225,1)	-	(0.14,1)	-	(0.13,1)
$WTP_{1,i}$	-	0.438	-	0.435	-	0.495	-	0.475
$\mathcal{Q}_{1,i}$	-	(0.25,0.8125)	-	(0.25,0.815)	-	(0.285,0.79)	-	(0.2725,0.7975)
$WTP_{1.25,i}$	-	1.000	-	1.000	-	1.000	-	1.000
$\mathcal{Q}_{1.25,i}$	-	(0,0)	-	(0,0)	-	(0,0)	-	(0,0)

[a] *RM:* Risk Management.
[b] PD: Probability of default. $Pr[\xi < \mathcal{T}_{max}]$.
[c] NPV: Net Present Value.
[d] WTP: Willingness to pay.

In the case that interest rates have an important downward trend, it will not be possible to offer fair guarantees to the new generation. Moreover, if a customer would be willing to accept an unfair guarantee, the insurer must decrease the guarantee level and the risky investment, as well as pay high equity contributions, to avoid wealth transfers between the shareholder and the policyholders. The joining generation, under the described conditions, would always be over-priced (i.e., $NPV_2 < 0$). Hence, the insurer should only expect to capture very risk averse policyholders.

Table 7illustrates an insurance portfolio in which the new generation does not consecutively join the insurance portfolio (i.e., $\tau_2 - \tau_1 > 1$). Hence, the difference in the time elapsed between the first generation and the second generation would affect the wealth transfer among the generations. If the difference, in time to maturity between the in-force portfolio and the new generation, increases, the wealth transfer among generations also increases, such that the insurer proposes the same guarantee level. If the new generation joins the insurance portfolio in a subsequent year and the expected lifetime of the portfolio does not change, then the wealth transfer is reduced , given that both contracts would be similar when considering the long-term perspective. On the contrary, if the difference between the time to maturity of the generations in the insurance portfolio increases, then offering the same guarantee level would create a greater wealth transfer among the generations. For example, in this case, one generation only remains as part of the portfolio for 15 years, while the other would still be part of the portfolio for 20 years. The adjustment in the guarantee level, to offer a fair contract to the new generation, depends on the magnitude of the wealth transfer among generations.

Table 7: Fair Pricing and Willingness to Pay for a Portfolio with $n = 2$. Non Subsequent Generations under Upward trend in a Scenario of Positive Interest Rates.

		$\tau 1 = -1$, $\tau_2 = 0$				$\tau 1 = -3$, $\tau_2 = 0$				$\tau 1 = -5$, $\tau_2 = 0$			
		Same Guarantee		Fair Guarantee		Same Guarantee		Fair Guarantee		Same Guarantee		Fair Guarantee	
Initial Portfolio													
	EC_{ini}	130.891		130.891		148.360		148.360		169.197		169.197	
	V_0	106.267		106.267		340.734		340.734		609.357		609.357	
RM Parameters[a]													
	γ_S	17.777%		17.162%		19.562%		16.596%		19.454%		13.254%	
	γ_B	74.636%		75.558%		70.393%		75.120%		71.297%		81.519%	
	EC	112.105		109.742		41.208		162.372		28.560		36.489	
	PD^b	0.315%		0.218%		0.458%		0.218%		0.430%		0.218%	
Portfolio NPVs[c]													
	NPV_E	0.000		0.001		-0.005		0.004		-0.010		0.000	
	NPV_P	0.000		-0.001		0.005		-0.004		0.010		0.000	
Individual NPVs [c]													
		PH_1	PH_2	PH_1	PH_2	PH_1	PH_2	PH_1	PH_2	PH_1	PH_2	PH_1	PH_2
	T_i	19	20	19	20	17	20	17	20	15	20	15	20
	g_i	1.250%	1.250%	1.250%	1.401%	1.250%	1.250%	1.250%	1.782%	1.250%	1.250%	1.250%	2.075%
	$V_{i,0}$	106.267	0.000	106.267	0.000	340.734	0.000	340.734	0.000	609.357	0.000	609.357	0.000
	NPV_i	2.105	-2.105	0.017	-0.017	6.569	-6.564	0.035	-0.039	9.580	-9.570	0.069	-0.069
	ΔNPV_i	0.169%	-0.181%	0.001%	-0.001%	0.472%	-0.581%	0.002%	-0.003%	0.605%	-0.856%	0.004%	-0.006%
	$E[W^{(I)}]$	-	4,239.014	-	4,233.269	-	4,142.905	-	4,218.438	-	4,192.599	-	4,124.032
	$\sigma_{W^{(I)}}$	-	446.608	-	428.873	-	641.887	-	412.871	-	500.793	-	420.244
Individual WTP[d]													
	a'	-	0.956	-	0.9550	-	0.976	-	0.956	-	0.961	-	0.962
	$WTP_{0,i}$	-	0.288	-	0.285	-	0.258	-	0.280	-	0.273	-	0.253
	$\mathcal{Q}_{0,i}$	-	(0.2875,1)	-	(0.285,1)	-	(0.2575,1)	-	(0.28,1)	-	(0.2725,1)	-	(0.2525,1)
	$WTP_{0.75,i}$	-	0.313	-	0.313	-	0.270	-	0.308	-	0.295	-	0.275
	$\mathcal{Q}_{0.75,i}$	-	(0.3125,1)	-	(0.3125,1)	-	(0.27,1)	-	(0.3075,1)	-	(0.295,1)	-	(0.275,1)
	$WTP_{1,i}$	-	1.000	-	1.000	-	1.000	-	1.000	-	1.000	-	1.000
	$\mathcal{Q}_{1,i}$	-	(0,0)	-	(0,0)	-	(0,0)	-	(0,0)	-	(0,0)	-	(0,0)
	$WTP_{1.25,i}$	-	1.000	-	1.000	-	1.000	-	1.000	-	1.000	-	1.000
	$\mathcal{Q}_{1.25,i}$	-	(0,0)	-	(0,0)	-	(0,0)	-	(0,0)	-	(0,0)	-	(0,0)

[a] *RM:* Risk Management.
[b] PD: Probability of default. $Pr[\xi < \mathcal{T}_{max}]$.
[c] NPV: Net Present Value.
[d] WTP: Willingness to pay.

The WTP results in Table 7suggest that it is not always the case that the second generation perceives a higher utility when the insurance company offers fair guarantees. For instance, in the case in which the second generation joins the portfolio five years later than the first generation, the policyholder shows a higher WTP for the same guarantee level than for the fair guarantee. In this case, customers with a risk aversion coefficient of greater than $a = 0.961$ would always prefer to buy insurance when $g_2 = 1.25\%$. In contrast, if the insurer adjusts the guarantee to be fair, it should convince customers with a risk aversion coefficient of greater than $a = 0.962$to join the insurance portfolio. This slight decrease in the second generation WTP for fair guarantees arises from a more conservative investment strategy (i.e., lower share in risky assets and higher share in bonds), that the insurer should adopt, given that the fair guarantee is significantly higher than the guarantee that was offered to the first generation.

3.4 n+1 policyholders

This section intends to assess the willingness to pay of the new generation that enters a more populated portfolio of insurance policies, in which the intergenerational effects are more accentuated. This section explores, in a more general way, the impact of larger portfolios on the fair pricing and willingness to pay of new generations joining the insurance portfolio. In particular, we assume that the company takes into account the latest information on the interest rate environment to design the insurance guarantee and the risk management strategy to merge the new generation with the outstanding insurance portfolio. We also assume that policyholders, at the same time, made their decisions by taking into account the latest available information about the financial market environment. Hence, new generations make more informed choices than previous generations (outstanding portfolio).

Tables 8 and9 correspond to the recursive constitution of the fair insurance portfolio. A comparison of the possible dynamic investment strategies, and the equity contributions, such that the entire insurance portfolio is fairly priced in the case of a level guarantee to all generations, or a fair guarantee for the joining generation, is presented. A comparison of the differences in the WTP of the new generations of policyholders, according to the contract features and portfolio composition, is also made.

Table 8: Positive Scenario IR. Recursive Fair Pricing and Willingness to Pay Policyholder n=4.

Same Guarantee

	Portfolio		PH_1	PH_2	PH_3	PH_4
		T_i	18	20		
$r(0)$	5.00%	g_i	1.25%	1.25%		
θ	4.48%	$V_{i,0}$	219.61	0.00		
EC_{ini}	139.23	NPV_i	4.3770	-4.3753		
V_0	219.61	ΔNPV_i	0.34%	-0.39%		
γ_S	19.50%	$E[W^{(I)}]$	-	4178.28		
γ_B	70.00%	$\sigma_{W^{(I)}}$	-	570.53		
EC	42.46	$WTP_{0,i}$	-	0.27		
PD	0.49%	$\mathcal{Q}_{0,i}$	-	(0.27,1)		
NPV_E	-0.0017	$WTP_{0.75,i}$	-	0.285		
ΔNPV_E	-0.001%	$\mathcal{Q}_{0.75,i}$	-	(0.285,1)		
NPV_P	0.0017	$WTP_{0.97,i}$	-	1		
ΔNPV_P	0.000%	$\mathcal{Q}_{0.97,i}$	-	(0,0)		
		T_i	1.250%	1.250%	1.250%	
$r(0)$	4.88%	g_i	773.2311	481.0721	0.000	
θ	4.48%	$V_{i,0}$	14	16	20	
EC_{ini}	288.09	NPV_i	10.1772	1.5326	-10.0884	
V_0	1254.30	ΔNPV_i	0.60%	0.10%	-0.91%	
γ_S	22.00%	$E[W^{(I)}]$	-	-	4,141.99	
γ_B	0.6227146	$\sigma_{W^{(I)}}$	-	-	663.56	
EC	27.28	$WTP_{0,i}$	-	-	0.26	
PD	0.47%	$\mathcal{Q}_{0,i}$	-	-	(0.26,1)	
NPV_E	-1.6215	$WTP_{0.75,i}$	-	-	0.27	
ΔNPV_E	-0.514%	$\mathcal{Q}_{0.75,i}$	-	-	(0.27,1)	
NPV_P	1.6215	$WTP_{0.98,i}$	-	-	1	
ΔNPV_P	0.038%	$\mathcal{Q}_{0.98,i}$	-	-	(0,0)	
		T_i	10	12	16	20
$r(0)$	5.10%	g_i	1.250%	1.250%	1.250%	1.250%
θ	4.48%	$V_{i,0}$	1499.0244	1115.1216	479.3108	0.000
EC_{ini}	470.89	NPV_i	12.087	4.486	-4.853	-11.716
V_0	3093.46	ΔNPV_i	0.539%	0.230%	-0.333%	-1.08%
γ_S	23.22%	$E[W^{(I)}]$	-	-	-	4,155.65
γ_B	60.00%	$\sigma_{W^{(I)}}$	-	-	-	854.54
EC	1.00	$WTP_{0,i}$	-	-	-	0.26
PD	0.42%	$\mathcal{Q}_{0,i}$	-	-	-	(0.26,1)
NPV_E	-0.0035	$WTP_{0.75,i}$	-	-	-	0.265
ΔNPV_E	-0.001%	$\mathcal{Q}_{0.75,i}$	-	-	-	(0.265,1)
NPV_P	0.0036	$\boldsymbol{WTP_{1.01,i}}$	-	-	-	**1**
ΔNPV_P	0.0000	$\boldsymbol{\mathcal{Q}_{1.01,i}}$	-	-	-	$(\mathbf{0},\mathbf{0})$

Fair Guarantee

	Portfolio		PH_1	PH_2	PH_3	PH_4
		T_i	18	20		
$r(0)$	5.00%	g_i	1.25%	1.56%		
θ	4.48%	$V_{i,0}$	219.61	0.00		
EC_{ini}	139.23	NPV_i	0.0001	-0.0006		
V_0	219.61	ΔNPV_i	0.00%	0.00%		
γ_S	18.68%	$E[W^{(I)}]$	-	4224.70		
γ_B	70.00%	$\sigma_{W^{(I)}}$	-	470.68		
EC	71.95	$WTP_{0,i}$	-	0.285		
PD	0.31%	$\mathcal{Q}_{0,i}$	-	(0.285,1)		
NPV_E	0.0004	$WTP_{0.75,i}$	-	0.31		
ΔNPV_E	0.000%	$\mathcal{Q}_{0.75,i}$	-	(0.31,1)		
NPV_P	-0.0005	$WTP_{0.96,i}$	-	1.000		
ΔNPV_P	0.000%	$\mathcal{Q}_{0.96,i}$	-	(0,0)		
		T_i	14	16	20	
$r(0)$	4.48%	g_i	1.25%	1.56%	2.06%	
θ	4.88%	$V_{i,0}$	773.2311	481.0721	0.00	
EC_{ini}	288.09	NPV_i	-0.5860	0.7705	-0.1696	
V_0	1254.30	ΔNPV_i	-0.03%	0.05%	-0.01%	
γ_S	18.06%	$E[W^{(I)}]$	-	-	4,152.26	
γ_B	69.04%	$\sigma_{W^{(I)}}$	-	-	476.98	
EC	53.93	$WTP_{0,i}$	-	-	0.265	
PD		$\mathcal{Q}_{0,i}$	-	-	(0.265,1)	
NPV_E	-0.0149	$WTP_{0.75,i}$	-	-	0.285	
ΔNPV_E	-0.004%	$\mathcal{Q}_{0.75,i}$	-	-	(0.285,1)	
NPV_P	0.0149	$WTP_{0.965,i}$	-	-	1	
ΔNPV_P	0.000%	$\mathcal{Q}_{0.965,i}$	-	-	(0,0)	
		T_i	10	12	16	20
$r(0)$	5.10%	g_i	1.25%	1.56%	2.06%	2.72%
θ	4.48%	$V_{i,0}$	1499.0244	1115.1216	479.3108	0.0000
EC_{ini}	470.89	NPV_i	0.1871	-0.6339	-0.8377	1.2854
V_0	3093.46	ΔNPV_i	0.01%	-0.03%	-0.06%	0.11%
γ_S	11.02%	$E[W^{(I)}]$	-	-	-	4,033.54
γ_B	87.42%	$\sigma_{W^{(I)}}$	-	-	-	363.31
EC	17.00	$WTP_{0,i}$	-	-	-	0.225
PD		$\mathcal{Q}_{0,i}$	-	-	-	(0.225,1)
NPV_E	-0.0010	$WTP_{0.75,i}$	-	-	-	0.25
ΔNPV_E	0.000%	$\mathcal{Q}_{0.75,i}$	-	-	-	(0.25,1)
NPV_P	0.0010	$\boldsymbol{WTP_{0.97,i}}$	-	-	-	**1**
ΔNPV_P	0.0000	$\boldsymbol{\mathcal{Q}_{0.97,i}}$	-	-	-	$(\mathbf{0},\mathbf{0})$

PD: Probability of default. $Pr[\xi < \mathcal{T}_{max}]$.
NPV: Net Present Value.
WTP: Willingness To Pay.

Table 9: Negative Scenario IR. Recursive Fair Pricing and Willingness to Pay Policyholder n=4.

Same Guarantee

	Portfolio		PH_1	PH_2	PH_3	PH_4
		T_i	18	20		
$r(0)$	1.00%	g_i	-0.625%	-0.625%		
θ	2.48%	$V_{i,0}$	210.0504	0		
EC_{ini}	145.3193	NPV_i	6.8441	-6.8442		
V_0	210.0504	ΔNPV_i	0.45%	-0.49%		
γ_S	9.982%	$E[W^{(I)}]$	-	3,077.88		
γ_B	70.759%	$\sigma_{W^{(I)}}$	-	353.89		
EC	16.00	$WTP_{0,i}$	-	0.16		
PD	0.46%	$\mathcal{Q}_{0,i}$	-	(0.16,1)		
NPV_E	0.0002	$WTP_{0.75,i}$	-	0.17		
ΔNPV_E	0.000%	$\mathcal{Q}_{0.75,i}$	-	(0.17,1)		
NPV_P	-0.0001	$WTP_{1.025,i}$	-	1		
ΔNPV_P	0.000%	$\mathcal{Q}_{1.025,i}$	-	(0,0)		
		T_i	14	16	20	
$r(0)$	0.88%	g_i	-0.625%	-0.625%	-0.625%	
θ	2.48%	$V_{i,0}$	686.8593	443.1928	0	
EC_{ini}	217.9451	NPV_i	14.5526	2.4668	-16.6600	
V_0	1130.0521	ΔNPV_i	0.82%	0.15%	-1.24%	
γ_S	13.679%	$E[W^{(I)}]$	-	-	3,038.28	
γ_B	60.000%	$\sigma_{W^{(I)}}$	-	-	533.68	
EC	16.00	$WTP_{0,i}$	-	-	0.145	
PD	0.46%	$\mathcal{Q}_{0,i}$	-	-	(0.145,1)	
NPV_E	-0.3594	$WTP_{0.75,i}$	-	-	0.15	
ΔNPV_E	-0.154%	$\mathcal{Q}_{0.75,i}$	-	-	(0.15,1)	
NPV_P	0.3594	$WTP_{1.105,i}$	-	-	1	
ΔNPV_P	0.008%	$\mathcal{Q}_{1.105,i}$	-	-	(0,0)	
		T_i	10	12	16	20
$r(0)$	1.10%	g_i	-0.625%	-0.625%	-0.625%	-0.625%
θ	2.48%	$V_{i,0}$	1228.6777	951.802	441.7728	0
EC_{ini}	391.0094	NPV_i	22.17	10.38	-8.60	-23.95
V_0	2622.2525	ΔNPV_i	1.06%	0.54%	-0.52%	-1.72%
γ_S	13.753%	$E[W^{(I)}]$	-	-	-	3,147.39
γ_B	60.000%	$\sigma_{W^{(I)}}$	-	-	-	508.00
EC	0.01	$WTP_{0,i}$	-	-	-	0.18
PD	0.20%	$\mathcal{Q}_{0,i}$	-	-	-	(0.18,1)
NPV_E	-0.0005	$WTP_{0.75,i}$	-	-	-	0.185
ΔNPV_E	0.000%	$\mathcal{Q}_{0.75,i}$	-	-	-	(0.185,1)
NPV_P	0.0005	$WTP_{1.045,i}$	-	-	-	1
ΔNPV_P	0.000%	$\mathcal{Q}_{1.045,i}$	-	-	-	(0,0)

Fair Guarantee

	Portfolio		PH_1	PH_2	PH_3	PH_4
		T_i	18	20		
$r(0)$	1.00%	g_i	-0.625%	-0.089%		
θ	2.48%	$V_{i,0}$	210.0504	0.00		
EC_{ini}	145.3193	NPV_i	0.0032	-0.0022		
V_0	38.0000	ΔNPV_i	0.00%	0.00%		
γ_S	7.982%	$E[W^{(I)}]$	-	3,088.48		
γ_B	70.938%	$\sigma_{W^{(I)}}$	-	249.30		
EC	48.61	$WTP_{0,i}$	-	0.165		
PD	0.31%	$\mathcal{Q}_{0,i}$	-	(0.165,1)		
NPV_E	-0.0010	$WTP_{0.75,i}$	-	0.18		
ΔNPV_E	0.000%	$\mathcal{Q}_{0.75,i}$	-	(0.18,1)		
NPV_P	0.0010	$WTP_{1.01,i}$	-	1		
ΔNPV_P	0.000%	$\mathcal{Q}_{1.01,i}$	-	(0,0)		
		T_i	14.000	16.000	20.000	
$r(0)$	0.88%	g_i	-0.625%	-0.089%	0.743%	
θ	2.48%	$V_{i,0}$	686.8593	443.1928	0.0000	
EC_{ini}	217.9451	NPV_i	-1.384	1.292	-0.376	
V_0	1130.0521	ΔNPV_i	-0.08%	0.08%	-0.03%	
γ_S	7.445%	$E[W^{(I)}]$	-	-	3,041.14	
γ_B	64.738%	$\sigma_{W^{(I)}}$	-	-	267.82	
EC	136.96	$WTP_{0,i}$	-	-	0.15	
PD		$\mathcal{Q}_{0,i}$	-	-	(0.15,1)	
NPV_E	0.4679	$WTP_{0.75,i}$	-	-	0.165	
ΔNPV_E	0.132%	$\mathcal{Q}_{0.75,i}$	-	-	(0.165,1)	
NPV_P	-0.4679	$WTP_{1.015,i}$	-	-	1	
ΔNPV_P	-0.009%	$\mathcal{Q}_{1.015,i}$	-	-	(0,0)	
		T_i	10	12	16	20
$r(0)$	1.10%	g_i	-0.63%	-0.09%	0.67%	1.28%
θ	2.48%	$V_{i,0}$	1228.6777	951.802	441.7728	0
EC_{ini}	391.0094	NPV_i	-2.4391	0.1863	4.1073	-2.1742
V_0	2622.2525	ΔNPV_i	-0.12%	0.01%	0.24%	-0.15%
γ_S	0.01977709	$E[W^{(I)}]$	-	-	-	2,921.86
γ_B	0.67146776	$\sigma_{W^{(I)}}$	-	-	-	268.77
EC	16.00	$WTP_{0,i}$	-	-	-	0.105
PD	0.44%	$\mathcal{Q}_{0,i}$	-	-	-	(0.105,1)
NPV_E	0.3197	$WTP_{0.75,i}$	-	-	-	0.12
ΔNPV_E	0.079%	$\mathcal{Q}_{0.75,i}$	-	-	-	(0.12,1)
NPV_P	-0.3197	$WTP_{1.03,i}$	-	-	-	1
ΔNPV_P	-0.004%	$\mathcal{Q}_{1.03,i}$	-	-	-	(0,0)

PD: Probability of default. $Pr[\xi < \mathcal{T}_{max}]$.
NPV: Net Present Value.
WTP: Willingness To Pay.

The interactions among generations becomes rather complex as the number of participating generations increases. If all generations that join the insurance portfolio earn the same guarantee level, the wealth transfer among generations is unavoidable. The wealth transfer also increases as the size of the portfolio increases. The new generation joining the insurance portfolio is overpriced under the apriori assumption that no future generations would join the insurance portfolio (i.e., outstanding portfolio size n at τ_{n+1} then $NPV_{n+1} < 0$). In contrast, older generations seem to improve their NPV position as the size of the portfolio increases.

The generations with a shorter time-to-maturity at τ_{n+1} have the highest NPV, while the generations that are expected to last longer in the portfolio have the lowest NPV. In general, it is possible to fairly price the new generation by taking into consideration the outstanding portfolio. A wealth transfer between the outstanding portfolio and the joining generation could be avoided (diminish) at the issuance date by adjusting the guarantee level and the risk management strategies. Tables 8 and 9 suggest that the fair guarantee offered to the new joining generation is expected to have the longest time-to-maturity; it is also higher than the guarantee of the outstanding portfolio.

Some wealth transfers might occur among the older generations part of the outstanding portfolio. The wealth transfer could be explained by the changes, or dynamics, in the financial scenarios (i.e., different investment opportunities at the issuance of the contract), as well as uncertain interactions with the new generations that were omitted at the issuance date. Additionally, periodic premium payments imply new asset acquisitions under updated market conditions, which usually differ from the pricing assumptions at the issuance date. Hence, fair pricing cannot be assured during the entire life of the insurance policy, but typically only during its issuance date. The dynamics in the portfolio imply that managers should continuously revise their investment strategies and actively adapt them. Active risk management results are then essential to account for new interactions among generations to minimize the wealth transfer.

The WTP of the new generation decreases as the size of the portfolio increases. The decrease in the customers WTP for insurance can be observed when comparing the a'_{n+1} of the new generation with the a'_n of the previous generation for the case of similar and fair guarantees (last row of WTP for each generation). For example, in the case of a positive interest rate (respectively, negative interest rates) with fair guarantees, customers joining a portfolio of size $(n+1) = 3$ would have a risk aversion coefficient that is higher than 0.965 (respectively, 1.015). In contrast, four years later, the customers joining the insurance portfolio would be more risk averse by having a risk-aversion coefficient that is higher than 0.97 (respectively, 1.03).The decrease in the new generations WTP is explained by the reduction in the risky share of the investments. Larger portfolios require more conservative investment strategies, given that the insurer must cope with the promises to the previous generations. The insurer must also avoid wealth transfers between the shareholders and the policyholders. In general, this implies offering higher guarantees to the new generations.

4 SUMMARY

This paper has developed a model to price insurance portfolios in a voluntary framework. Because of the voluntary features of the private insurance portfolios (e.g., third pillar in Switzerland), the portfolio size is unknown a priori. We build the insurance portfolio recursively, as proposed in Hieber et al. (2016).In general, portfolio sizes in voluntary schemes are determined by the conditions in the financial market. Additionally, insurance companies allocated a higher proportion of their investments to very safe investments, a decision that may constrain the fair pricing of the insurance portfolio, as well as influence the new customer's decision concerning purchasing or not purchasing the insurance coverage.

The interest rate levels would set the feasible minimum interest rate guarantees to offer to new generations, adequate investment strategies and equity contributions that minimize the wealth transfer among generations in the insurance portfolio. In particular, the fair pricing of the insurance portfolios of participating life insurance policies, in a voluntary framework, requires the implementation of continuous risk management controls. Once the contract conditions are established (i.e., the guarantee level and the interaction with the outstanding portfolio), it is possible to evaluate the policyholder's willingness to participate in the insurance portfolio.

The w ealth transfer among subsequent generations, with the same guarantee level, can be minimized. In most cases, this wealth transfer is unavoidable. The amount of the wealth transfer would depend on the interest rate scenario and the investment strategy. Fair prices lead to proposed higher guarantees to the entering generation. The new generation's fair guarantees are higher than the fair guarantees of the older generations (outstanding portfolio) in a scenario of stable non-decreasing interest rates.

In the case that the insurer decides to offer fair guarantees, the new informational assumption plays a key role in determining the fair guarantees and the new generation's willingness to join the insurance portfolio. Policyholders with a risk-aversion coefficient of greater than 0.98 would, in general, be part of a portfolio of insurance policies in the scenario of positive interest rates. On the contrary, risk-averse policyholders with a risk-aversion coefficient of $a > 1.00$would compose the insurance portfolio under a scenario of negative interest rates. Portfolios offering the same guarantee level to all participants could also be offered and allow the insurer to adopt riskier investment strategies. However, only their very risk-averse customers would join the portfolio under unfair conditions.

Appendix

Stochastic interest rate from real-world measure to risk-neutral measure

$$\begin{aligned}
dr(t) &= \kappa(\theta - r(t))dt - \sigma_r \sqrt{k_1 r(t) + k_2} dW_r(t) \\
&= \kappa(\theta - r(t))dt - \sigma_r \sqrt{k_1 r(t) + k_2} (dW_r^Q(t) - \lambda_r \sqrt{k_1 r(t) + k2} dt) \\
&= \kappa(\theta - r(t))dt - \sigma_r \sqrt{k_1 r(t) + k_2} dW_r^Q(t) + \sigma_r \lambda_r (k_1 r(t) + k_2)dt \\
&= \kappa\theta dt - \kappa r(t)dt + \sigma_r \lambda_r k_1 r(t)dt + \sigma_r \lambda_r k_2 dt - \sigma_r \sqrt{k_1 r(t) + k_2} dW_r^Q(t) \\
&= (\kappa\theta + \sigma_r \lambda_r k_2)dt - (\kappa - \sigma_r \lambda_r k_1) r(t)dt - \sigma_r \sqrt{k_1 r(t) + k_2} dW_r^Q(t) \\
&= (\kappa - \sigma_r \lambda_r k_1) \left[\frac{\kappa\theta + \sigma_r \lambda_r k_2}{(\kappa - \sigma_r \lambda_r k_1)} - r(t) \right] dt - \sigma_r \sqrt{k_1 r(t) + k_2} dW_r^Q(t)
\end{aligned}$$

References

BajeuxBesnainou, I., Jordan, J. V., & Portait, R. (2003). Dynamic Asset Allocation for Stocks, Bonds, and Cash. *The Journal of Business*, *76*(2), 263–287.

Braun, A., Fischer, M., & Schmeiser, H. (2015). How to Derive Optimal Guarantee Levels in Participating Life Insurance Contracts. *I.VW-HSG Working Paper, University of St. Gallen*.

Butsic, R. P. (1994). Solvency Measurement for Property-Liability Risk-Based Capital Applications. *The Journal of Risk and Insurance*, *61*(4), 656–690.

Doherty, N. A., & Garven, J. R. (1986). Price Regulation in Property-Liability Insurance: A Contingent-Claims Approach. *The Journal of Finance*, *41*(5), 1031–1050.

Gao, J. (2008). Stochastic Optimal Control of DC Pension Funds. *Insurance: Mathematics and Economics*, *42*(3), 1159–1164.

Gerstner, T., Griebel, M., Holtz, M., Goschnick, R., & Haep, M. (2008). A General Asset-Liability Management Model for the Efficient Simulation of Portfolios of Life Insurance Policies. *Insurance: Mathematics and Economics*, *42*(2), 704–716.

Grosen, A., & Jørgensen, P. L. (2002). Life Insurance Liabilities at Market Value: An Analysis of Insolvency Risk, Bonus Policy, and Regulatory Intervention Rules in a Barrier Option Framework. *Journal of Risk and Insurance*, *69*(1), 63–91.

Hieber, P., Natolski, J., & Werner, R. (2016). Fair Valuation of Cliquet-Style Return Guarantees in (Homogeneous and) Heterogeneous Life Insurance Portfolios.
URL `https://ssrn.com/abstract=2715571`

Ibragimov, R., Jaffee, D., & Walden, J. (2010). Pricing and Capital Allocation for Multiline Insurance Firms. *Journal of Risk and Insurance*, *77*(3), 551–578.

Orozco-Garcia, C., & Schmeiser, H. (2017). Is Fair Pricing Possible? An Analysis of Participating Life Insurance Portfolios. *Journal of Risk and Insurance*.

Saha, A. (1997). Risk Preference Estimation in the Nonlinear Mean Standard Deviation Approach. *Economic Inquiry*, *35*(4), 770–782.

Zaglauer, K., & Bauer, D. (2008). Risk-Neutral Valuation of Participating Life Insurance Contracts in a Stochastic Interest Rate Environment. *Insurance: Mathematics and Economics*, *43*(1), 29–40.