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The Merits of Pooling Claims: Mutual vs. Stock Insurer

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Structure

1. Motivation (I)

Text books and lectures in insurance theory typically start with risk pooling as the «production law» of the business model (diversification of unsystematic risk)

For instance, for i.i.d. risks we have

A) For a fixed (and small) ruin probability, the premium for each risk decreases and reaches the expected claim for n $\rightarrow$

B) For a fixed "unfair" premium, the ruin probability reaches 0 for n $\rightarrow$

Cf., e.g., Cummins (1991); Albrecht (1982, 1990); Smith and Kane (1994)

1. Motivation (II)

Modeling setup may be misleading from the economic point of view: Framework can give the impression that …

… risk pooling generates an added value for the policyholder

… that the premium level (or the ruin probability) plays an important role

Concept is to some extend incomplete:

If a mutual is considered, the policyholder's owner stake and the recovery option must be taken into account

If a shareholder company is considered, the setup allows arbitrage opportunities

The fulfillment of A) or B) is not a necessary or sufficient condition that pooling is beneficial for a policyholder

Structure

1. **Motivation**
2. **Introduction and research question**
3. **Model description**
4. **Numerical illustration**
5. **Conclusion**

2. Introduction and research question

Under which conditions is risk pooling beneficial for policyholders in the context of mutual insurer?

Cf. Gatzert and Schmeiser (2012); Albrecht and Huggenberger (2017)

Aim of this paper:

Compare the utility gain of risk pooling in mutual and shareholder driven companies

Identify conditions under which a policyholder is indifferent between purchasing an insurance contract with a mutual or a shareholder insurance company

Major finding: Risk-averse policyholder accepts an unfair premium in a shareholder insurance company to obtain the same utility level provided by the mutual insurance company (even if the underwriting risk is of pure unsystematic nature)

Related literature: Cf. working paper

Structure

3. Model description: Pooling claims

The pool's claims distribution consists of n i.i.d. risks with distribution X:

$$S = \sum_{i=1}^{n} X_i$$

The premium payment π_i is fair if it equals the policyholder's expected payoffs in t = 1

We assume an incomplete market setting in which policyholders are not able to replicate future cash flows

Policyholders value their wealth position W based on exponential utility function

$$U(W) = -e^{-a(W+K)}$$

(Other utility functions are introduced later)

3. Model description: Case of mutual insurer (I)

Policyholder's wealth position at t = 1:

$$W_i^M = A_i(1+r) - X_i - \pi_{i,M}(1+r) + I_{i,M} + E_{i,M}$$

In what follows we set r = 0

The debtholder stake is given by

$$I_{i,M} = X_i - \frac{1}{n} max(S - \pi_M, 0)$$

Note: $I_{i,M}$ can be positive or negative (because of the recovery option)

and the owner's position is

$$E_{i,M} = \frac{1}{n} max(\pi_M - S, 0)$$

3. Model description: Case of mutual insurer (II)

In any case, we have

$$
\begin{aligned}
E(I_{i,M} + E_{i,M}) &= E\left(X_i - \frac{1}{n}max(S - \pi_M, 0) + \frac{1}{n}max(\pi_M - S, 0)\right) \\
&= E\left(X_i + \frac{1}{n}(\pi_M - S)\right) \\
&= E\left(X_i + \frac{1}{n}(n\pi_{i,M} - S)\right) \\
&= \pi_{i,M}
\end{aligned}
$$

In this sense, any premium level in the mutual company is «fair» with respect to our definition

However, the premium is in general not equal to the expected value of the claims

3. Model description: Case of mutual insurer (III)

The expected utility of policyholder i does not depend on the premium level and policyholders are identically treated (we assume that credit risk plays no role)

For instance, the two central moments of the wealth distribution are not influenced by π_i:

$$E(W_i^M) = A_i - E(X_i) - \pi_{i,M} + E(I_{i,M} + E_{i,M}) = A_i - E(X_i)$$

$$\begin{aligned}
\sigma^2(W_i^M) &= \sigma^2(-X_i + I_{i,M} + E_{i,M}) = \sigma^2\left(-X_i + X_i + \frac{1}{n}(\pi_M - S)\right) \\
&= \sigma^2\left(\frac{1}{n}(\pi_M - S)\right) \\
&= \sigma^2\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2(X_i) = \frac{1}{n}\sigma^2(X_i)
\end{aligned}$$

3. Model description: Case of a shareholder insurer (I)

Policyholder's wealth position at t = 1

$$W_i^S = A_i(1+r) - X_i - \pi_{i,S}(1+r) + I_{i,S}$$

Again, we set in what follows r = 0

The debt holder position is

$$I_{i,S} = B_{i,S} \cdot \left(X_i - \frac{1}{n} max\left(\mathcal{S} - \pi_S - EC_0, 0\right)\right)$$

$$B_{i,S} = \begin{cases} 1 & \text{if } \frac{1}{n}\left(\mathcal{S} - \pi_S - EC_0\right) \leq X_i \\ 0 & \text{otherwise} \end{cases}$$

Note: Shareholders and policyholders enjoy limited liability

3. Model description: Case of a shareholder insurer (II)

The payoff to the shareholders is given by

$$E_S = max(EC_0 + \pi_S - S, 0)$$

The expected wealth position equals the one in the mutual only if fair pricing takes place

The policyholder's expected utility depends on

the premium setting and

the amount of equity capital of the insurer

3. Model description: Summary

Policyholder's wealth position in the two company forms:

$$W_i^M = A_i - \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$W_i^S = A_i - X_i - \pi_{i,S} + B_{i,S}\left[X_i - \max\left(\frac{1}{n}\sum_{i=1}^{n} X_i - \pi_{i,S} - \frac{1}{n}EC_0, 0\right)\right]$$

Closed form solutions can only be provided if the sum of the individual claims are normally distributed

Hence, in general, approximations must be applied

3. Model description: Initial considerations (I)

Assumptions:

(a) Fair pricing

$\pi_{i,M} = E[I_{i,M} + E_{i,M}]$ $\qquad$ $\pi_{i,S} = E(I_{i,S})$

Note: In the mutual, any premium setting is "fair" according to our definition

(b) Equal default probabilities ε

Note that the consequences of a default situation are different in the two legal forms

Hence, we have

$\pi_{i,M} = \pi_{i,S} + EC_{0,i}$ $\qquad$ $EC_{0,i} = EC_0/n$

3. Model description: Initial considerations (II)

Differences in the distribution of W?

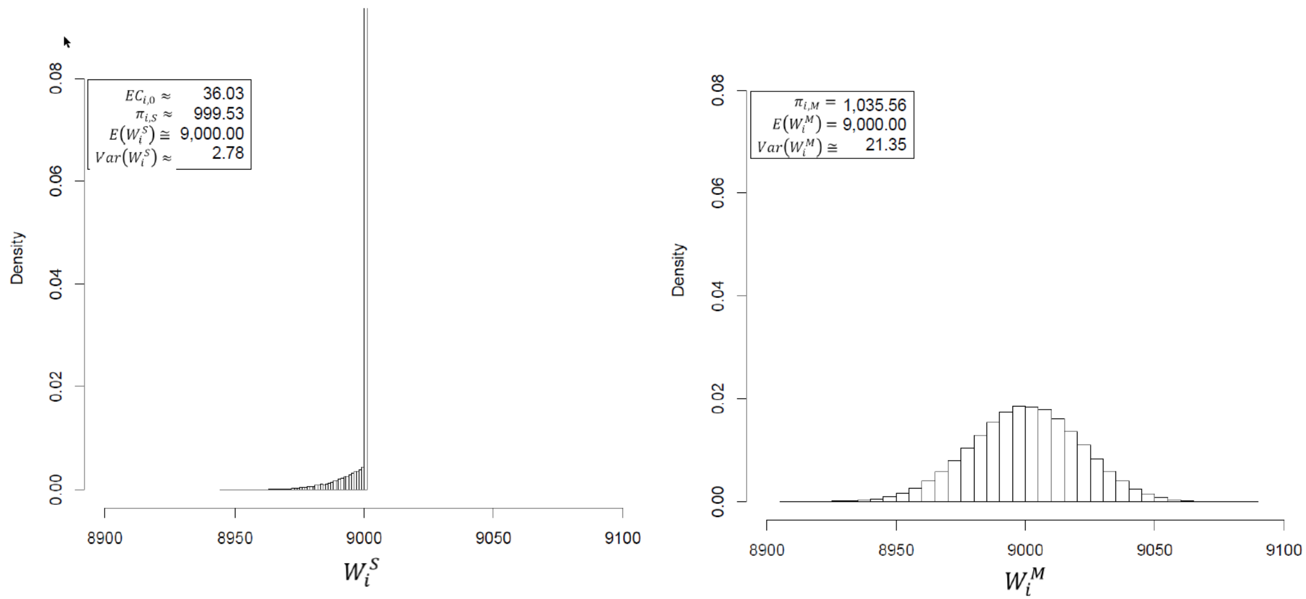

Figure 1: Comparison of the distribution of the policyholder's wealth at time $t = 1$ for the case of the shareholder insurance company W_i^S (left) and mutual insurance company W_i^M (right). Please note that $n = 1,000$, $E[X_i] = 1,000$, $\sigma(X_i) = 675$, $A_i = 10,000$ and $\varepsilon = 5\%$.

3. Model description: Initial considerations (III)

Differences regarding the policyholder's utility?

	Mutual Company				Shareholder Company		
n	$\pi_{i,M}$	$\Phi(W_i^M)$	$\sigma(W_i^M)$	$EC_{0,i}$	$\pi_{i,S}$	$\Phi(W_i^S)$	$\sigma(W_i^S)$
			$\varepsilon = 5\%$				
100	1,114.94	-10,782,793.5648	67.50	116.59	998.35	-1,719.4769	9.84
1,000	1,035.56	-3.7922	21.35	36.03	999.53	-1.0466	2.78
10,000	1,011.76	-1.1537	6.75	11.91	999.85	-1.0027	0.89
100,000	1,004.97	-1.0261	2.13	5.04	999.94	-1.0004	0.38
			$\varepsilon = 1\%$				
100	1,168.08	-10,782,793.5648	67.50	168.36	999.72	-36.3435	3.90
1,000	1,050.75	-3.7922	21.35	50.83	999.92	-1.0059	1.07
10,000	1,016.72	-1.1537	6.75	16.75	999.98	-1.0004	0.33
100,000	1,007.07	-1.0261	2.13	7.08	999.99	-1.0001	0.14
			$\varepsilon = 0.5\%$				
100	1,188.68	-10,782,793.5648	67.50	188.82	999.86	-8.6113	2.64
1,000	1,056.52	-3.7922	21.35	56.56	999.96	-1.0024	0.71
10,000	1,018.49	-1.1537	6.75	18.51	999.99	-1.0002	0.22
100,000	1,007.84	-1.0261	2.13	7.85	1,000.00	-1.0000	0.09

3. Model description: Initial considerations (IV)

First results:

A risk-averse policyholder ($a > 0$) prefers under identical conditions in respect to fair pricing, default probability and pool size of the shareholder company over the mutual

Result is stable for different distributional assumptions (log-normal, gamma, Pareto)

Result is stable for three different utility functions (exponential, downside variance, power utility)

Would rational shareholders provide the necessary equity capital?

Yes, if they are risk-neutral or we have a risk neutral market in respect to insurance risk (i.i.d. risk)

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4. Numerical illustration: Overview

Next step: Derive conditions under which a policyholder is indifferent between the two legal forms

In formal terms:

$$\Phi\Big(W_i^M(n_M)\Big) = \Phi\Big(W_i^S(n_S, \pi_{i,S}, EC_{0,i})\Big)$$

Necessary to explain the existence of the two legal forms in the long run

Three Cases are considered:

Case A – Fixed default probability

Case B – Fixed pool size

Case C – Fixed pool size and default probability

Sensitivity of the results is tested for different utility functions and claim distributions

4. Numerical illustration: Case C (I)

Fixed pool size n *and* fixed default probability ε

	Mutual Company			Shareholder Company				
n	$\pi_{i,M}$	$\Phi(W_i^M)$	$\sigma(W_i^M)$	$EC_{0,i}$	$\pi_{i,S}$	$\Phi(W_i^S)$	$\sigma(W_i^S)$	$E[I_{i,S}]$
$\varepsilon = 5\%$								
100	1,114.94	-10,782,793.5648	67.50	0.01	1,114.93	-10,782,793.5648	9.84	998.35
1,000	1,035.56	-3.7922	21.35	18.87	1,016.70	-3.7922	2.78	999.53
10,000	1,011.76	-1.1537	6.75	10.04	1,001.72	-1.1537	0.89	999.85
100,000	1,004.97	-1.0261	2.13	4.70	1,000.27	-1.0261	0.38	999.94
$\varepsilon = 1\%$								
100	1,168.08	-10,782,793.5648	67.50	0.358	1,167.72	-10,782,793.5648	3.90	999.72
1,000	1,050.75	-3.7922	21.35	33.134	1,017.61	-3.7922	1.07	999.92
10,000	1,016.72	-1.1537	6.75	14.846	1,001.88	-1.1537	0.33	999.98
100,000	1,007.07	-1.0261	2.13	6.742	1,000.33	-1.0261	0.14	999.99
$\varepsilon = 0.5\%$								
100	1,188.68	-10,782,793.5648	67.50	1.613	1,187.07	-10,782,793.5648	2.64	999.86
1,000	1,056.52	-3.7922	21.35	38.822	1,017.70	-3.7922	0.71	999.96
10,000	1,018.49	-1.1537	6.75	16.601	1,001.89	-1.1537	0.22	999.99
100,000	1,007.84	-1.0261	2.13	7.504	1,000.34	-1.0261	0.09	1,000.00

Data cf. chapter 3

Structure

5. Conculsion

The classical risk pooling framework must be specified in several ways in order to explain the rationality of pooling claims from the policyholder's perspective – and the business model of insurance companies per se

In this paper, we compare the merits of pooling claims for mutual and stock insurance companies

At first glance and under the assumptions taken (in particular fair pricing), shareholder companies seem to offer higher utility levels to their policyholders than mutual insurer

However, we must assume that in competitive markets, parameter conditions in respect to premium and equity contributions take place that lead to (approximately) the same utility levels for both legal forms

If utility levels are the same and n and ε are fixed, the premium level in the shareholder company is below the mutual, but exceeds the fair premium