A Note on the Merits of Pooling Claims

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1. Introduction

- Risk pooling – "product law" of insurance
  - Conveys the impression that risk pooling does generate an additional value for the policyholders
  - More precisely, premiums decrease / safety level increases
  - Aim is to extend and combine previous work by focusing on the merits of pooling from the policyholder's perspective

- Result
2. Pooling Claims: The Base Case

- Total claim amount

\[ S = \sum_{i=1}^{n} X_i \]

- Premium of risk i

\[ \pi_i = E(X_i) + c \]

- Safety loading \( c > 0 \)
Merits of Pooling Claims

- **Case A: Fixed ruin probability**

\[
R = P(S > \pi) = \varepsilon \iff P(S > E(S) + nc(n)) = \varepsilon
\]

\[
R = 1 - N\left(\frac{E(S) + nc(n) - E(S)}{\sigma(S)}\right) = 1 - N\left(\frac{c(n)}{\sigma(X_i)} \cdot \sqrt{n}\right) = \varepsilon
\]

\[
\frac{c}{\sigma(X_i)} \cdot \sqrt{n} = z_{1-\varepsilon} \iff c = \frac{z_{1-\varepsilon} \cdot \sigma(X_i)}{\sqrt{n}}
\]

- Interpretation: Fixed safety level, premiums decrease
Merits of Pooling Claims

- **Case B: Fixed premium**

\[ R = 1 - N \left( \frac{E(S) + nc - E(S)}{\sigma(S)} \right) = 1 - N \left( \frac{c}{\sigma(X_i)} \cdot \sqrt{n} \right) \rightarrow 0 \]

- Interpretation: Fixed premium, safety level increases

- Summary (at first glance):

  Policyholder seem to benefit from pooling claims

  Premiums seem to play a role
3. Policyholder's Point of View

- Wealth position (with $r = 0$) with / without risk pooling

\[ W_i = A_i - X_i - \pi_i + I_i + E_i \]

1) Frictionless and efficient market

- Debtholder position

\[ PV(I_i) = PV(X_i) - \frac{1}{n} PV(\max[S - \pi, 0]) \]

- Equityholder position

\[ PV(E_i) = \frac{1}{n} PV(\max[\pi - S, 0]) \]
Merits of Pooling Claims

- Fair premium calculation

\[ PV(I_i + E_i) = PV(I_i) + PV(E_i) \]
\[ = PV(X_i) - \frac{1}{n} PV(\max[S - \pi, 0]) + \frac{1}{n} PV(\max[\pi - S, 0]) \]
\[ = PV(X_i) + \frac{1}{n} PV(\pi - S) \]
\[ = PV(X_i) + \pi_i - \frac{1}{n} \sum_{i=1}^{n} PV(X_i) \]
\[ = \pi_i. \]

- Fulfilled for all "fair" premium principles

\( \pi_i = E(X_i) + c \) \( c \in \mathbb{R} \)

- Hence, whether risk pooling in the sense of Case A or B is fulfilled or not is of no importance. No additional value can be created from the diversification of unsystematic risk
Merits of Pooling Claims

- Same holds true for risk-neutral policyholders

- Example

Table 1: Premiums $\pi_i$ and payouts ($I_i + E_i$) for the case of pooling claims for a given ruin probability of 1%; $E(X_i) = 30$ and $\sigma(X_i) = 10$; risk neutral market

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>1000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_i$</td>
<td>53.32</td>
<td>37.39</td>
<td>33.28</td>
<td>32.26</td>
<td>30.74</td>
<td>30.23</td>
</tr>
<tr>
<td>$c(n)$</td>
<td>23.32</td>
<td>7.39</td>
<td>3.28</td>
<td>2.36</td>
<td>0.74</td>
<td>0.23</td>
</tr>
<tr>
<td>$I_i$</td>
<td>29.97</td>
<td>29.99</td>
<td>29.99</td>
<td>30.00</td>
<td>30.00</td>
<td>30.00</td>
</tr>
<tr>
<td>$E_i$</td>
<td>23.36</td>
<td>7.40</td>
<td>3.29</td>
<td>2.36</td>
<td>0.74</td>
<td>0.23</td>
</tr>
<tr>
<td>$I_i + E_i$</td>
<td>53.32</td>
<td>37.39</td>
<td>33.28</td>
<td>32.26</td>
<td>30.74</td>
<td>30.23</td>
</tr>
</tbody>
</table>
Table 2: Premiums $\pi_i$ and payouts ($I_i + E_i$) for the case of pooling claims for a fixed premium level per of $\pi_i = 29.00$ (*Case B*).

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>1000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_i$</td>
<td>29.00</td>
<td>29.00</td>
<td>29.00</td>
<td>29.00</td>
<td>29.00</td>
<td>29.00</td>
</tr>
<tr>
<td>$c$</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
</tr>
<tr>
<td>$R = P(S &gt; \pi)$</td>
<td>53.98%</td>
<td>62.41%</td>
<td>76.02%</td>
<td>84.13%</td>
<td>99.92%</td>
<td>100.00%</td>
</tr>
<tr>
<td>$I_i$</td>
<td>27.02</td>
<td>28.92</td>
<td>29.00</td>
<td>29.00</td>
<td>29.00</td>
<td>29.00</td>
</tr>
<tr>
<td>$E_i$</td>
<td>1.98</td>
<td>0.08</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$I_i + E_i$</td>
<td>29.00</td>
<td>29.00</td>
<td>29.00</td>
<td>29.00</td>
<td>29.00</td>
<td>29.00</td>
</tr>
</tbody>
</table>
2) Risk-averse policyholder

- Hybrid model with \( a > 0 \)

\[
\Phi = E(W_i) - \frac{a}{2} \cdot \sigma^2(W_i)
\]

- Expected wealth independent of \( c \) and \( n \)

- Variance independent of \( c \)

\[
\sigma^2(X_i + I_i + E_i) < \sigma^2(X_i) \quad \text{for } n \geq 2
\]

- Utility increases c.p. with an increasing number of pool members
4. Summary

- Merits of risk pooling in the definition used in chapter 2 under Case A and Case B gives no hint whether risk pooling is beneficial for policyholders or not.

- Used model set up: Valuation of both stakes, debtholder and equityholder position.

- Situation in which risk pooling is beneficial for policyholders is easy to derive.

- However, the use of the definitions of Case A and B is not clear to us.