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A Note on the Merits of Pooling Claims

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1. Introduction

- Risk pooling – "product law" of insurance

- Conveys the impression that risk pooling does generate an additional value for the policyholders

- More precisely, premiums decrease / safety level increa-ses

- Aim is to extend and combine previous work by focusing on the merits of pooling from the policyholder's per-spective

- Result

2. Pooling Claims: The Base Case

- Total claim amount

$$S = \sum_{i=1}^{n} X_i$$

- Premium of risk i

$$\pi_i = E\left(X_i\right) + c$$

- Safety loading c > 0

Merits of Pooling Claims

- Case A: Fixed ruin probability

$$R = P\left(S > \pi\right) \overset{!}{=} \varepsilon \quad \Leftrightarrow \quad P\left(S > E\left(S\right) + nc\left(n\right)\right) \overset{!}{=} \varepsilon$$

$$R = 1 - N\left(\frac{E\left(S\right) + nc\left(n\right) - E\left(S\right)}{\sigma\left(S\right)}\right) = 1 - N\left(\frac{c\left(n\right)}{\sigma\left(X_i\right)} \cdot \sqrt{n}\right) \overset{!}{=} \varepsilon$$

$$\frac{c}{\sigma\left(X_i\right)} \cdot \sqrt{n} = z_{1-\varepsilon} \quad \Leftrightarrow \quad c = \frac{z_{1-\varepsilon} \cdot \sigma\left(X_i\right)}{\sqrt{n}}$$

- Interpretation: Fixed safety level, premiums decrease

Merits of Pooling Claims

- Case B: Fixed premium

$$R = 1 - N\left(\frac{E(S) + nc - E(S)}{\sigma(S)}\right) = 1 - N\left(\frac{c}{\sigma(X_i)} \cdot \sqrt{n}\right) \underset{n\to\infty}{\rightarrow} 0$$

- Interpretation: Fixed premium, safety level increases

- Summary (at first glance):

Policyholder seem to benefit from pooling claims

Premiums seem to play a role

3. Policyholder's Point of View

- Wealth position (with $r = 0$) with / without risk pooling

$$W_i = A_i - X_i - \pi_i + I_i + E_i$$

1) Frictionless and efficient market

- Debtholder position

$$PV(I_i) = PV(X_i) - \frac{1}{n} PV\left(\max\left[S - \pi, 0\right]\right)$$

- Equityholder position

$$PV(E_i) = \frac{1}{n} PV\left(\max\left[\pi - S, 0\right]\right)$$

Merits of Pooling Claims

- Fair premium calculation

$$
\begin{aligned}
PV(I_i + E_i) &= PV(I_i) + PV(E_i) \\
&= PV(X_i) - \frac{1}{n} PV\left(\max[S-\pi, 0]\right) + \frac{1}{n} PV\left(\max[\pi - S, 0]\right) \\
&= PV(X_i) + \frac{1}{n} PV(\pi - S) \\
&= PV(X_i) + \pi_i - \frac{1}{n}\sum_{i=1}^{n} PV(X_i) \\
&= \pi_i .
\end{aligned}
$$

- Fulfilled for all "fair" premium principles

$$\pi_i = E(X_i) + c \qquad c \in \mathbb{R}$$

- Hence, whether risk pooling in the sense of Case A or B is fulfilled or not is of no importance. No additional value can be created from the diversification of unsystematic risk

Merits of Pooling Claims

- Same holds true for risk-neutral policyholders

- Example

Table 1: Premiums π_i and payouts $(I_i + E_i)$ for the case of pooling claims for a given ruin probability of 1%; $E(X_i)$ = 30 and $\sigma(X_i)$ = 10; risk neutral market

n	1	10	50	100	1000	10000
π_i	53.32	37.39	33.28	32.26	30.74	30.23
$c(n)$	23.32	7.39	3.28	2.36	0.74	0.23
I_i	29.97	29.99	29.99	30.00	30.00	30.00
E_i	23.36	7.40	3.29	2.36	0.74	0.23
$I_i + E_i$	53.32	37.39	33.28	32.26	30.74	30.23

Merits of Pooling Claims

- Example

Table 2: Premiums π_i and payouts ($I_i + E_i$) for the case of pooling claims for a fixed premium level per of π_i = 29.00 (*Case B*)

n	1	10	50	100	1000	10000
π_i	29.00	29.00	29.00	29.00	29.00	29.00
c	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
$R = P(S > \pi)$	53.98%	62.41%	76.02%	84.13%	99.92%	100.00%
I_i	27.02	28.92	29.00	29.00	29.00	29.00
E_i	1.98	0.08	0.00	0.00	0.00	0.00
$I_i + E_i$	29.00	29.00	29.00	29.00	29.00	29.00

Merits of Pooling Claims

2) Risk-averse policyholder

- Hybrid model with a > 0

$$\Phi = E\left(W_i\right) - \frac{a}{2} \cdot \sigma^2\left(W_i\right)$$

- Expected wealth independent of c and n

- Variance independent of c

$$\sigma^2\left(X_i + I_i + E_i\right) < \sigma^2\left(X_i\right) \quad \textit{for } n \geq 2$$

- Utility increases c.p. with an increasing number of pool members

4. Summary

- Merits of risk pooling in the definition used in chapter 2 under Case A and Case B gives no hint whether risk pooling is beneficial for policyholders or not
- Used model set up: Valuation of both stakes, debtholder and equityholder position
- Situation in which risk pooling is beneficial for policyhol-ders is easy to derive
- However, the use of the definitions of Case A and B is not clear to us