Institute of Insurance Economics

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Combing Fair Pricing and Capital Requirements for Non-Life Insurance Companies

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1. Introduction

- New solvency capital requirements in Europe (Solvency II, Swiss Solvency Test SST):

  Available economic capital (risk-bearing capital RBC) should exceed the required solvency capital (SC)

- Risk measurement level: solvency capital requirements are derived using the VaR (Solvency II) or the TVaR (SST) concept

- Risk valuation level: competitive market conditions should lead to risk adequate return for the stakeholders (net present value of zero; "fair" pricing)

Fair Pricing and Capital Requirements

- Literature: Fair pricing, capital structure, and solvency requirements are usually studied individually

- Aim of this paper: Combining fair pricing and solvency capital requirements in order to gain a deeper understanding of the effect of solvency regulation on the cost of insolvency

- More precisely: Minimum safety levels are identified using the default put option value for fair conditions that satisfy solvency capital requirements

- Organization of the presentation

2. Model Framework

a) A simplicities model of a non-life insurer (t = 0,1)

- Premiums and equity capital are invested

$$P_0^\tau + E_0^\tau = A_0^\tau$$

- Payoff to the policyholders

$$P_1^\tau = L_1 - max\left(L_1 - A_1^\tau, 0\right) + T_1$$

- Payoff to the shareholders

$$E_1^\tau = max\left(A_1^\tau - L_1, 0\right) - T_1$$

Fair Pricing and Capital Requirements

- Corporate taxes for a tax level τ

$$T_1 = \tau \cdot max\left(\left(E_0^\tau + P_0^\tau\right)\left(\frac{A_1^\tau}{A_0^\tau} - 1\right) + P_0^\tau - L_1, 0\right)$$

b) Risk valuation (under the risk-neutral measure $\mathbf{Q}$)

$$\Pi^P = E^{\mathbf{Q}}\left(exp(-r)P_1^\tau\right) = \Pi^L - \Pi^{DPO} + \Pi^{T_1}$$

$$\Pi^E = E^{\mathbf{Q}}\left(exp(-r)E_1^\tau\right) = E^{\mathbf{Q}}\left(exp(-r)max\left(A_1^\tau - L_1, 0\right)\right) - \Pi^{T_1}$$

- Fair pricing

$$\Pi^P = P_0^\tau$$

$$\Pi^E = E_0^\tau$$

c) Modeling asset and liabilities

- Assets: GBM
- Liabilities: Merton's jump-diffusion-process

$$dA_t = \mu_A A_t dt + \sigma_A A_t dW_{A,t}^{\mathbf{P}}$$

$$\frac{dL_t}{L_{t-}} = \mu_L dt + \sigma_L dW_{L,t}^{\mathbf{P}} + dJ_t \qquad dW_A dW_L = \rho\, dt \qquad J_t = \sum_{j=1}^{N_t} \left(Y_j - 1 \right)$$

- In the Merton case, solutions for the stochastic differential equations can be derived under $\mathbf{P}$ and $\mathbf{Q}$

3. Solvency Capital Requirements

a) Risk-bearing capital RBC and solvency capital SC

$$RBC_t = A_t^{\tau} - L_t$$

$$RBC_0 = A_0^{\tau} - L_0 \geq SC_{\alpha}$$

- Excursion: Fair conditions lead to

$$RBC_0 = \left(E_0^{\tau} + P_0^{\tau}\right) - L_0 = \left(E_0^{\tau} + L_0 - \Pi^{DPO} + \Pi^{T_1}\right) - L_0$$

$$= E_0^{\tau} + \Pi^{T_1} - \Pi^{DPO}$$

b) Swiss Solvency Test SST

- SC is derived using the TVaR concept for α = 1%

$$SC_\alpha = TVaR_\alpha = -E\left(X \middle| X \le VaR_\alpha\right) \qquad X = \exp(-r)\, RBC_1 - RBC_0$$

c) Solvency II

- SC is derived using the VaR concept with α = 0.5%

$$\mathrm{P}\left(X < VaR_\alpha\right) = \alpha \qquad SC_\alpha = -VaR_\alpha$$

d) Shortfall propability SP

$$SP = \mathrm{P}\left(A_1^\tau < L_1\right)$$

4. Numerical Results

- Aim: Identifying fair equity capital / premium combinations that satisfy solvency capital requirements

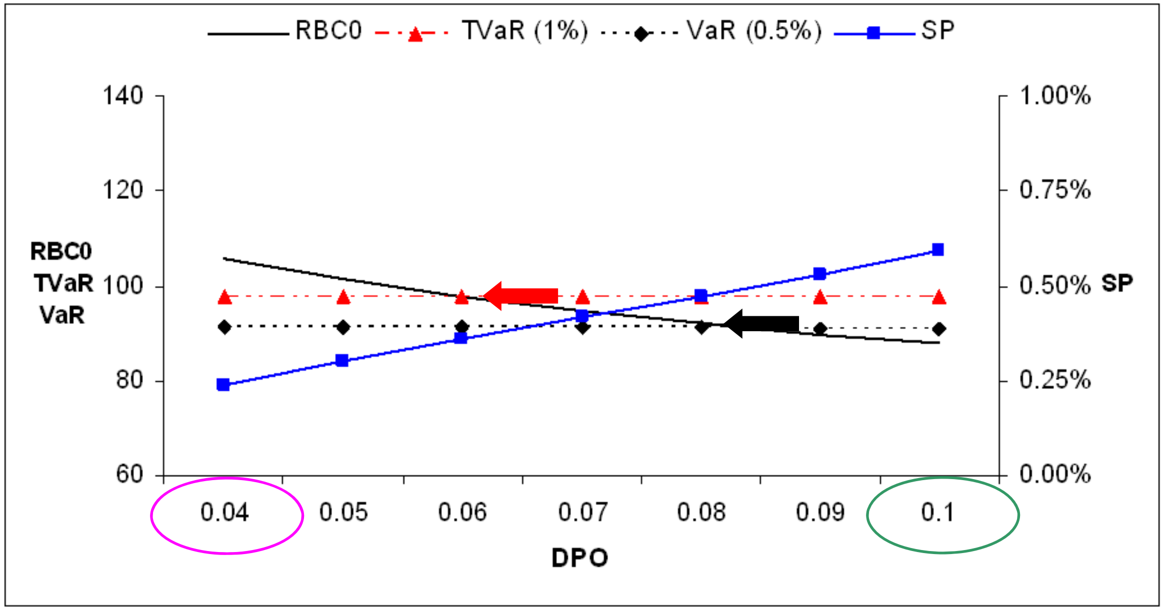
- More precisely: A fixed nominal value of L_0, input parameters for the fair pricing conditions, and a fixed DPO value leads to a ...

  ... fair capital structure and to a value for the available economic capital RBC_0

- RBC_0 is compared with SC under the different solvency regimes

Fair Pricing and Capital Requirements

- Reference case

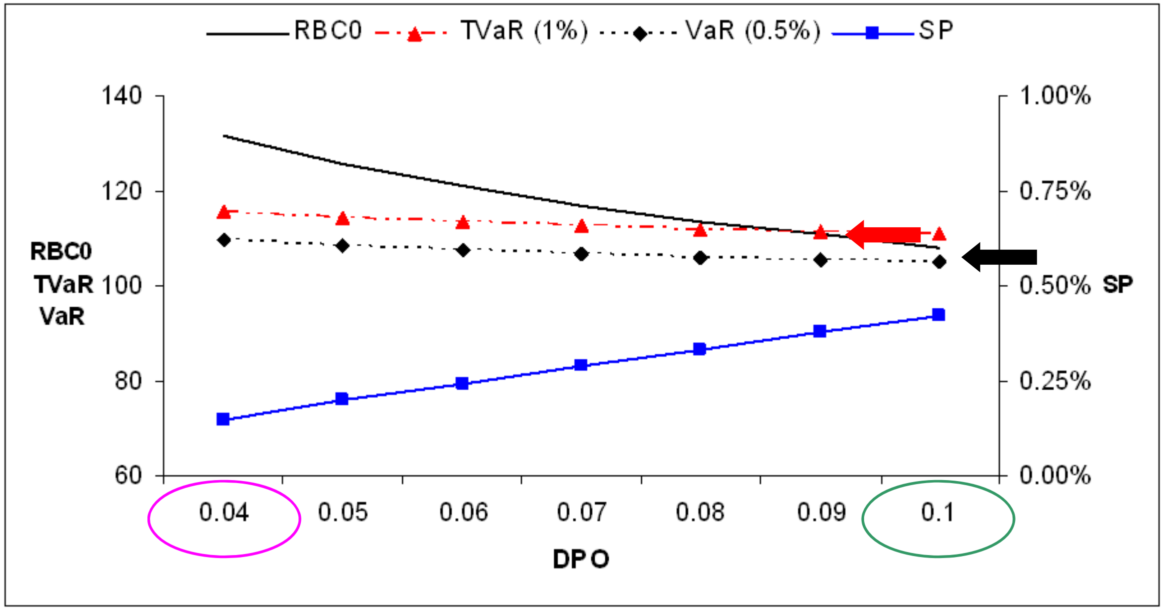


Fair equity-premium combinations for given DPO values for $L_0 = 100$, $r = 3\%$, $\sigma_A = 10\%$, $\mu_A = 8\%$, $\sigma_L = 20\%$, $\mu_L = 1.5\%$, $\rho = 0.2$, $E(Y) = 1.15$, $\sigma(Y) = 10\%$, ($a = 0.1360$, $b = 0.0868$), $\lambda = 0.5$

Π^{DPO}	0.04		0.1	
τ	0	30%	0	30%
$P_0^\tau - L_0 - \Pi^{DPO}$	99.96	104.64	99.90	104.31
E_0^τ	105.87	101.19	87.83	83.42
Π^{T_1}	0	4.68	0	4.41
A_0^τ	205.83	205.83	187.73	187.73

Fair Pricing and Capital Requirements

- Risky investment case

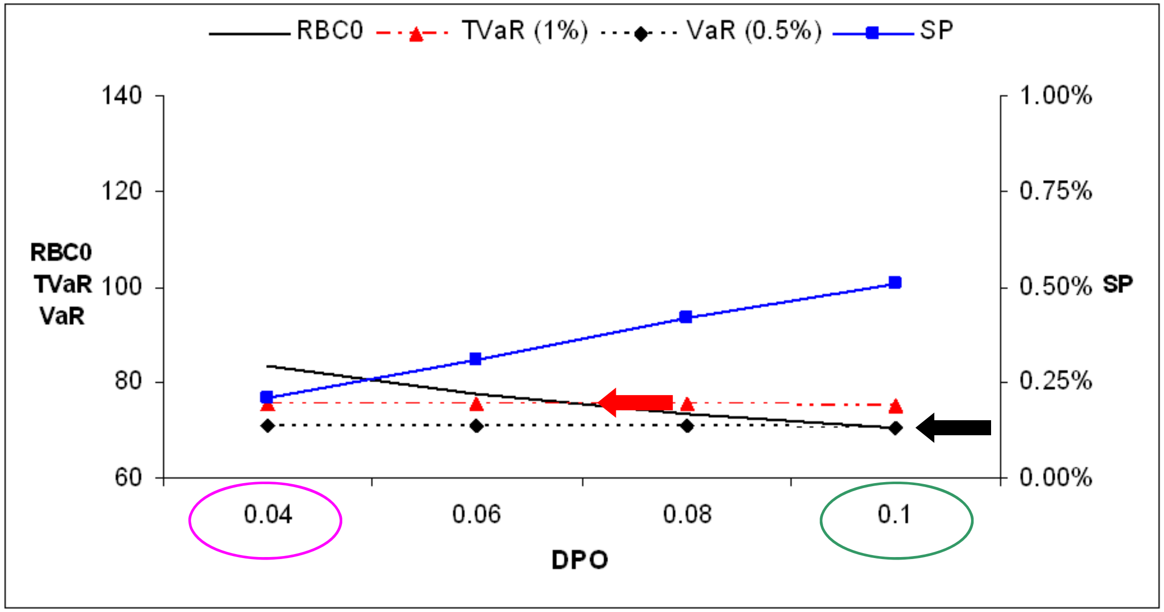


Fair equity-premium combinations for given DPO values for $L_0 = 100$, $r = 3\%$, $\sigma_A = 20\%$, $\mu_A = 12\%$, $\sigma_L = 20\%$, $\mu_L = 1.5\%$, $\rho = 0.2$, $E(Y) = 1.15$, $\sigma(Y) = 10\%$, ($a = 0.1360$, $b = 0.0868$), $\lambda = 0.5$

Π^{DPO}	0.04		0.1	
τ	0	30%	0	30%
$P_0^\tau - L_0 - \Pi^{DPO}$	99.96	107.44	99.90	106.67
E_0^τ	131.63	124.16	108.28	101.51
Π^{T_1}	0	7.48	0	6.77
A_0^τ	231.59	231.59	208.18	208.18

Fair Pricing and Capital Requirements

- Jump case (reduced liabilities' risk)



Fair equity-premium combinations for given DPO values for $L_0 = 100$, $r = 3\%$, $\sigma_A = 20\%$, $\mu_A = 12\%$, $\sigma_L = 20\%$, $\mu_L = 1.5\%$, $\rho = 0.2$, $E(Y) = 1.10$, $\sigma(Y) = 10\%$, $(a = 0.0912, b = 0.0907)$, $\lambda = 0.33$

Π^{DPO}	0.04		0.1	
τ	0	30%	0	30%
$P_0^\tau - L_0 - \Pi^{DPO}$	99.96	104.08	99.90	103.83
E_0^τ	83.26	79.14	70.37	66.45
Π^{T_1}	0	4.12	0	3.93
A_0^τ	183.22	183.22	170.27	170.27

5. Conclusion

- Implications for insurance regulators:

  Even if capital requirements are fulfilled (Solvency II / SST), corresponding price for the default risk can differ substantially

  However, costs of insolvency is the important factor

- Financial crisis: large companies' default put a much higher thread to the market

- Should we demand a limit for the DPO value (too?)