Combing Fair Pricing and Capital Requirements for Non-Life Insurance Companies

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1. Introduction

- New solvency capital requirements in Europe (Solvency II, Swiss Solvency Test SST):
  
  Available economic capital (risk-bearing capital RBC) should exceed the required solvency capital (SC)

- Risk measurement level: solvency capital requirements are derived using the VaR (Solvency II) or the TVaR (SST) concept

- Risk valuation level: competitive market conditions should lead to risk adequate return for the stakeholders (net present value of zero; "fair" pricing)
Fair Pricing and Capital Requirements

- Literature: Fair pricing, capital structure, and solvency requirements are usually studied individually

- Aim of this paper: Combining fair pricing and solvency capital requirements in order to gain a deeper understanding of the effect of solvency regulation on the cost of insolvency

  More precisely: Minimum safety levels are identified using the default put option value for fair conditions that satisfy solvency capital requirements

- Organization of the presentation
2. Model Framework

a) A simplicities model of a non-life insurer \( t = 0,1 \)

- Premiums and equity capital are invested

\[
P_0^\tau + E_0^\tau = A_0^\tau
\]

- Payoff to the policyholders

\[
P_1^\tau = L_1 - max(L_1 - A_1^\tau, 0) + T_1
\]

- Payoff to the shareholders

\[
E_1^\tau = max(A_1^\tau - L_1, 0) - T_1
\]
Fair Pricing and Capital Requirements

- Corporate taxes for a tax level $\tau$

$$T_1 = \tau \cdot \max \left( \left( E_0^\tau + P_0^\tau \right) \left( \frac{A_1^\tau}{A_0^\tau} - 1 \right) + P_0^\tau - L_1, 0 \right)$$

b) Risk valuation (under the risk-neutral measure $Q$)

$$\Pi^P = E^Q \left( \exp(-r) P_1^\tau \right) = \Pi^L - \Pi^{DPO} + \Pi^{T_1}$$

$$\Pi^E = E^Q \left( \exp(-r) E_1^\tau \right) = E^Q \left( \exp(-r) \max \left( A_1^\tau - L_1, 0 \right) \right) - \Pi^{T_1}$$

- Fair pricing

$$\Pi^P = P_0^\tau$$

$$\Pi^E = E_0^\tau$$
c) Modeling asset and liabilities

- **Assets**: GBM

- **Liabilities**: Merton’s jump-diffusion-process

\[
dA_t = \mu_A A_t dt + \sigma_A A_t dW_{A,t}^p
\]

\[
\frac{dL_t}{L_t} = \mu_L dt + \sigma_L dW_{L,t}^p + dJ_t \quad dW_A dW_L = \rho \, dt \quad J_t = \sum_{j=1}^{N_i} (Y_j - 1)
\]

- In the Merton case, solutions for the stochastic differential equations can be derived under \( P \) and \( Q \)
3. Solvency Capital Requirements

a) Risk-bearing capital RBC and solvency capital SC

\[ RBC_t = A_t^\tau - L_t \]

\[ RBC_0 = A_0^\tau - L_0 \geq SC_\alpha \]

Excursion: Fair conditions lead to

\[ RBC_0 = \left( E_0^\tau + P_0^\tau \right) - L_0 = \left( E_0^\tau + L_0 - \Pi^{DPO} + \Pi^{T_i} \right) - L_0 \]

\[ = E_0^\tau + \Pi^{T_i} - \Pi^{DPO} \]
b) Swiss Solvency Test SST

• SC is derived using the TVaR concept for $\alpha = 1\%$

$$SC_\alpha = TVaR_\alpha = -E( X \mid X \leq VaR_\alpha ) \quad X = \exp(-r) RBC_1 - RBC_0$$

c) Solvency II

• SC is derived using the VaR concept with $\alpha = 0.5\%$

$$P( X < VaR_\alpha ) = \alpha \quad SC_\alpha = -VaR_\alpha$$

d) Shortfall propability SP

$$SP = P( A_i^\tau < L_i )$$
4. Numerical Results

• Aim: Identifying fair equity capital / premium combinations that satisfy solvency capital requirements

• More precisely: A fixed nominal value of $L_0$, input parameters for the fair pricing conditions, and a fixed DPO value leads to a ...

... fair capital structure and to a value for the available economic capital $RBC_0$

• $RBC_0$ is compared with SC under the different solvency regimes
Fair Pricing and Capital Requirements

- **Reference case**

Fair equity premium combinations for given DPO values for $L_0 = 100$, $r = 3\%$, $\sigma_a = 10\%$, $\mu_a = 8\%$, $\sigma_1 = 20\%$, $\mu_1 = 1.5\%$, $\rho = 0.2$, $E(Y) = 1.15$, $\sigma(Y) = 10\%$, $(a = 0.1360, b = 0.0868)$, $\lambda = 0.5$

<table>
<thead>
<tr>
<th>$\Pi^{DPO}$</th>
<th>0.04</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>0</td>
<td>30%</td>
</tr>
<tr>
<td>$P_0^L - L_0 - \Pi^{DPO}$</td>
<td>99.96</td>
<td>104.64</td>
</tr>
<tr>
<td>$E_{X}$</td>
<td>105.87</td>
<td>101.19</td>
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<tr>
<td>$\Pi^{T_1}$</td>
<td>0</td>
<td>4.68</td>
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<tr>
<td>$A_{t}$</td>
<td>205.83</td>
<td>205.83</td>
</tr>
</tbody>
</table>
Fair Pricing and Capital Requirements

- Risky investment case

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• **Jump case (reduced liabilities’ risk)**

![Graph showing fair pricing and capital requirements](image)

**Table: Fair equity-premium combinations for given DPO values**

<table>
<thead>
<tr>
<th>DPO</th>
<th>0.04</th>
<th>0.1</th>
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</thead>
<tbody>
<tr>
<td>$\Pi^{\text{DPO}}$</td>
<td>99.96</td>
<td>99.90</td>
</tr>
<tr>
<td>$\pi$</td>
<td>30%</td>
<td>30%</td>
</tr>
<tr>
<td>$P_{0}^{\text{a}} - L_{0} - \Pi^{\text{DPO}}$</td>
<td>104.08</td>
<td>103.83</td>
</tr>
<tr>
<td>$\mathcal{E}_{0}^{\text{a}}$</td>
<td>83.26</td>
<td>70.37</td>
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<td>$\hat{\Pi}^{\text{a}}$</td>
<td>79.14</td>
<td>66.45</td>
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<tr>
<td>$A^{\text{a}}_{0}$</td>
<td>4.12</td>
<td>3.93</td>
</tr>
<tr>
<td>$A^{\text{a}}_{0}$</td>
<td>183.22</td>
<td>170.27</td>
</tr>
</tbody>
</table>
5. Conclusion

• Implications for insurance regulators:

Even if capital requirements are fulfilled (Solvency II / SST), corresponding price for the default risk can differ substantially

However, costs of insolvency is the important factor

- Financial crisis: large companies’ default put a much higher thread to the market

- Should we demand a limit for the DPO value (too?)