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Creating Customer Value in Participating Life Insurance

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1. Introduction

- Participating life insurance contracts contain numerous guarantees and options:

- E.g., minimum interest rate guarantee, guaranteed annual surplus participation, terminal bonus payment,…

- Appropriate pricing is crucial for insurer's stability

- Two perspectives for valuation: policyholder and insurer

- Derived prices on contract level (including embedded options) must meet customer demand

Creating Customer Value

- Insurer perspective:

- Present value calculation (risk-neutral or fair valuation): based on insurer's ability to duplicate cash flows

- Policyholder perspective:

- May not be able to duplicate claims via capital market instruments

- Contract valuation is generally based on individual pre-ferences

- Willingness to pay (customer value) will be different from fair premium calculated by insurance company

Creating Customer Value

- Literature

- Aim of this paper

– Combine perspective of insurer and policyholder

– Identify contract parameters – guaranteed interest rate, annual and terminal surplus participation – leading to a 1) fixed fair contract value (from the insurer's perspective) and 2) will maximize customer value

– Distinguish between deterministic and stochastic basis wealth of policyholder (different degree of diversification)

2. Life Insurance Contract and Valuation

- Standard Black-Scholes setting

- Assets A(t) follow geometric Brownian motion
- Policyholders pay up-front premium $P_0 \left(= \beta \cdot A_0\right)$
- Equityholders make initial contribution $Eq_0 \left(= \left(1-\beta\right) \cdot A_0\right)$
- Total initial payments invested in assets $A_0 = P_0 + Eq_0$
- P(t) policyholder account

$$P\left(t\right) = P\left(t-1\right) \cdot \left(1+g\right) + max\left[\alpha \cdot \gamma \cdot \left(A\left(t\right) - A\left(t-1\right)\right) - g \cdot P\left(t-1\right), 0\right]$$

Guaranteed interest rate

Annual surplus participation rate

Relation of book to market values: build up hidden reserves

Creating Customer Value

- Terminal bonus participation:

$$B(T) = \delta \cdot max\left(\beta \cdot A(T) - P(T), 0\right)$$

- Default put option (insurer may get insolvent):

$$D(T) = max\left(P(T) - A(T), 0\right)$$

- Final payoffs

- Policyholders' payoff:

$$L(T) = P(T) + \delta \cdot B(T) - D(T)$$

- Equityholders' payoff:

$$Eq(T) = A(T) - L(T) = max\left(A(T) - P(T), 0\right) - \delta \cdot B(T)$$

Creating Customer Value

- Insurer perspective

- Risk-neutral valuation (under pricing measure Q)

$$\Pi^* = E^{\mathbb{Q}}\left(e^{-rT} \cdot L(T)\right) = E^{\mathbb{Q}}\left[e^{-rT} \cdot \left(P(T) + \delta \cdot B(T)\right)\right] - E^{\mathbb{Q}}\left(e^{-rT} \cdot D(T)\right) = \Pi - \Pi^{DPO}$$

- Fair contracts: calibrate (g, α, δ) to satisfy $\Pi^* = P_0$

- Provides lower end of premium agreement range

- Necessary premium to conduct adequate risk management measurers (e.g., equity capital)

- Infinite number of contract specifications with same fair value, but <u>different customer value</u>

Creating Customer Value

- Customer perspective

- In general, there a very different ways to derive the customer value of a life insurance contract

- E.g., mean-variance preferences (under real-world measure P)

– Determine upper end of premium agreement area, i.e., customer's willingness to pay P_0^{Φ} using the following preference function:

$$\Phi = E(Z_T) - \frac{a}{2} \cdot \sigma^2(Z_T)$$

– Z is policyholder's wealth, a is the degree of risk aversion (a > 0)

Creating Customer Value

- Derivation of customer's willingness to pay P_0^{Φ}

- Compare preference functions for case

  without insurance (WI)

$$\Phi^{NI} = E\left(Z_T^{NI}\right) - \frac{a}{2} \cdot \sigma^2\left(Z_T^{NI}\right), \quad Z_0^{NI} = Z_0$$

  and with insurance (NI)

$$\Phi^{WI} = E\left(Z_T^{WI} + L(T)\right) - \frac{a}{2} \cdot \sigma^2\left(Z_T^{WI} + L(T)\right), \quad Z_0^{WI} = Z_0 - P_0^{\Phi}$$

- Maximum willingness to pay is price at which customer becomes indifferent between the two cases:

$$\Phi^{WI} = \Phi^{NI}$$

Creating Customer Value

- Deterministic wealth (policyholder cannot diversify)

- Invest in risk-free asset or purchase life insurance

$$Z(t) = Z_0 e^{rt}$$

$$P_0^{\Phi} = e^{-rT} \cdot \left[E\left(L(T)\right) - \frac{a}{2} \cdot \sigma^2\left(L(T)\right) \right]$$

- Stochastic wealth (policyholder can diversify):

- Invest in stochastic assets or purchase life insurance

$$Z(t) = Z(t-1) \cdot \exp\left[\mu_Z - \sigma_Z{}^2 / 2 + \sigma_Z \left(W_Z(t) - W_Z(t-1) \right) \right], \qquad dW_A dW_Z = \rho dt$$

$$P_0^{\Phi} = \left[\left[\frac{\frac{1}{a} \cdot E\left(\tilde{Z}_T\right) - Cov\left(Z_T{}^{NI}, \tilde{Z}_T\right) - Cov\left(\tilde{Z}_T, L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)} \right]^2 - \frac{\sigma^2\left(L(T)\right) + 2 \cdot Cov\left(Z_T{}^{NI}, L(T)\right) - \frac{2}{a} \cdot E\left(L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)} \right]^{\frac{1}{2}} - \frac{\frac{1}{a} \cdot E\left(\tilde{Z}_T\right) - Cov\left(Z_T{}^{NI}, \tilde{Z}_T\right) - Cov\left(\tilde{Z}_T, L(T)\right)}{\sigma^2\left(\tilde{Z}_T\right)}.$$

3. Creating Customer Value

- Customer value differs, even though contracts have the same fair value (based on risk-neutral valuation)
- Idea: keep contracts fair from insurer perspective:

$$\underbrace{P_0^{\Phi} \to \max_{g,\alpha,\delta}}_{\substack{\text{Customer value} \\ \text{under the real-world measure } \mathbb{P}}} \qquad \text{such that} \qquad \underbrace{P_0 = \Pi^*(g,\alpha,\delta) = E^{\mathbb{Q}}\left(e^{-rT} \cdot L(T)\right).}_{\substack{\text{Fair contract} \\ \text{under the risk-neutral measure } \mathbb{Q}}}$$

– For fixed nominal premium, choose fair combination (g,α,δ) that leads to highest customer value, while providing at least a risk-adequate returns for shareholders

4. Numerical Examples

- Input parameters

- Risk-free rate $r = 4.46\%$
- Asset drift $\mu_A = 7\%$,
- Volatility of assets $\sigma_A = 6\%$
- Fair premium $P_0 = 100$
- Contribution of equityholders $Eq_0 = 30$
- Relation of book to market values $\gamma = 50\%$
- Time to maturity $T = 10$

Creating Customer Value

Table 1: Fair contracts and corresponding customer value for deterministic and stochastic wealth.

Fair contract parameters (insurer perspective)					Customer value P_0^{Φ} (policyholder perspective)					
Guaranteed interest rate (g)	Terminal participation rate (δ)	Annual participation rate (α)	Π^*	*Shortfall probability*	Part A: deterministic (a = 0.0685)	Part B: stochastic (a = 0.0105)	ρ = 0.9	σ_Z = 8%	Z_0 = 200	a = 0.0685
Panel A: Contract with regulatory restrictions:										
2.25%	68%	90%	100	0.02%	100.0	100.0	96.6	106.7	104.2	133.7
Panel B: Simple contracts with one parameter only:										
4.56%	0%	0%	100	0.69%	100.9	85.4	85.3	91.7	89.0	133.8
0.00%	99.89%	0%	100	0.02%	96.5	**104.1**	**100.3**	**110.9**	**108.4**	130.6
0.00%	0%	130%	100	0.09%	**101.3**	99.9	96.7	106.7	104.1	**134.7**
Panel C: Maximizing customer value:										
2.00%	0%	113%	100	0.03%	**103.5**	96.9	94.1	103.5	101.0	**136.6**
	40%	105%	100	0.02%	101.8	98.7	95.5	105.4	102.8	135.1
	80%	85%	100	0.01%	98.8	**101.3**	**97.7**	**108.0**	**105.5**	132.5
4.00%	0%	77%	100	0.26%	**104.3**	91.0	89.3	97.4	94.8	**137.2**
	40%	58%	100	0.22%	104.2	94.0	91.6	100.5	97.9	**137.2**
	50%	47%	100	0.21%	104.0	**94.6**	**92.2**	**101.2**	**98.6**	137.1
4.30%	0%	67%	100	0.39%	103.7	89.3	88.1	95.7	93.0	136.9
	10%	62%	100	0.39%	104.1	90.0	88.6	96.5	93.8	137.2
	30%	45%	100	0.37%	**104.6**	**91.3**	89.6	97.8	95.1	**137.6**
4.40%	0%	62%	100	0.42%	103.3	87.7	87.5	95.0	92.3	136.6
	25%	37%	100	0.38%	104.4	88.3	88.6	96.4	93.7	137.5
	27%	9%	100	0.38%	**104.7**	**90.1**	**88.8**	**96.6**	**93.9**	**137.7**
4.50%	0%	56%	100	0.45%	102.8	87.7	86.9	94.1	91.4	136.2
	15%	39%	100	0.42%	103.6	**88.4**	**87.4**	**94.8**	**92.1**	136.8
	17%	26%	100	0.42%	**103.7**	-	-	-	-	**136.9**

5. Summary

- Examine how insurers can generate customer value for participating life insurance contracts
- Insurer: preference-free approach of risk-neutral valuation
- Customer value defined as policyholder willingness to pay
  - Calculated based on mean-variance preferences
- First calibrate contract parameters to the same fair value, then derive corresponding customer value of these contracts

Creating Customer Value

- Find that customer value varies substantially, even if contracts have same value
- Customer segmentation is a viable tool for increasing insurer profit and achieving shareholder return above risk-adequate rate
- Design contracts to specifically increase customer value compared to standard contracts
- Preferred contracts may be simple ones (e.g., only one instead of three embedded options)
- Further research: Analyzing the impact of different valuation techniques for policyholders (e.g., regarding the shortfall risk of a contract)