Creating Customer Value in Participating Life Insurance

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1. Introduction

- Participating life insurance contracts contain numerous guarantees and options:
  - e.g., minimum interest rate guarantee, guaranteed annual surplus participation, terminal bonus payment,…

- Appropriate pricing is crucial for insurer's stability

- Two perspectives for valuation: policyholder and insurer

- Derived prices on contract level (including embedded options) must meet customer demand
Creating Customer Value

- Insurer perspective:
  - Present value calculation (risk-neutral or fair valuation): based on insurer's ability to duplicate cash flows

- Policyholder perspective:
  - May not be able to duplicate claims via capital market instruments
  - Contract valuation is generally based on individual preferences
  - Willingness to pay (customer value) will be different from fair premium calculated by insurance company
• Literature

• Aim of this paper
  – Combine perspective of insurer and policyholder
  – Identify contract parameters – guaranteed interest rate, annual and terminal surplus participation – leading to a 1) fixed fair contract value (from the insurer's perspective) and 2) will maximize customer value
  – Distinguish between deterministic and stochastic basis wealth of policyholder (different degree of diversification)
2. Life Insurance Contract and Valuation

- Standard Black-Scholes setting
- Assets $A(t)$ follow geometric Brownian motion
- Policyholders pay up-front premium $P_0 (= \beta \cdot A_0)$
- Equityholders make initial contribution $Eq_0 (= (1 - \beta) \cdot A_0)$
- Total initial payments invested in assets $A_0 = P_0 + Eq_0$
- $P(t)$ policyholder account

$$
P(t) = P(t-1) \cdot (1 + g) + \max\left[ \alpha \cdot \gamma \cdot (A(t) - A(t-1)) - g \cdot P(t-1), 0 \right]
$$
• Terminal bonus participation:
\[ B(T) = \delta \cdot \max(\beta \cdot A(T) - P(T), 0) \]

• Default put option (insurer may get insolvent):
\[ D(T) = \max(P(T) - A(T), 0) \]

• Final payoffs
- Policyholders' payoff:
\[ L(T) = P(T) + \delta \cdot B(T) - D(T) \]
- Equityholders' payoff:
\[ Eq(T) = A(T) - L(T) = \max(A(T) - P(T), 0) - \delta \cdot B(T) \]
Creating Customer Value

- Insurer perspective

- Risk-neutral valuation (under pricing measure Q)

\[ \Pi^* = E^Q \left( e^{-rT} \cdot L(T) \right) = E^Q \left[ e^{-rT} \cdot \left( P(T) + \delta \cdot B(T) \right) \right] - E^Q \left( e^{-rT} \cdot D(T) \right) = \Pi - \Pi^{DPO} \]

- Fair contracts: calibrate \((g, \alpha, \delta)\) to satisfy \(\Pi^* = P_0\)

- Provides lower end of premium agreement range

- Necessary premium to conduct adequate risk management measurers (e.g., equity capital)

- Infinite number of contract specifications with same fair value, but different customer value
Customer perspective

- In general, there are very different ways to derive the customer value of a life insurance contract

- E.g., mean-variance preferences (under real-world measure P)

  - Determine upper end of premium agreement area, i.e., customer's willingness to pay $P_0^\Phi$ using the following preference function:

    $$\Phi = E(Z_T) - \frac{a}{2} \cdot \sigma^2(Z_T)$$

  - $Z$ is policyholder's wealth, $a$ is the degree of risk aversion ($a > 0$)
Creating Customer Value

• Derivation of customer's willingness to pay $P_0^\Phi$

- Compare preference functions for case without insurance (WI)

$$\Phi^{WI} = E(Z_{T}^{WI}) - \frac{a}{2} \cdot \sigma^2(Z_{T}^{WI}), \quad Z_{0}^{NI} = Z_{0}$$

and with insurance (NI)

$$\Phi^{NI} = E(Z_{T}^{NI} + L(T)) - \frac{a}{2} \cdot \sigma^2(Z_{T}^{WI} + L(T)), \quad Z_{0}^{WI} = Z_{0} - P_0^\Phi$$

- Maximum willingness to pay is price at which customer becomes indifferent between the two cases:

$$\Phi^{WI} = \Phi^{NI}$$
Creating Customer Value

- Deterministic wealth (policyholder cannot diversify)
  - Invest in risk-free asset or purchase life insurance
    \[ Z(t) = Z_0 e^{rt} \]
    \[ P_0^\Phi = e^{-rT} \cdot \left[ E(L(T)) - \frac{a}{2} \cdot \sigma^2(L(T)) \right] \]

- Stochastic wealth (policyholder can diversify):
  - Invest in stochastic assets or purchase life insurance
    \[ Z(t) = Z(t-1) \cdot \exp \left[ \mu_Z - \sigma_Z^2 / 2 + \sigma_Z \left( W_Z(t) - W_Z(t-1) \right) \right], \quad dW_A dW_Z = \rho dt \]
    \[ P_0^\Phi = \left[ \frac{1}{\sigma^2(Z_t)} \cdot \frac{E(Z_t) - \text{Cov}(Z_t, Z_t) - \text{Cov}(Z_t, L(T))}{\sigma^2(Z_t)} \right]^{\frac{3}{2}} \cdot \frac{1}{\sigma^2(Z_t)} \cdot \frac{E(Z_t) - \text{Cov}(Z_t, Z_t) - \text{Cov}(Z_t, L(T))}{\sigma^2(Z_t)} \]
3. Creating Customer Value

- Customer value differs, even though contracts have the same fair value (based on risk-neutral valuation)

- Idea: keep contracts fair from insurer perspective:

\[
P_0^{\Phi} \rightarrow \max_{g,\alpha,\delta} \text{ such that } P_0 = \Pi^*(g,\alpha,\delta) = E^Q\left(e^{-rT} \cdot L(T)\right).
\]

- For fixed nominal premium, choose fair combination \((g,\alpha,\delta)\) that leads to highest customer value, while providing at least a risk-adequate returns for shareholders.
4. Numerical Examples

- Input parameters
  - Risk-free rate $r = 4.46\%$
  - Asset drift $\mu_A = 7\%$
  - Volatility of assets $\sigma_A = 6\%$
  - Fair premium $P_0 = 100$
  - Contribution of equityholders $Eq_0 = 30$
  - Relation of book to market values $\gamma = 50\%$
  - Time to maturity $T = 10$
### Table 1: Fair contracts and corresponding customer value for deterministic and stochastic wealth.

<table>
<thead>
<tr>
<th>Fair contract parameters (insurer perspective)</th>
<th>Customer value $P^w$ (policyholder perspective)</th>
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</thead>
<tbody>
<tr>
<td>Guaranteed interest rate ($g$)</td>
<td>Part A: deterministic $(a = 0.00685)$</td>
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<tr>
<td>Terminal participation rate ($\delta$)</td>
<td>Shortfall probability</td>
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<td>Annual participation rate ($\alpha$)</td>
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<tr>
<td>$\Pi^*$</td>
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<td>Panel A: Contract with regulatory restrictions:</td>
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<td>2.25%</td>
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<td>Panel B: Simple contracts with one parameter only:</td>
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<td>Panel C: Maximizing customer value:</td>
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5. Summary

- Examine how insurers can generate customer value for participating life insurance contracts
- Insurer: preference-free approach of risk-neutral valuation
- Customer value defined as policyholder willingness to pay
  - Calculated based on mean-variance preferences
- First calibrate contract parameters to the same fair value, then derive corresponding customer value of these contracts
Creating Customer Value

• Find that customer value varies substantially, even if contracts have same value

• Customer segmentation is a viable tool for increasing insurer profit and achieving shareholder return above risk-adjusted rate

• Design contracts to specifically increase customer value compared to standard contracts

• Preferred contracts may be simple ones (e.g., only one instead of three embedded options)

• Further research: Analyzing the impact of different valuation techniques for policyholders (e.g., regarding the shortfall risk of a contract)