Minimum Standards for Investment Performance: 
A New Perspective on Non-Life Insurer Solvency

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Outline

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1. Introduction

• New RBC standards in the EU (Solvency II):
  - Minimum Capital Requirements (MCR)
  - Target Capital Requirements (TCR), derived via a standard model or an internal model

• Proposal for a standard model

• New perspective: Instead of deriving a certain amount of capital, minimum standards for the investment performance are obtained ("solvency line")

• Literature
2. Model Framework

\[ U_1 = A_1 - L_1 = A_0 \cdot R - L_1 = (U_0 + L_0) \cdot R - (S_1 + B_1) \]

- **U** = Risk-bearing capital
- **A** = Assets
- **L** = Liabilities
- **R** = 1 + r = Rate of return
- **S** = Market value of net claims
- **B** = Operating expenses
• For the distribution of the risk-bearing capital in $t = 1$, $U_1$, a Normal-Power-Approximation is used

Skewness of the distribution can be taken into account

Analytical expressions for different risk measures can be derived

• Ruin Probability (RP):

$$RP = P(U_1 \leq 0) = P(A_1 \leq L_1) = \varepsilon$$

$$0 = E(U_1) + z_\varepsilon \cdot \sigma(U_1) + \frac{z_\varepsilon^2 - 1}{6} \cdot \gamma(U_1) \cdot \sigma(U_1)$$
• **Expected Policyholder Deficit (EPD):**

\[
EPD = E\left(\max(0 - U_1, 0)\right)
\]

\[
EPD = -E(U_1) \cdot \Phi(-\gamma(0)) + \left(\sigma(U_1) + \frac{\gamma(U_1) \cdot \sigma(U_1)}{6} \cdot \varphi(0)\right) \cdot \varphi(\gamma(0))
\]

• **Tail Value at Risk (TVaR):**

\[
TVaR_\alpha(U_1) = E(-U_1 \mid U_1 \leq q_\alpha) \quad \quad \quad \quad \quad VaR_\alpha(U_1) = q_\alpha
\]

\[
TVaR_\alpha(U_1) = \frac{-E(U_1) \cdot \Phi(\gamma(q_\alpha))}{\alpha} + \frac{\left(\sigma(U_1) + \frac{\gamma(U_1) \cdot \sigma(U_1)}{6} \cdot \varphi(q_\alpha)\right) \cdot \varphi(\gamma(q_\alpha))}{\alpha} + q_\alpha (1 - \alpha)
\]
3. Numerical Example Based on Empirical Data

- Given a fixed safety level, a risk-bearing capital $U_0$, and the insurer’s liabilities, minimum requirements for the investment portfolio in an $E(r)-\sigma(r)$ relationship can be derived.

- Data: Medium-size German non-life insurer

\[
\begin{align*}
A_0 &= U_0 + L_0 = \text{€ 1.582 billion} \\
E(S_1) &= \text{€ 1.171 billion} \\
\sigma(S_1) &= \text{€ 66 billion} \\
\gamma(S_1) &= 0.3 \\
B_1 &= \text{€ 245 billion}
\end{align*}
\]
• RP: \( \varepsilon = 0.5\% \) (Solvency II safety level)
• TVaR: \( \alpha = 1\% \) (Swiss Solvency Test safety level)
• \( \text{TVAR}_{1\%}(U_1) = \€ 25.72 \text{ billion}; \text{ EPD} = \€ 122 \text{ million} \)
• RP: ε = 0.1%, 1%, 0.5% (Solvency II)
• Solvency Line and Capital Market Line

<table>
<thead>
<tr>
<th>Asset class</th>
<th>Index</th>
<th>Illustration</th>
<th>E((r)) annualized</th>
<th>(\sigma((r))) annualized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money</td>
<td>JPM Euro Cash 3 Month</td>
<td>Money market in the EMU = (r_f)</td>
<td>3.95%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Stocks</td>
<td>MSCI World ex EMU</td>
<td>Worldwide stocks without the EMU</td>
<td>8.92%</td>
<td>13.41%</td>
</tr>
<tr>
<td></td>
<td>MSCI EMU ex Germany</td>
<td>Stocks from the EMU without Germany</td>
<td>12.16%</td>
<td>17.86%</td>
</tr>
<tr>
<td></td>
<td>MSCI Germany</td>
<td>Stocks from Germany</td>
<td>10.46%</td>
<td>22.12%</td>
</tr>
<tr>
<td>Bonds</td>
<td>MSCI SDI World ex EMU</td>
<td>Worldwide government bonds without the EMU</td>
<td>4.84%</td>
<td>5.78%</td>
</tr>
<tr>
<td></td>
<td>MSCI SDI EMU ex Germany</td>
<td>Gov. bonds from the EMU without Germany</td>
<td>6.54%</td>
<td>3.82%</td>
</tr>
<tr>
<td></td>
<td>MSCI SDI Germany</td>
<td>Government bonds from Germany</td>
<td>5.59%</td>
<td>3.46%</td>
</tr>
<tr>
<td></td>
<td>MSCI Euro Credit Corporate</td>
<td>Corporate bonds from the EMU</td>
<td>5.84%</td>
<td>3.41%</td>
</tr>
<tr>
<td>Real estate</td>
<td>GPR General PSI Global</td>
<td>Real estate worldwide</td>
<td>8.45%</td>
<td>11.37%</td>
</tr>
<tr>
<td></td>
<td>GPR General PSI Europe</td>
<td>Real estate in Europe</td>
<td>8.19%</td>
<td>7.16%</td>
</tr>
<tr>
<td></td>
<td>DIMAX</td>
<td>Real estate in Germany</td>
<td>7.65%</td>
<td>13.15%</td>
</tr>
</tbody>
</table>

JPM: J.P. Morgan; MSCI: Morgan Stanley Capital International; EMU: European Monetary Union; SDI: Sovereign Debt Index; GPR: Global Property Research; PSI: Property Share Index; DIMAX: Deutscher Immobilien Aktienindex
• Solvency Lines from Figure 1 and Capital Market Line

Figure 3

![Graph showing the Efficient Frontier, TVaR, EPD, RP, and annualized returns vs. annualized standard deviation.](image)
## Asset Allocation and RP

### Examples of asset allocation

<table>
<thead>
<tr>
<th>Asset class</th>
<th>Index</th>
<th>Asset allocation Example 1</th>
<th>Asset allocation Example 2</th>
<th>Asset allocation Example 3</th>
<th>Asset allocation Example 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money</td>
<td>JPM Euro Cash 3 Month</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Stocks</td>
<td>MSCI World ex EMU</td>
<td>0%</td>
<td>5%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>MSCI EMU ex Germany</td>
<td>0%</td>
<td>5%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>MSCI Germany</td>
<td>0%</td>
<td>5%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>Bonds</td>
<td>MSCI SDI World ex EMU</td>
<td>0%</td>
<td>15%</td>
<td>10%</td>
<td>0%</td>
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<td>0%</td>
<td>15%</td>
<td>10%</td>
<td>0%</td>
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<tr>
<td></td>
<td>MSCI Euro Credit Corporate</td>
<td>0%</td>
<td>10%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td>Real estate</td>
<td>GPR General PSI Global</td>
<td>0%</td>
<td>10%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>GPR General PSI Europe</td>
<td>0%</td>
<td>10%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>DIMAX</td>
<td>0%</td>
<td>10%</td>
<td>10%</td>
<td>20%</td>
</tr>
</tbody>
</table>

\[ E(r) \text{ annualized} \]

- Example 1: 3.95%
- Example 2: 7.14%
- Example 3: 7.86%
- Example 4: 9.48%

\[ \sigma(r) \text{ annualized} \]

- Example 1: 0%
- Example 2: 4.58%
- Example 3: 6.64%
- Example 4: 11.96%

**Corresponding RP of the insurer**

- Example 1: 0.12%
- Example 2: 0.31%
- Example 3: 1.07%
- Example 4: 5.80%

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4. Summary

- Proposal for a standard model under Solvency II
- Applicability is tested using data from a German non-life insurer
- Consequences of the framework:
  - One regulatory framework instead of two separate regulatory tool for assets and equity capital
  - Insurer’s do have different degrees of freedom in how to choose their asset allocation