Institute of Insurance Economics

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An Analysis of Pricing and Basis Risk for Industry Loss Warranties

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Outline

1. Introduction

- Industry loss warranties (ILWs) are innovative index-based reinsurance instruments that have become increasingly popular in recent years (Guy Carpenter, 2006)
- Payment on these contracts is triggered by an industry loss, where the contracted trigger amount varies by geographic region and type of catastrophic event
- Two predominant ILW forms are binary and indemnity-based contracts: binary contracts pay a fixed amount if the industry loss is triggered; indemnity-based contracts take into consideration the reinsured company's loss (SwissRe, 2006)

1. Introduction

Aim of this paper:

- simultaneously examine basis risk and pricing of an indemnity-based industry loss warranty contract

- give valuable insights for reinsurers and industry loss warranty buyers

2. ILW Contract

- Contract design

$$X_1 = \min\left(\max\left(S_1 - A, 0\right), L\right) \cdot 1\left\{I_1 > Y\right\}$$

- *A* attachment of the company loss
- *L* layer limit
- S_1 company's loss distribution in t = 1
- *Y* industry loss trigger
- I_1 industry loss distribution in t =1

3. Pricing approaches

In general: $\Pi(X_1) = \exp\left(-r_f\right) \cdot CE(X_1)$

Actuarial:

- Expected value principle $CE = E(X_1) + \delta_E \cdot E(X_1)$
- Standard deviation principle $CE = E(X_1) + \delta_S \cdot \sigma(X_1)$
- Variance principle $CE = E(X_1) + \delta_V \cdot \sigma^2(X_1)$
- Investment equivalent reinsurance pricing $CE = E(X_1) + R$

with $R = \max\left(\frac{\left(E(y) - r_f{}^d\right)\left(q_\alpha^X - E(X_1)\right)}{1 + E(y)}, \frac{\left(E(y) - r_f{}^d\right)\sigma(X_1)}{\sigma(y)} \right)$

Financial

- Capital Asset Pricing Model $CE = E(X_1) - \lambda \cdot Cov(X_1, r_m)$ $\lambda = \frac{E(r_m) - r_f{}^d}{\sigma^2(r_m)}$
- Contingent Claims Approach $CE = E^Q\left(X_1\right)$

4. Basis Risk Measures

- Type I basis risk

$$P\left(S_1 > A \mid I_1 < Y\right) = \frac{P\left(S_1 > A, I_1 < Y\right)}{P\left(I_1 < Y\right)}$$

$$E\left(\min\left(\max\left(S_1 - A, 0\right), L\right) \mid I_1 < Y\right)$$

- Type II basis risk

$$P\left(I_1 < Y \mid S_1 > A\right) = \frac{P\left(I_1 < Y, S_1 > A\right)}{P\left(S_1 > A\right)}$$

$$E\left(\min\left(\max\left(S_1 - A, 0\right), L\right) \cdot 1\left\{I_1 < Y\right\}\right).$$

- Relationship between traditional reinsurance contract and ILW

$$E\left(X_1\right) = E\left(X^{trad}\right) - E\left(\min\left(\max\left(S_1 - A, 0\right), L\right) \cdot 1\left\{I_1 < Y\right\}\right)$$

5. Comparison of pricing approaches

- Actuarial methods considered evaluate individual contracts without consideration of diversification in the market or within insurer's portfolio.
- Financial methods assume perfect diversification of unsystematic risk in the market and insurer's portfolio composition has no impact on pricing.
- Risk-free interest rate has no impact on calculation of certainty equivalent for almost all actuarial pricing methods.
- Only investment equivalent reinsurance pricing can account for higher moments of contract's payoff distribution

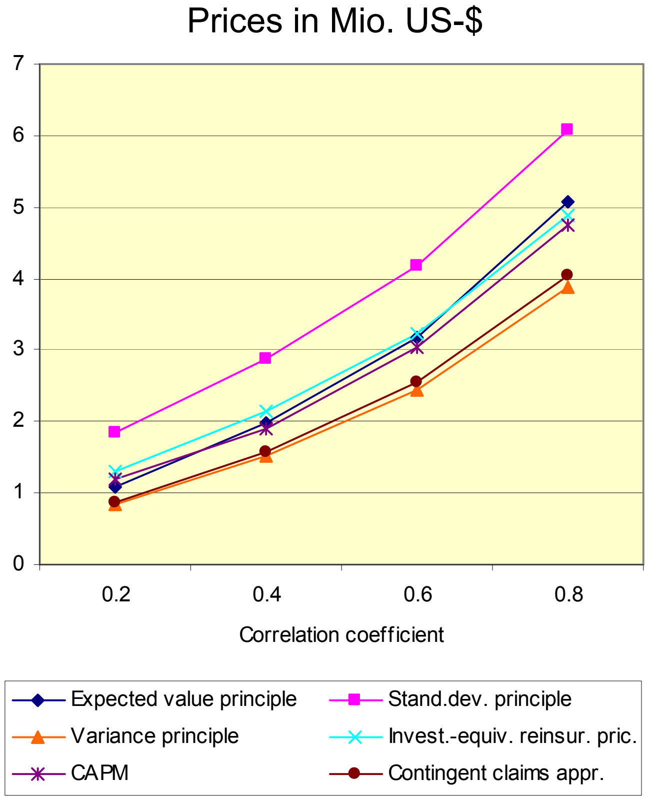
6. Numerical Analysis

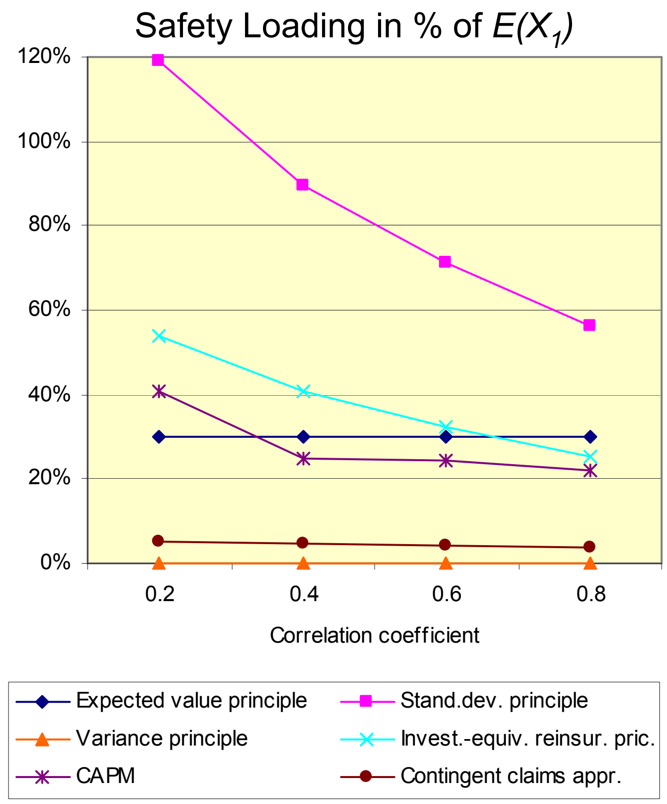
- Parametrization based on available data and literature
- Results based on Monte Carlo simulation with 50,000 paths

Contract parameters	$E(S_1)$	\$58 million
	$\sigma(S_1)$	\$134 million
	$E(I_1)$	\$1,450 million
	$\sigma(I_1)$	\$3,550 million
	μ_S, μ_I	2.50%
	$\rho(S_1, I_1)$	0.60
	L	\$150 million
	Y	\$5,000 million
	A	\$150 million
	r_f^d	4.92%
	r_f	4.80%
Expected value principle	δ_E	30.00%
Standard deviation principle	δ_S	10.00%
Variance principle	δ_V	$1.5x10^{-7}$
Investment-equivalent	E(y)	5.30%
reinsurance pricing	σ(y)	8.40%
	q_α^X	99%-quantile of X_1
CAPM	$\rho(r_m, I_1)$	-0.20
	$\rho(r_m, S_1)$	-0.10
	$E(r_m)$	8.00%
	$\sigma(r_m)$	4.00%

6. Numerical Analysis

- Prices and safety loadings as functions of the correlation coefficient $\rho(S_1, I_1)$





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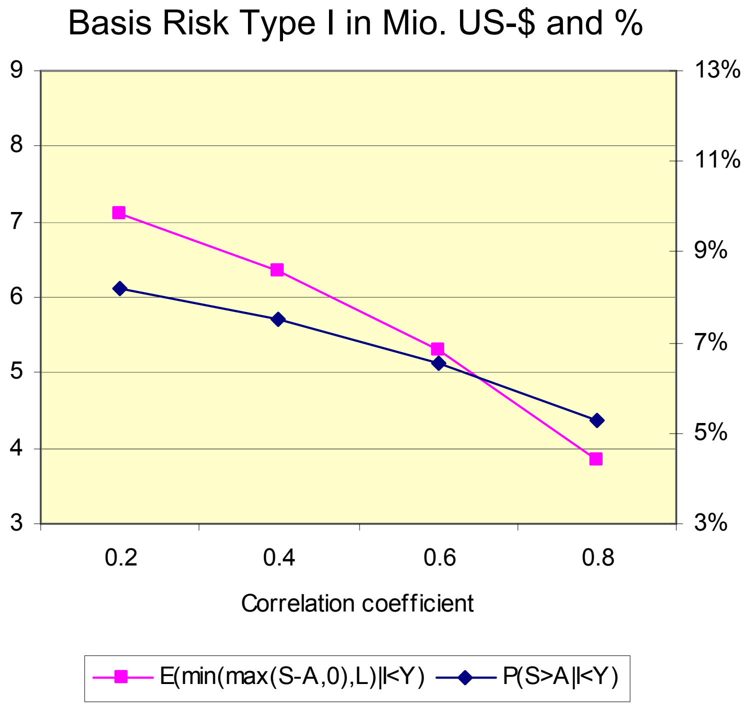
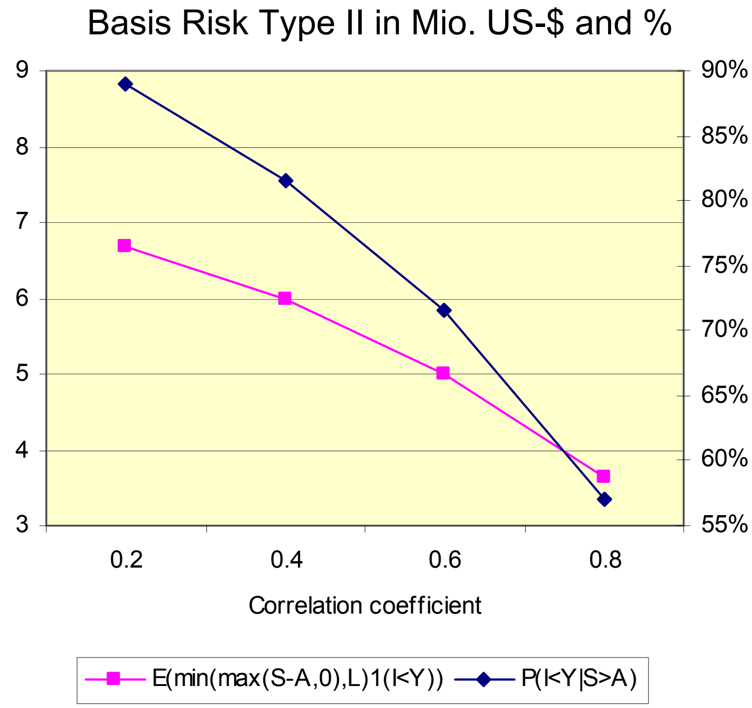
- Basis risk as function of the correlation $\rho(S_1, I_1)$

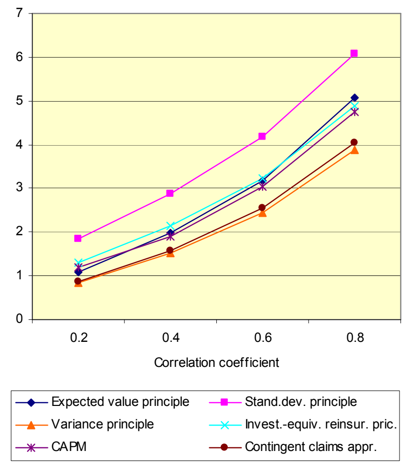




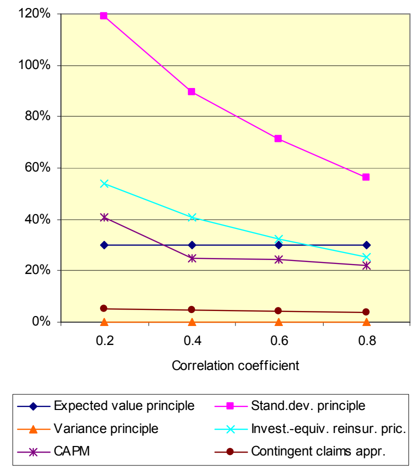
6. Numerical Analysis

- Prices, safety loadings, and basis risk as functions of the correlation coefficient $\rho(S_1, I_1)$

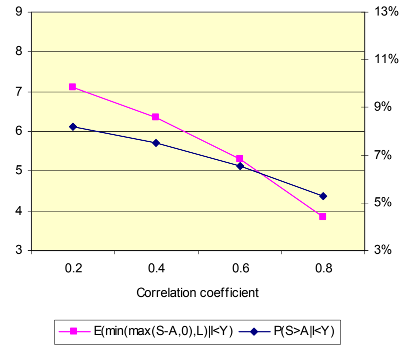
a) Prices in Mio. US-$



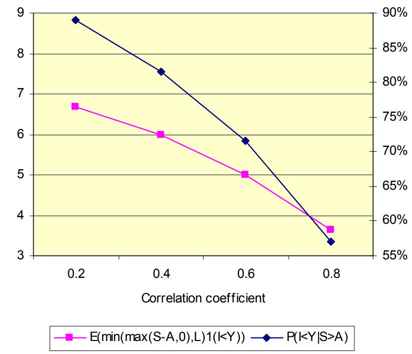
b) Safety Loading in % of $E(X_1)$



c) Basis Risk Type I in Mio. US-$ and %

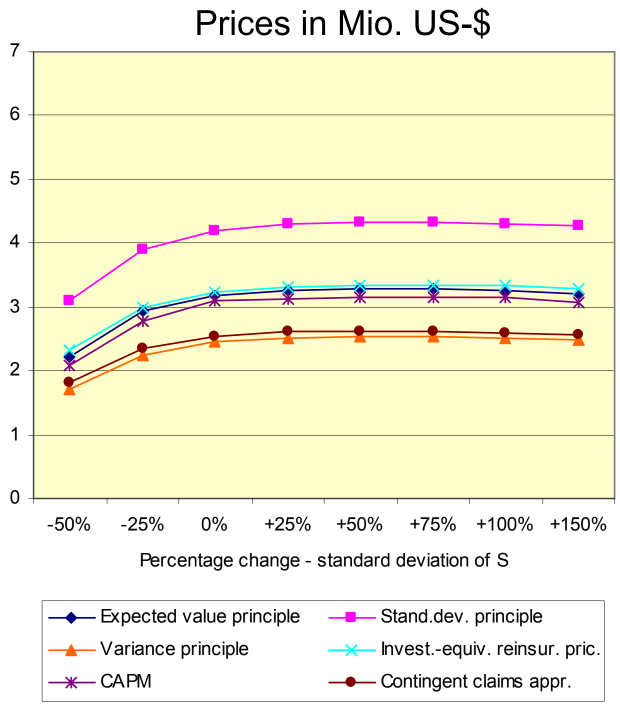


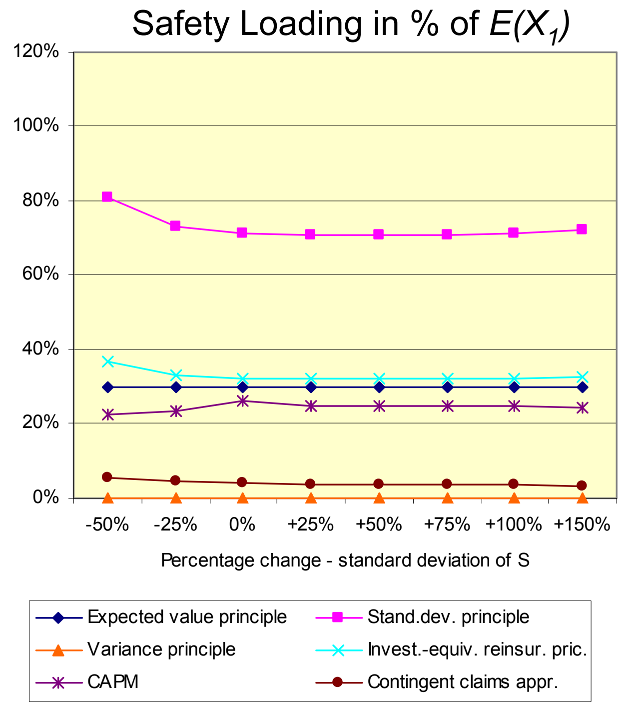
d) Basis Risk Type II in Mio. US-$ and %



6. Numerical Analysis

- Prices and safety loading as functions of percentage change in the company loss volatility $\sigma(S_1)$





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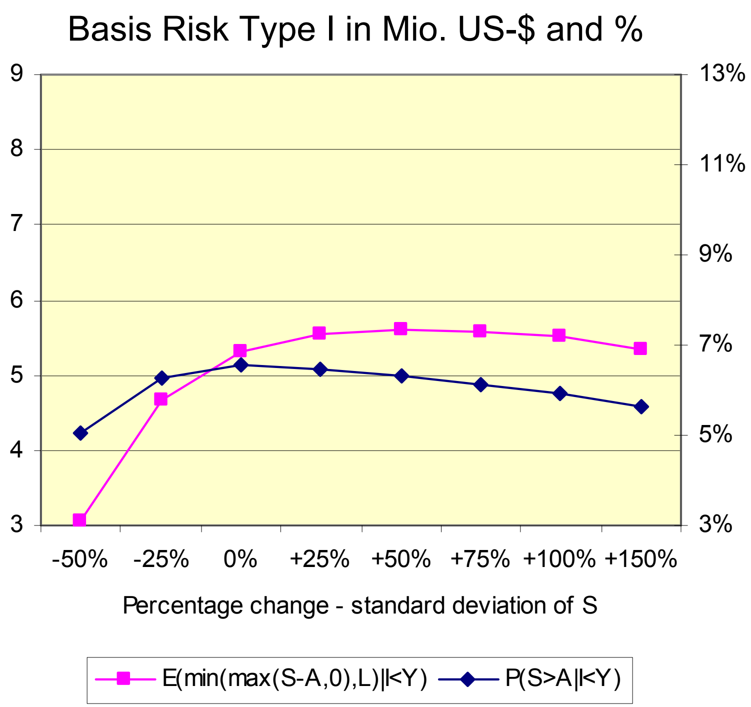
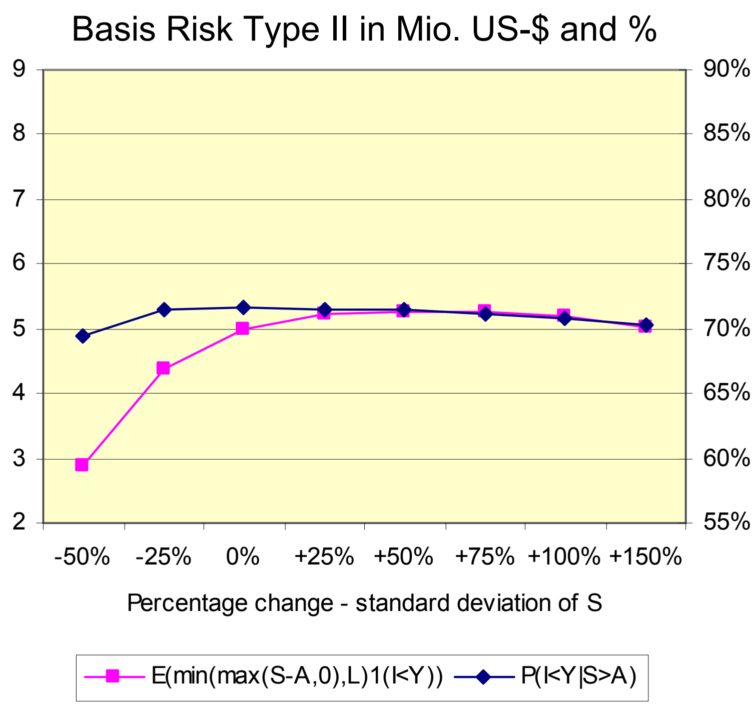
- Basis risk as function of percentage change in the company loss volatility $\sigma(S_1)$

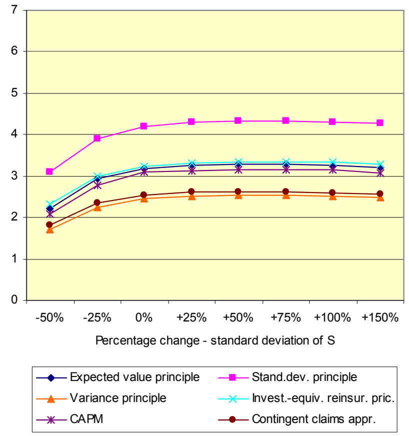




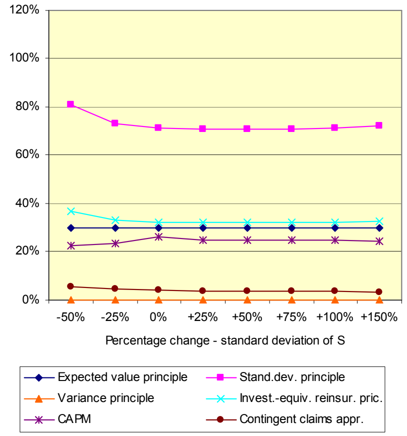
6. Numerical Analysis

- Prices, safety loadings, and basis risk as functions of percentage changes in the company loss volatility $\sigma(S_1)$

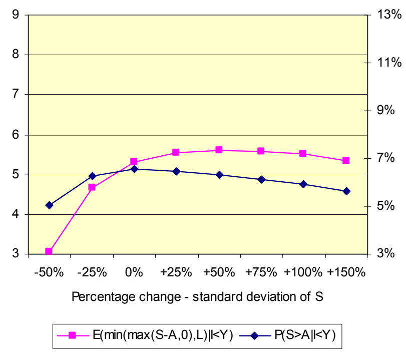
a) Prices in Mio. US-$



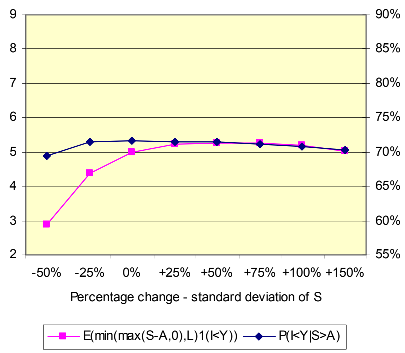
b) Safety Loading in % of $E(X_1)$



c) Basis Risk Type I in Mio. US-$ and %



d) Basis Risk Type II in Mio. US-$ and %



6. Numerical Analysis

- Also analyzed prices, safety loadings, and basis risk with respect to changes in industry loss trigger, company loss trigger and industry loss volatility
- For increasing industry loss trigger, prices are decreasing while basis risk is increasing
- Prices and basis risk are decreasing for increasing company loss trigger
- For industry loss volatility, basis risk has opposite shape of company loss volatility

7. Conclusion

- Analyzed pricing and basis risk of an indemnity-based ILW contract
- Conducted sensitivity analysis of price and basis risk for changes in contract parameter
- Compared six actuarial and financial pricing methods
- Discussed economic differences between pricing approa-ches
- Identified three groups of pricing methods leading to similar prices
- Provided safety loadings for the different pricing schemes