1. Introduction

- Increasing demand for investment products with financial guarantees

- We compare risk and return profiles of two actual products currently offered on the Swiss capital market

  - Interest rate guarantee of 2% on the premiums paid into the contract, given a traditionally managed underlying fund

  - A lookback guarantee secured by a Constant Proportion Portfolio Insurance (CPPI) strategy with respect to the underlying fund
2. Model Framework

- Number of units acquired:
  \[ n_i = \frac{P}{S_{t_i}}, \quad i \in \{0,\ldots,N-1\} \]

- Total number of units at time \( t_i \) before paying the \((i + 1)\)st premium:
  \[ N_{t_i} = \sum_{j=0}^{i-1} n_{t_j}, \quad i \in \{1,\ldots,N-1\} \]

\( \Delta t = t_j - t_{j-1} = 1/12 \)
Mutual Fund with Interest Rate Guarantee

- Guaranteed maturity payment:

\[ G_T = P \cdot \sum_{j=0}^{N-1} \epsilon^{g(T-t_j)} \]

- The value of the investment in \( T \):

\[ F_T = N_T \cdot S_T = P \cdot \sum_{j=0}^{N-1} \frac{S_T}{S_{t_j}} \]

- Value of the investments in \( t \):

\[ F_t = (F_{t-1} + P) \frac{S_t}{S_{t-1}} \]
Mutual Funds with Investment Guarantees

- Terminal payoff:

\[ L_T^G = \max \left( F_T, G_T \right) = \max \left( P \cdot \sum_{j=0}^{N-1} \frac{S_T}{S_{t_j}}, P \cdot \sum_{j=0}^{N-1} e^{g(T-t_j)} \right) \]

\[ = P \cdot \max \left( \sum_{j=0}^{N-1} \frac{S_T}{S_{t_j}}, \sum_{j=0}^{N-1} e^{g(T-t_j)} \right) = P \cdot \tilde{L}_T^G. \]

- Terminal payoff can be written as:

\[ L_T^G = \max \left( F_T, G_T \right) = F_T + \max \left( G_T - F_T, 0 \right) \]

- References
**Mutual Fund with Lookback Guarantee**

- The fund with the lookback option guarantees a payoff of the highest value (or peak) $H_T$ of the index that has been attained during the policy term:

$$H_T = \max_{j \in \{0,\ldots,N-1\}} S_{t_j}$$

- Terminal payoff:

$$L^H_T = N_T \cdot H_T = P \cdot \sum_{j=0}^{N-1} \frac{\max_{j \in \{0,\ldots,N-1\}} S_{t_j}}{S_{t_j}} = P \cdot \tilde{L}^H_T$$

- Minimum rate of return $g$ on premiums: 0%
3. Investment Strategies

Traditional Fund (Constant Average Rate of Return and Standard Derivation)

- Geometric Brownian motion

Constant Proportion Portfolio Insurance (CPPI)

- Risky investment $A$: geometric Brownian motion

$$dA_t = A_t(\mu dt + \sigma dW_t)$$

- Bond process $B$ with riskless rate of return $r$

$$dB_t = B_trdt$$
• Evolution of the underlying fund giving discrete monthly adjustments:

\[ S_{t_j} = S_{t_{j-1}} \cdot \left( \alpha_{t_{j-1}} \cdot \frac{A_{t_j}}{A_{t_{j-1}}} + (1 - \alpha_{t_{j-1}}) \cdot \frac{B_{t_j}}{B_{t_{j-1}}} \right) = S_{t_{j-1}} \cdot \left( \alpha_{t_{j-1}} \cdot e^{r_t^A} + (1 - \alpha_{t_{j-1}}) \cdot e^{r_t^{A \Delta}} \right) \]

\[ r_{t_j}^A = \mu_A \cdot \Delta t_j + \sigma_A \cdot \sqrt{\Delta t_j} \cdot Z_{t_j} \]

• Cushion \( C \) and stock limit \( \alpha \):

\[ C_{t_i} = (F_{t_i} + P) - e^{-r(T-t_i)} \cdot G_{t_i} \]

\[ \alpha_{t_i} = \min \left\{ \max \left( \frac{m \cdot C_{t_i}}{F_{t_i}}, 0 \right), \alpha_0 \right\} \]
4. Valuation and Performance Measurement

- Valuation of the investment guarantee using risk-neutral valuation:

\[
\Pi_0 = E^Q \left( e^{-rT} L_T \right) - P \cdot \sum_{j=0}^{N-1} e^{-rT_j} = P \left( E^Q \left( e^{-rT} \tilde{L}_T \right) - \sum_{j=0}^{N-1} e^{-rT_j} \right)
\]

\[
dS_t = S_t \left( rdt + \sigma dW_t^Q \right)
\]

- Analysis of maturity payoff:

\[
E \left( L_T \right) = P \cdot E \left( \tilde{L}_T \right)
\]

\[
\sigma \left( L_T \right) = P \cdot \sigma \left( \tilde{L}_T \right)
\]
Performance measurement of maturity payoff:

\[
SR(L_T) = \frac{E(L_T) - P \cdot \sum_{j=0}^{N-1} r(T-t_j)}{\sigma(L_T)} = \frac{E(\tilde{L}_T) - \sum_{j=0}^{N-1} e^{r(T-t_j)}}{\sigma(\tilde{L}_T)}
\]

\[
Omega(L_T) = \frac{E(L_T) - P \cdot \sum_{j=0}^{N-1} e^{r(T-t_j)}}{LPM_1(L_T, E(L_T))}
\]

\[
Sortino\ ratio(L_T) = \frac{E(L_T) - P \cdot \sum_{j=0}^{N-1} e^{r(T-t_j)}}{\sqrt{LPM_2(L_T, E(L_T))}}
\]

\[
LPM_k(L_T, E(L_T)) = E\left(\max\left(E(L_T) - L_T, 0\right)^k\right)
\]
Mutual Funds with Investment Guarantees

• „Shortfall probability“:

\[ SP^G = \text{Prob}(G_T > F_T) \]

\[ SP^H = \text{Prob}(L_T^H > F_T) \]

• Parameter configuration: \( T = 10 \) years, \( P = 100 \), monthly reallocation

- Traditional fund: \( \mu = 4.7\%; \sigma = 8.1\% \)

- CPPI strategy: \( m = 2; \alpha_0 = 50\%; r = 2.556\% \)

\[
\mu_A = 8.0\% \quad \sigma_A = 20.0\%
\]
• **Case 1:** Comparison of actual products
  – Lookback guarantee with CPPI managed underlying
  – Interest rate guarantee with traditional underlying fund

**Table 1:** Case 1—comparison of actual products: look-back guarantee with CPPI managed fund and interest rate guarantee with traditional underlying fund

<table>
<thead>
<tr>
<th></th>
<th>Interest rate guarantee</th>
<th>Lookback guarantee</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pi_0$</td>
<td>499.67</td>
<td>—</td>
</tr>
<tr>
<td>$g$</td>
<td>2.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>$SP$</td>
<td>17.61%</td>
<td>0.00%</td>
</tr>
<tr>
<td>$E(T)$</td>
<td>15,801.66</td>
<td>14,464.91</td>
</tr>
<tr>
<td>$\sigma(T)$</td>
<td>2,266.60</td>
<td>867.61</td>
</tr>
<tr>
<td>$L_T$ Minimum</td>
<td>13,295.24</td>
<td>12,000.00</td>
</tr>
<tr>
<td>$SR(T)$</td>
<td>0.9327</td>
<td>0.8959</td>
</tr>
<tr>
<td>$Omega(T)$</td>
<td>2.3274</td>
<td>2.2386</td>
</tr>
<tr>
<td>$Sortino,ratio(T)$</td>
<td>1.5493</td>
<td>1.3811</td>
</tr>
</tbody>
</table>

**Notes:** $\Pi_0$ = value of the guarantee in $t = 0$; $g$ = minimum rate of return; $SP$ = shortfall probability; $L_T$ = contract’s payoff at maturity $T$; $SR$ = Sharpe ratio; $Omega$ = Omega performance measure; $Sortino\,ratio$ = Sortino ratio performance measure.
• Case 1: Comparison of maturity payoffs:
• **Case 2:** Both products with traditional underlying fund, same guarantee costs

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<tr>
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<tbody>
<tr>
<td>$\Pi_0$</td>
<td>1,058.59</td>
<td>1,058.57</td>
</tr>
<tr>
<td>$g$</td>
<td>3.70%</td>
<td>0.00%</td>
</tr>
<tr>
<td>$SP$</td>
<td>35.84%</td>
<td>78.44%</td>
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<tr>
<td>$E(L_T)$</td>
<td>16,129.17</td>
<td>16,382.05</td>
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<tr>
<td>$\sigma(L_T)$</td>
<td>1,960.81</td>
<td>2,107.03</td>
</tr>
<tr>
<td>$L_T$ Minimum</td>
<td>14,543.52</td>
<td>12,000.00</td>
</tr>
<tr>
<td>$SR(L_T)$</td>
<td>1.2452</td>
<td>1.2788</td>
</tr>
<tr>
<td>$Omega(L_T)$</td>
<td>2.3077</td>
<td>2.1153</td>
</tr>
<tr>
<td>$Sortino\ ratio(L_T)$</td>
<td>3.1543</td>
<td>3.2411</td>
</tr>
</tbody>
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• **Case 2: Comparison of maturity payoffs:**

![Graph showing comparison of maturity payoffs for Interest Rate Guarantee and Lookback Guarantee](graph.png)
Mutual Funds with Investment Guarantees

**Case 3:**
- Both products have CPPI managed underlying fund
- Minimum interest rate guarantee of 0%

<table>
<thead>
<tr>
<th>Table 3: Both investment products have CPPI managed underlying fund</th>
</tr>
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<tbody>
<tr>
<td><strong>Variable</strong></td>
</tr>
<tr>
<td>( \Pi_0 )</td>
</tr>
<tr>
<td>( g )</td>
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<tr>
<td>( SP )</td>
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<tr>
<td>( E(L_T) )</td>
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*Notes: \( \Pi_0 \) = value of the guarantee in \( t = 0 \); \( g \) = minimum rate of return; \( SP \) = shortfall probability; \( L_T \) = contract’s payoff at maturity \( T \); \( SR \) = Sharpe ratio; \( Omega \) = Omega performance measure; \( Sortino ratio \) = Sortino ratio performance measure.*
• **Case 3: Comparison of maturity payoffs:**

![Diagram showing comparison of maturity payoffs for Interest Rate Guarantee and Lookback Guarantee.](image-url)
5. Summary

• Comparison of risk and return profiles of mutual funds with investment guarantees:
  – Lookback guarantee
  – Interest rate guarantee

• Different underlying investment strategy

• Showed interaction between type of guarantee, its costs, and underlying strategy of mutual fund

=> Interest rate guarantee has
  – Higher volatility
  – Higher expected maturity payoff
  – Lower guarantee costs
  – Performance measures depend on risk measure