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Mutual Funds with Investment Guarantees –

A Comparison of Guarantee Costs and Performance

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1. Introduction

- Increasing demand for investment products with financial guarantees

- We compare risk and return profiles of two actual products currently offered on the Swiss capital market

- Interest rate guarantee of 2% on the premiums paid into the contract, given a traditionally managed underlying fund

- A lookback guarantee secured by a Constant Proportion Portfolio Insurance (CPPI) strategy with respect to the underlying fund

2. Model Framework

- Number of units aquired:

$$n_{t_i} = \frac{P}{S_{t_i}}, \quad i \in \{0, \ldots, N-1\}$$

- Total number of units at time t_i before paying the $(i + 1)$st premium:

$$N_{t_i} = \sum_{j=0}^{i-1} n_{t_j}, \quad i \in \{1, \ldots, N-1\}$$

$$\Delta t = t_j - t_{j-1} = 1/12$$

Mutual Fund with Interest Rate Guarantee

- Guaranteed maturity payment:

$$G_T = P \cdot \sum_{j=0}^{N-1} e^{g(T-t_j)}$$

- The value of the investment in *T:*

$$F_T = N_T \cdot S_T = P \cdot \sum_{j=0}^{N-1} \frac{S_T}{S_{t_j}}$$

- Value of the investments in *t*:

$$F_t = (F_{t-1} + P)\frac{S_t}{S_{t-1}}$$

- Terminal payoff:

$$L_T^G = \max\left(F_T, G_T\right) = \max\left(P \cdot \sum_{j=0}^{N-1} \frac{S_T}{S_{t_j}}, P \cdot \sum_{j=0}^{N-1} e^{g\left(T-t_j\right)}\right)$$

$$= P \cdot \max\left(\sum_{j=0}^{N-1} \frac{S_T}{S_{t_j}}, \sum_{j=0}^{N-1} e^{g\left(T-t_j\right)}\right) = P \cdot \tilde{L}_T^G.$$

- Terminal payoff can be written as:

$$L_T^G = \max\left(F_T, G_T\right) = F_T + \max\left(G_T - F_T, 0\right)$$

- References

Mutual Fund with Lookback Guarantee

- The fund with the lookback option guarantees a payoff of the highest value (or peak) H_T of the index that has been attained during the policy term:

$$H_T = \max_{j \in \{0,\ldots,N-1\}} S_{t_j}$$

- Terminal payoff:

$$L_T^H = N_T \cdot H_T = P \cdot \sum_{j=0}^{N-1} \frac{\max_{j \in \{0,\ldots,N-1\}} S_{t_j}}{S_{t_j}} = P \cdot \tilde{L}_T^H$$

- Minimum rate of return g on premiums: 0%

3. Investment Strategies

Traditional Fund (Constant Average Rate of Return and Standard Derivation)

- Geometric Brownian motion

Constant Proportion Portfolio Insurance (CPPI)

- Risky investment *A*: geometric Brownian motion

$$dA_t = A_t\left(\mu dt + \sigma dW_t\right)$$

- Bond process *B* with riskless rate of return *r*

$$dB_t = B_t r dt$$

- Evolution of the underlying fund giving discrete monthly adjustments:

$$S_{t_j} = S_{t_{j-1}} \cdot \left(\alpha_{t_{j-1}} \cdot \frac{A_{t_j}}{A_{t_{j-1}}} + \left(1-\alpha_{t_{j-1}}\right) \cdot \frac{B_{t_j}}{B_{t_{j-1}}} \right) = S_{t_{j-1}} \cdot \left(\alpha_{t_{j-1}} \cdot e^{r_{t_j}^A} + \left(1-\alpha_{t_{j-1}}\right) \cdot e^{r\Delta t_j} \right)$$

$$r_{t_j}^A = \mu_A \cdot \Delta t_j + \sigma_A \cdot \sqrt{\Delta t_j} \cdot Z_{t_j}$$

- Cushion *C* and stock limit α:

$$C_{t_i} = \left(F_{t_i} + P\right) - e^{-r(T-t_i)} \cdot G_{t_i}$$

$$\alpha_{t_i} = \min\left\{ \max\left(\frac{m \cdot C_{t_i}}{F_{t_i}}, 0 \right), \alpha_0 \right\}$$

4. Valuation and Performance Measurement

- Valuation of the investment guarantee using risk-neutral valuation:

$$\Pi_0 = E^Q\left(e^{-rT} L_T\right) - P \cdot \sum_{j=0}^{N-1} e^{-rt_j} = P \cdot \left(E^Q\left(e^{-rT} \tilde{L}_T\right) - \sum_{j=0}^{N-1} e^{-rt_j} \right)$$

$$dS_t = S_t\left(r dt + \sigma dW_t^Q\right)$$

- Analysis of maturity payoff:

$$E\left(L_T\right) = P \cdot E\left(\tilde{L}_T\right)$$

$$\sigma\left(L_T\right) = P \cdot \sigma\left(\tilde{L}_T\right)$$

- Performance measurement of maturity payoff:

$$SR(L_T) = \frac{E(L_T) - P \cdot \sum_{j=0}^{N-1} e^{r(T-t_j)}}{\sigma(L_T)} = \frac{E(\tilde{L}_T) - \sum_{j=0}^{N-1} e^{r(T-t_j)}}{\sigma(\tilde{L}_T)}$$

$$Omega(L_T) = \frac{E(L_T) - P \cdot \sum_{j=0}^{N-1} e^{r(T-t_j)}}{LPM_1(L_T, E(L_T))}$$

$$Sortino\,ratio(L_T) = \frac{E(L_T) - P \cdot \sum_{j=0}^{N-1} e^{r(T-t_j)}}{\sqrt{LPM_2(L_T, E(L_T))}}$$

$$LPM_k(L_T, E(L_T)) = E\left(\max\left(E(L_T) - L_T, 0\right)^k\right)$$

Mutual Funds with Investment Guarantees

- „Shortfall probability“:

$$SP^G = \text{Prob}\left(G_T > F_T\right)$$

$$SP^H = \text{Prob}\left(L_T^H > F_T\right)$$

- Parameter configuration: T = 10 years, P = 100, monthly reallocation

- Traditional fund: μ = 4.7%; σ = 8.1%

- CPPI strategy: m = 2; α_0 = 50%; r = 2.556%

$$\mu_A = 8.0\% \quad \sigma_A = 20.0\%$$

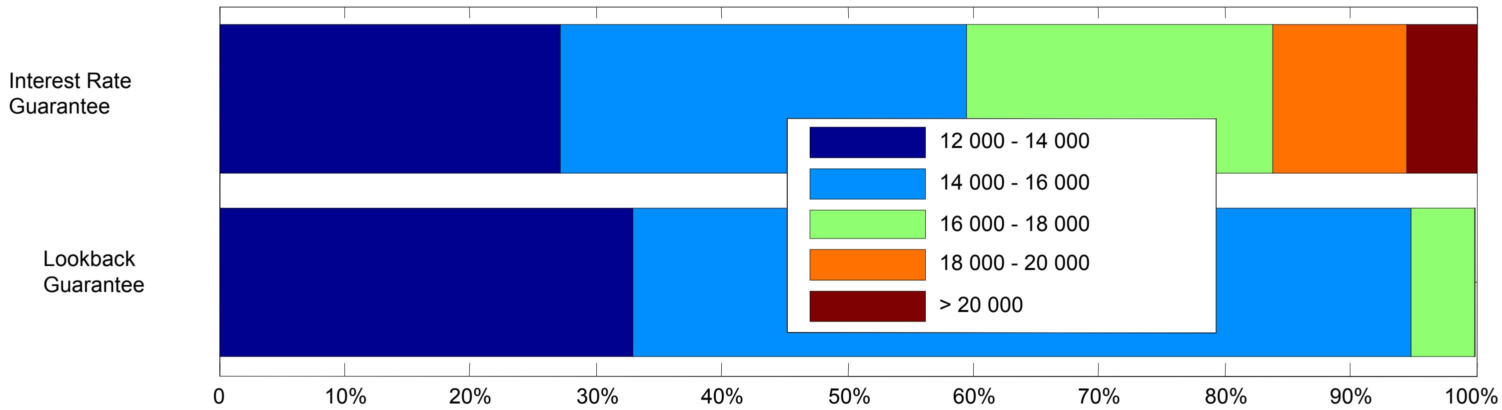
- Case 1: Comparison of actual products
  - Lookback guarantee with CPPI managed underlying
  - Interest rate guarantee with traditional underlying fund

Table 1: Case 1—comparison of actual products: look-back guarantee with CPPI managed fund and interest rate guarantee with traditional underlying fund

	Interest rate guarantee	Lookback guarantee
Π_0	499.67	—
g	2.00%	0.00%
SP	17.61%	0.00%
$E(L_T)$	15,801.66	14,464.91
$\sigma(L_T)$	2,266.60	867.61
L_T Minimum	13,295.24	12,000.00
$SR(L_T)$	0.9327	0.8959
$Omega(L_T)$	2.3274	2.2386
$Sortino\,ratio(L_T)$	1.5493	1.3811

Notes: Π_0 = value of the guarantee in t = 0; g = minimum rate of return; SP = shortfall probability; L_T = contract's payoff at maturity T; SR = Sharpe ratio; $Omega$ = Omega performance measure; $Sortino\,ratio$ = Sortino ratio performance measure.

- Case 1: Comparison of maturity payoffs:



- Case 2: Both products with traditional underlying fund, same guarantee costs

Table 2: Case 2—both investment products have traditional underlying fund

	Interest rate guarantee	Lookback guarantee
Π_0	1,058.59	1,058.57
g	3.70%	0.00%
SP	35.84%	78.44%
$E(L_T)$	16,129.17	16,382.05
$\sigma(L_T)$	1,960.81	2,107.03
L_T Minimum	14,543.52	12,000.00
$SR(L_T)$	1.2452	1.2788
$Omega(L_T)$	2.3077	2.1153
$Sortino\,ratio(L_T)$	3.1543	3.2411

Notes: Π_0 = value of the guarantee in $t = 0$; g = minimum rate of return; SP = shortfall probability; L_T = contract's payoff at maturity T; SR = Sharpe ratio; $Omega$ = Omega performance measure; $Sortino\,ratio$ = Sortino ratio performance measure.

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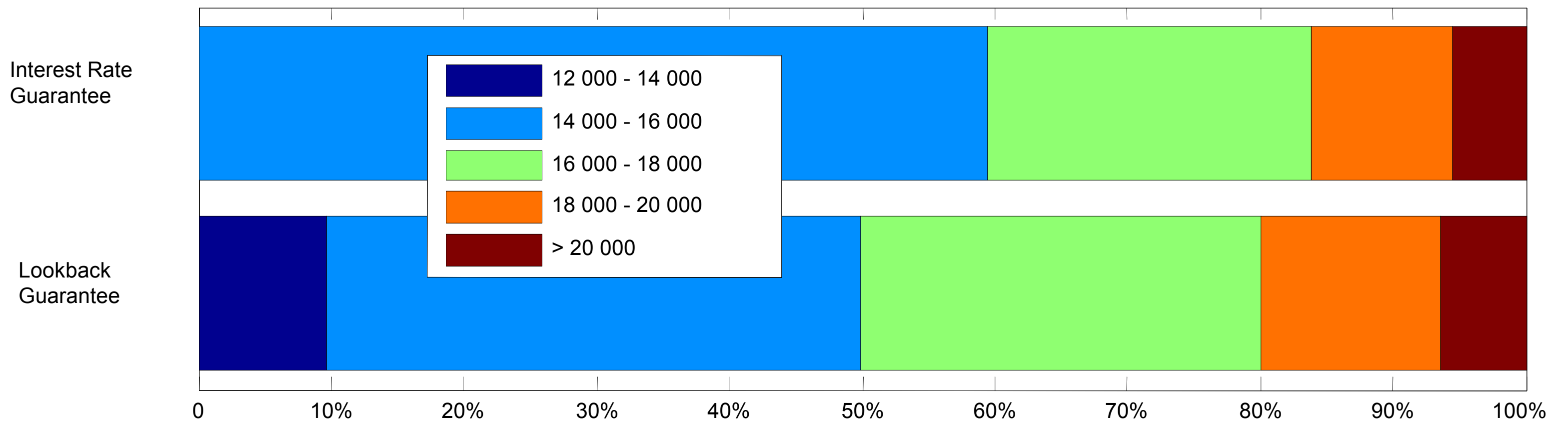

- Case 3:
  - Both products have CPPI managed underlying fund
  - Minimum interest rate guarantee of 0%

Table 3: Both investment products have CPPI managed underlying fund

	Interest rate guarantee	Lookback guarantee
Π_0	—	—
g	0.00%	0.00%
SP	0.00%	0.00%
$E(L_T)$	16,128.70	14,464.91
$\sigma(L_T)$	3,229.78	867.61
L_T Minimum	12,000.00	12,000.00
$SR(L_T)$	0.7558	0.8959
$Omega(L_T)$	1.8943	2.2386
$Sortino\,ratio(L_T)$	1.3233	1.3811

Notes: Π_0 = value of the guarantee in $t = 0$; g = minimum rate of return; SP = shortfall probability; L_T = contract's payoff at maturity T; SR = Sharpe ratio; $Omega$ = Omega performance measure; $Sortino\,ratio$ = Sortino ratio performance measure.

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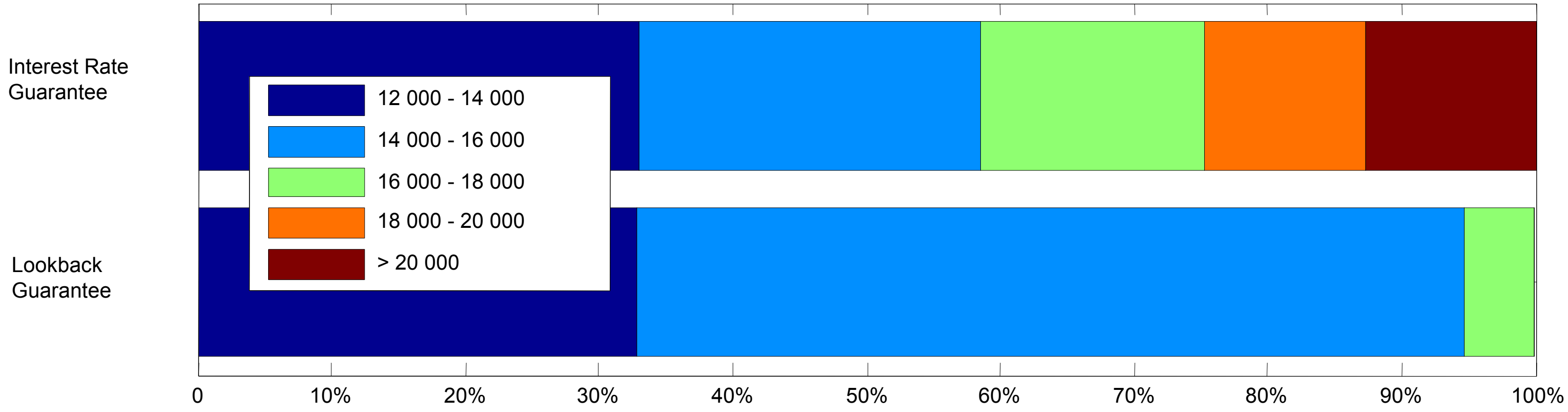

5. Summary

- Comparison of risk and return profiles of mutual funds with investment guarantees:
  - Lookback guarantee
  - Interest rate guarantee
- Different underlying investment strategy
- Showed interaction between type of guarantee, its costs, and underlying strategy of mututal fund

=> Interest rate guarantee has
- Higher volatility
- Higher expected maturity payoff
- Lower guarantee costs
- Performance measures depend on risk measure