Institute of Insurance Economics

University of St.Gallen

Portfolio Optimization under Solvency II – Implicit Constraints Imposed by the Market Risk Standard Formula

Alexander Braun, Hato Schmeiser, Florian Schreiber

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Overview – Research Question and Procedure

Research Question

- What are the influences of the market risk module of the Solvency II Standard Formula on the asset allocation of an insurer?
- Can the detected influences be economically justified?

Procedure

1. Starting point: Portfolio Selection in a mean-variance setting by using empirical data
2. Effects of a) the Solvency II Standard Formula and b) a proposed partial internal model for market risk on efficient and inefficient portfolios are derived
3. Results of a) and b) are compared

Policy implications

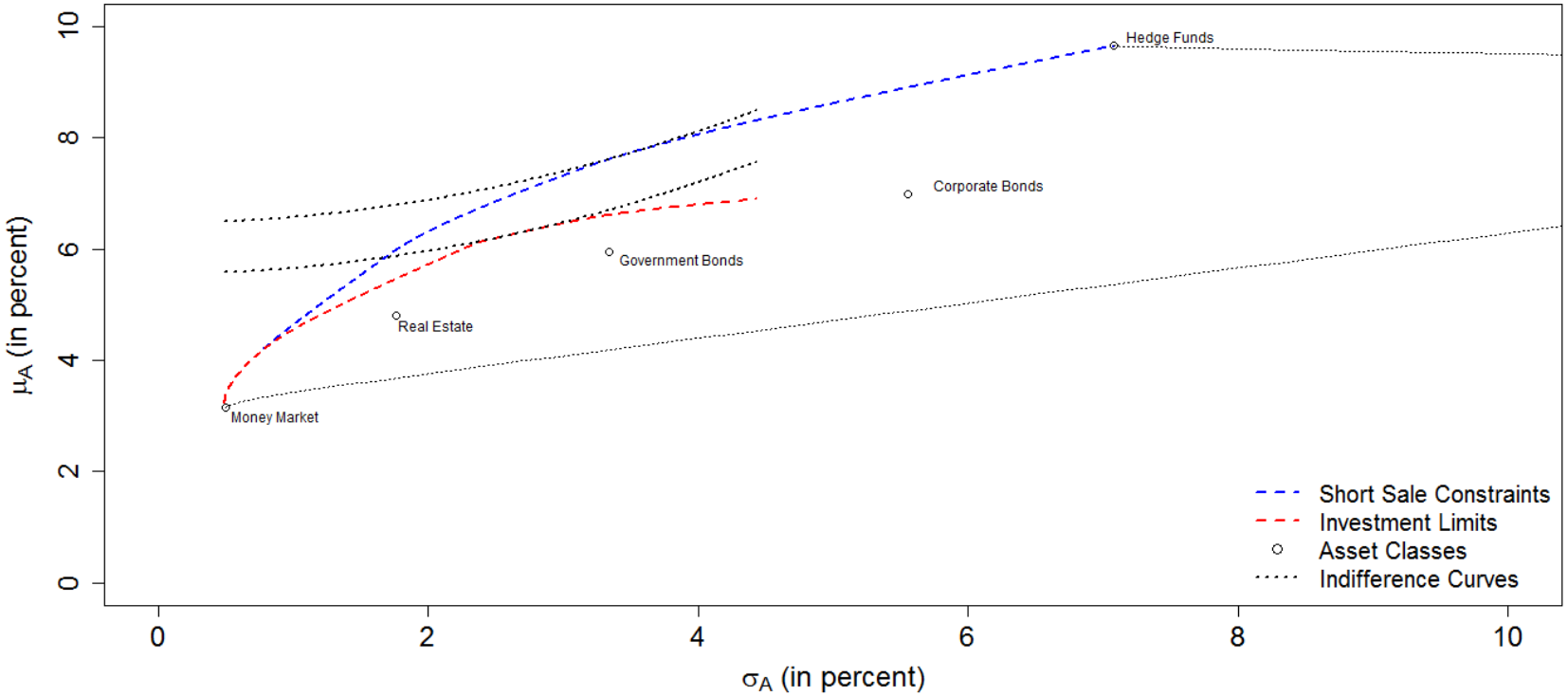
Empirical Data

Six asset classes

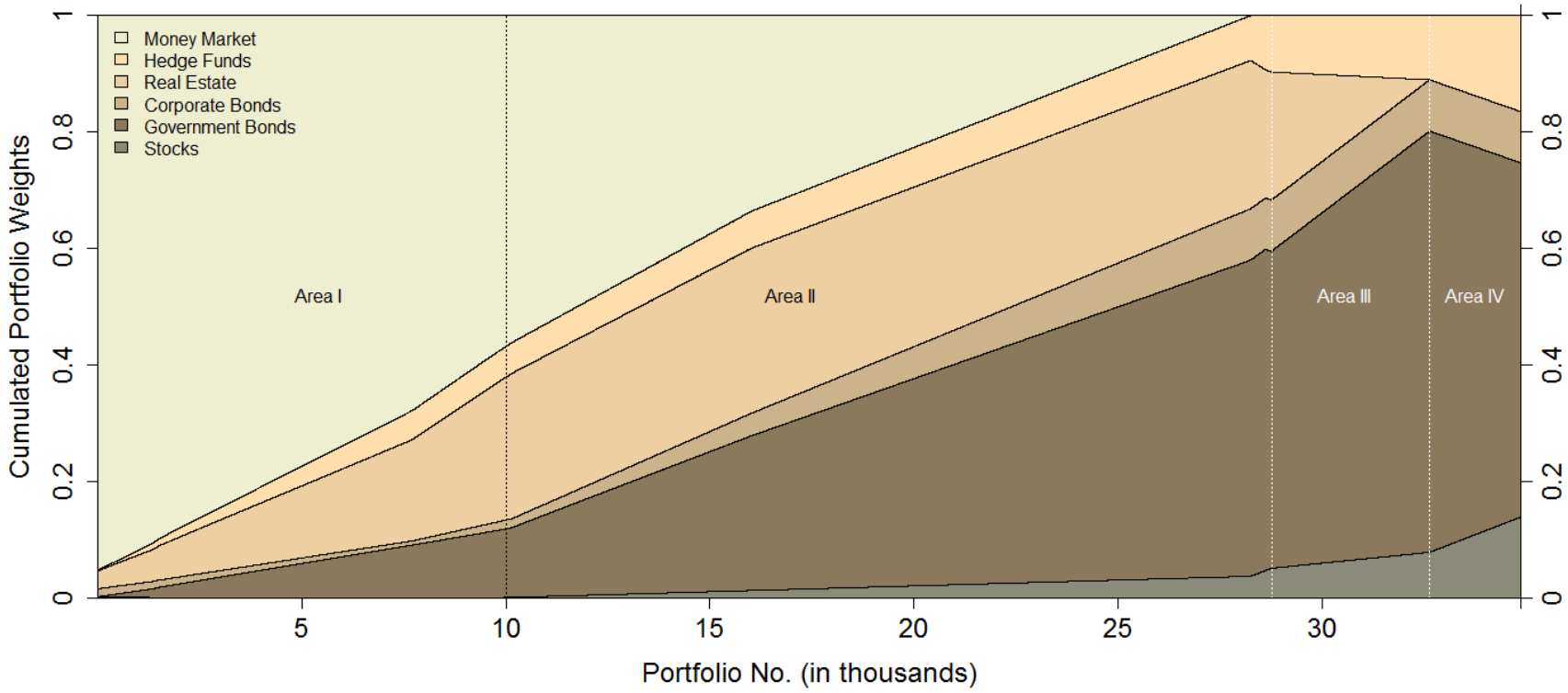
- Discrete returns for 20-year period from 01/1993 until 12/2012

Stocks	▪ EURO STOXX 50 Total Return Index
Government Bonds	▪ German Stock Exchange REX Performance Index (REXP)
Corporate Bonds	▪ Barclays U.S. Corporate Bond Total Return Index
Real Estate	▪ Real Estate *"Grundbesitz"* Europa Fund (adjusted for dividends)
Hedge Funds	▪ HFRI Fund Weighted Composite Index
Money Market	▪ 1 Month FIBOR/ EURIBOR

Results (1/5) – Efficient Frontiers in a mu-sigma-Space



Results (2/5) – Portfolio Compositions



Partial Internal Model (1/2)

Why normal distribution?

- Simply to have a benchmark that is consistent with what the Solvency II Standard Formula claims to use

Equations

- Basic own funds *BOF* at the end of the year / change within one year respectively:

$$B\tilde{O}F_1 = \tilde{A}_1 - \tilde{L}_1,$$

$$\Delta B\tilde{O}F = B\tilde{O}F_1 - BOF_0 = (\tilde{A}_1 - \tilde{L}_1) - (A_0 - L_0).$$

- Expected value of the change of *BOF* within one year

$$\begin{aligned}
\mu_{\Delta B\tilde{O}F} &= \mathrm{E}[\Delta B\tilde{O}F] = \mathrm{E}[B\tilde{O}F_1 - BOF_0] \\
&= \mathrm{E}[\tilde{A}_1] - \mathrm{E}[\tilde{L}_1] - (A_0 - L_0) \\
&= \mathrm{E}[A_0(1+\tilde{r}_A)] - \mathrm{E}[L_0(1+\tilde{g}_L)] - (A_0 - L_0) \\
&= A_0\mu_A - L_0\mu_L \\
&= A_0\mathbf{w}'\mathbf{M} - L_0\mu_L,
\end{aligned}$$

Partial Internal Model (2/2)

Equations

- Standard deviation of the change of *BOF* within one year

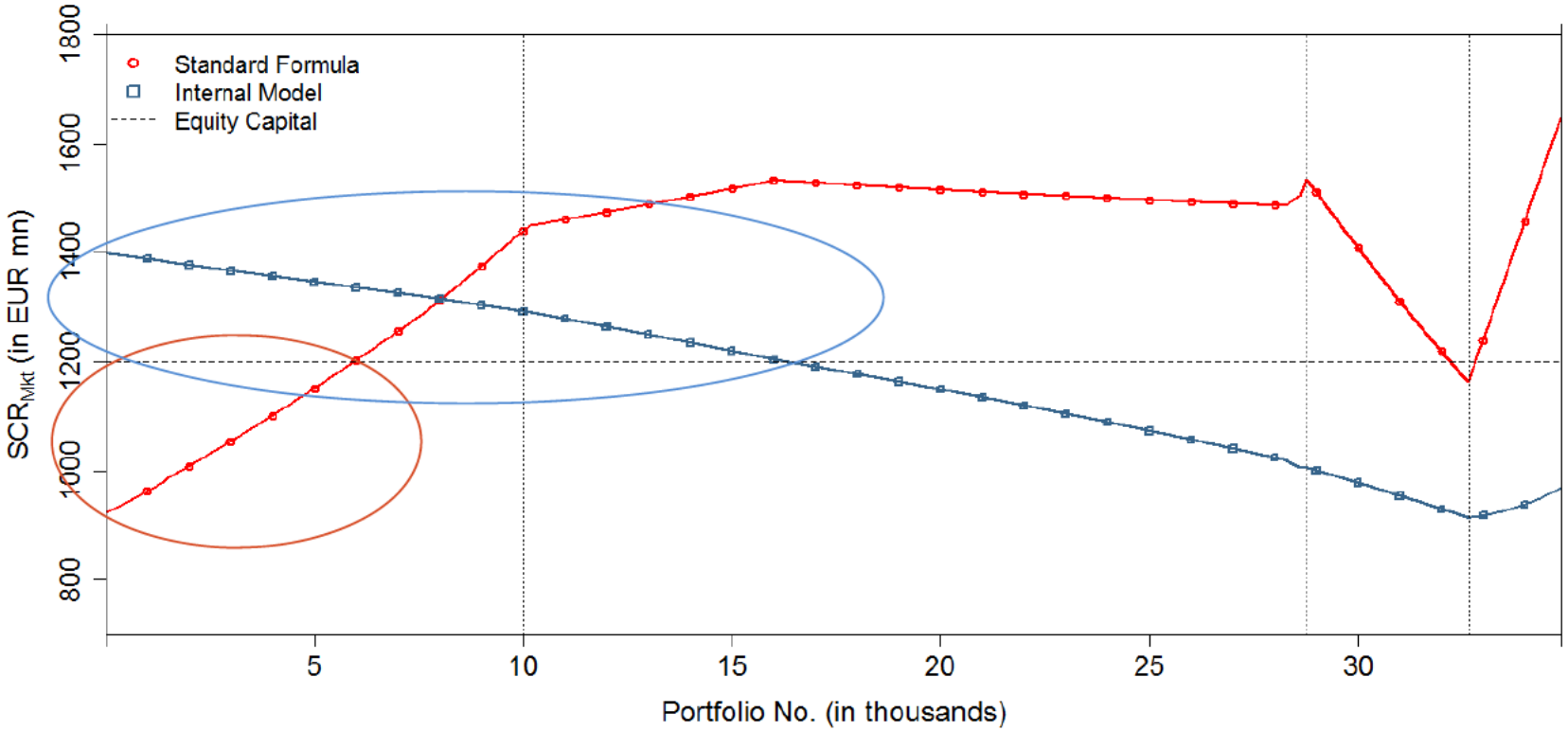
$$\begin{aligned}\sigma^2_{\Delta\tilde{BOF}} &= \mathrm{Var}[\Delta\tilde{BOF}] = \mathrm{Var}[\tilde{BOF}_1 - BOF_0] \\ &= \mathrm{Var}[\tilde{A}_1] + \mathrm{Var}[\tilde{L}_1] - 2\mathrm{cov}[\tilde{A}_1, \tilde{L}_1] \\ &= \mathrm{Var}[A_0(1+\tilde{r}_A)] + \mathrm{Var}[L_0(1+\tilde{g}_L)] - 2\mathrm{cov}[A_0(1+\tilde{r}_A), L_0(1+\tilde{g}_L)] \\ &= A_0^2\sigma_A^2 + L_0^2\sigma_L^2 - 2A_0L_0\sigma_{A,L} = A_0^2\sigma_A^2 + L_0^2\sigma_L^2 - 2A_0L_0\rho_{A,L}\sigma_A\sigma_L \\ &= A_0^2\mathbf{w}'\Sigma\mathbf{w} + L_0^2\sigma_L^2 - 2A_0L_0\mathbf{w}'\mathrm{cov}[\mathbf{R}, \tilde{g}_L].\end{aligned}$$

- Solvency Capital Requirement *SCR* based on the 0,5% confidence level

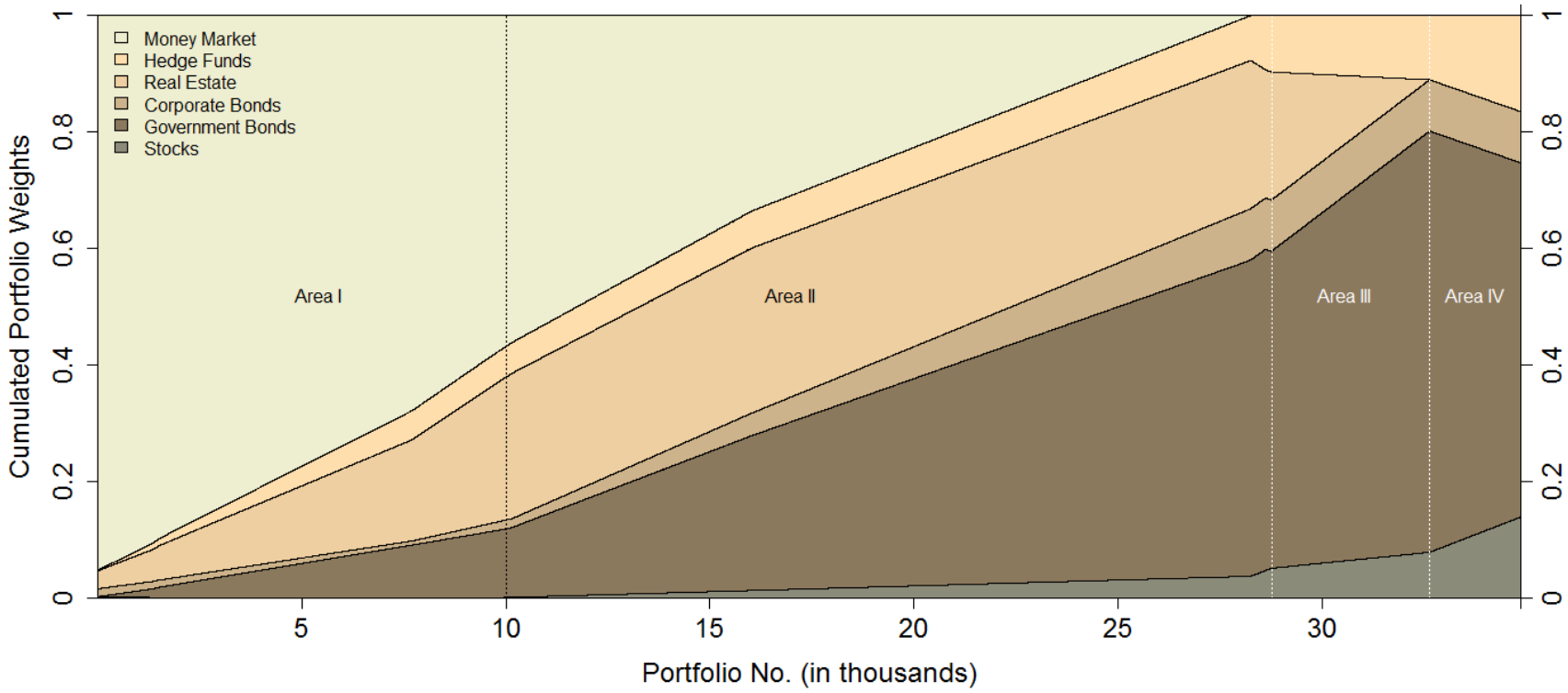
$$SCR_{Mkt} = |(\mu_{\Delta\tilde{BOF}} + z_{0.5\%}\sigma_{\Delta\tilde{BOF}})|$$

- Equity capital of the insurer should exceed *SCR*

Results (3/5) – Capital Requirements for Individual Portfolios

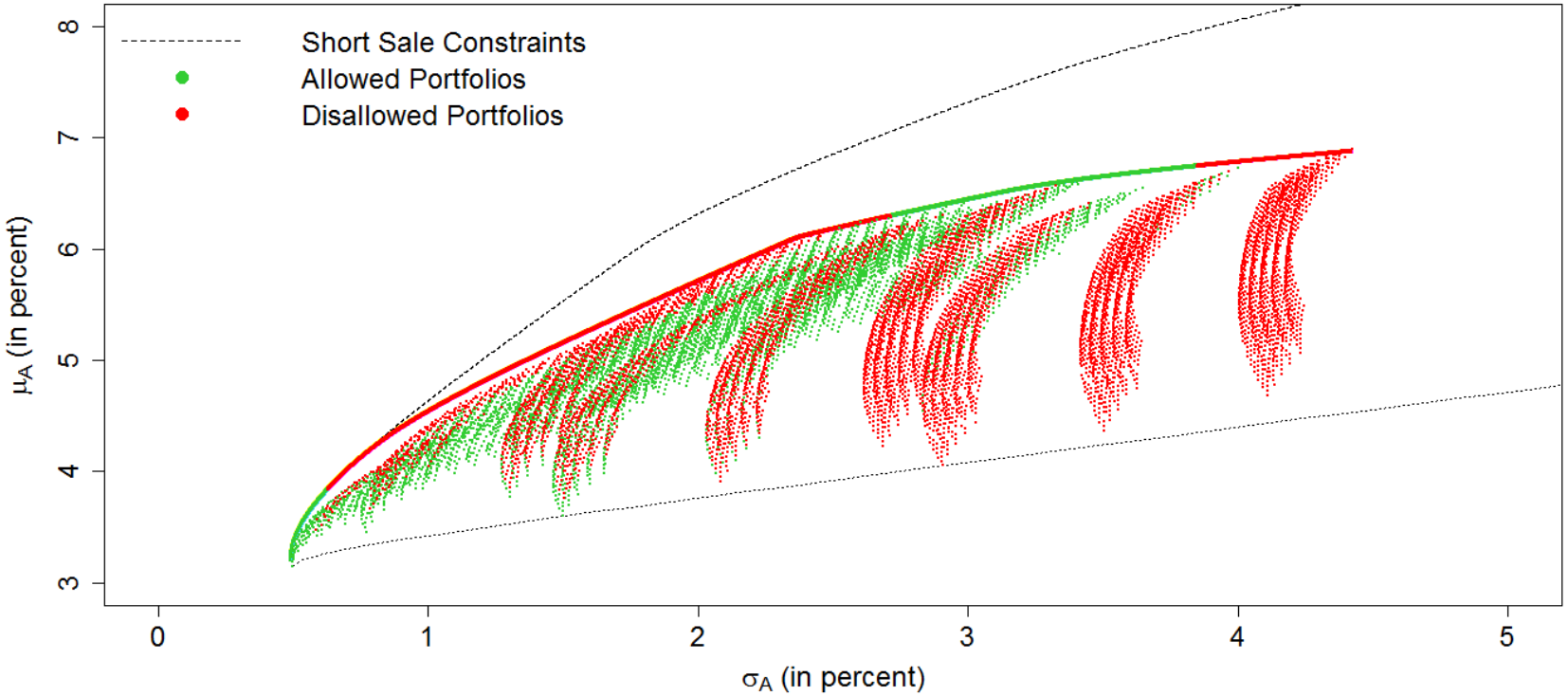


Results (2/5) – Portfolio Compositions

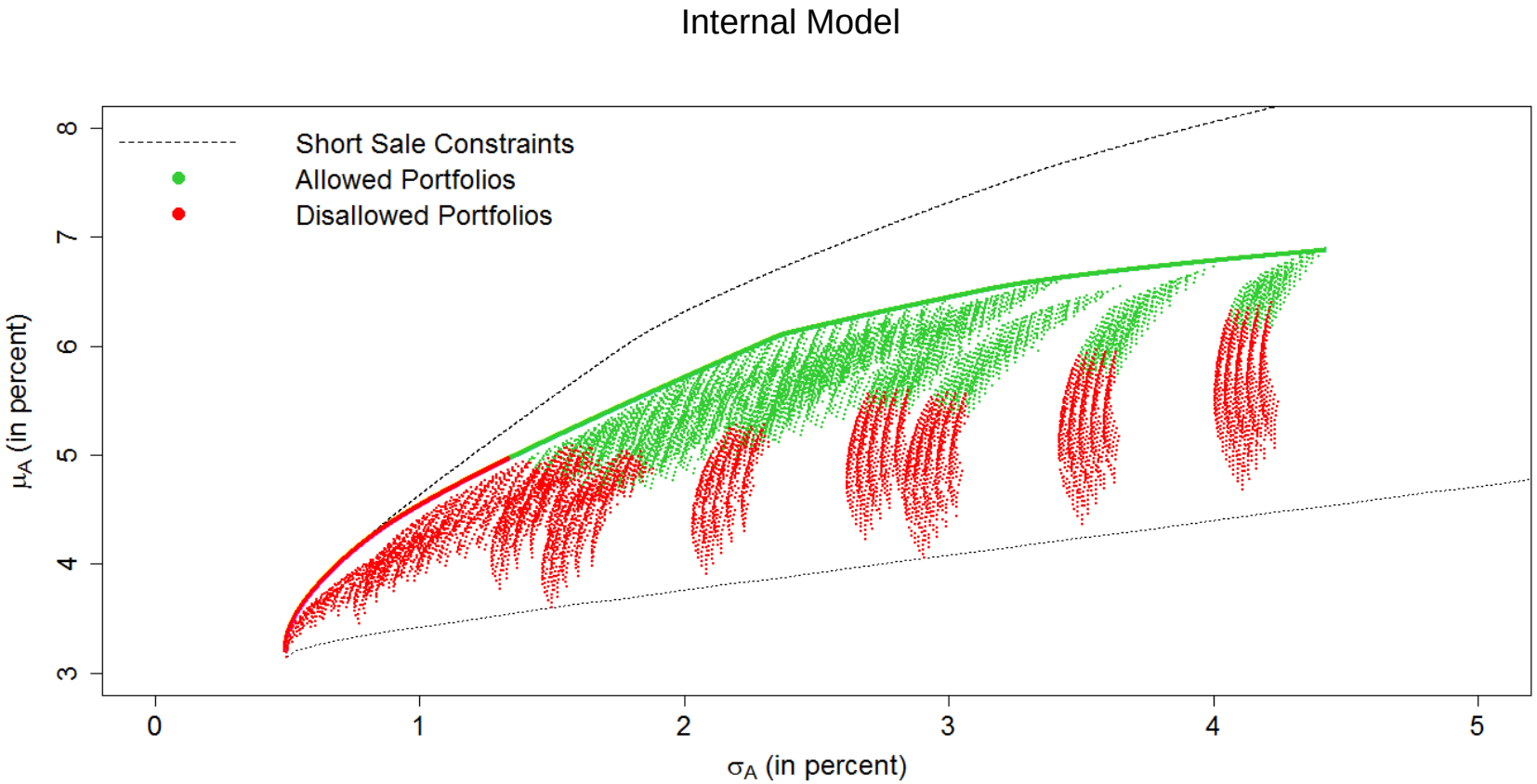


Results (4/5) – Admissibility of Inefficient Portfolios

Solvency II Standard Formula



Results (5/5) – Admissibility of Inefficient Portfolios



Implications

Initial Research Question

- What are the influences of the market risk module of the Solvency II Standard Formula on the asset allocation of an insurer?

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  - Low risk portfolios are chosen, even though they lead to a high ruin probability for the insurance companies
  - The Standard Formula and the used stress factors are not at all consistent with the “mü-sigma-world” (even though this is claimed)
  - The ruin probability of the insurer in a mü-sigma-setting is not at all consistent with the ruin probability of 0,5% proposed by the Standard Formula (cf. back-up)

- Can the detected influences be economically justified?

  →
  - Market risk module of the Solvency II Standard Formula is economically inconsistent
  - Since a majority of European insurers may apply the Standard Formula, substantial impacts (demand and pricing of asset classes on the financial market) can be expected

Summary

- The analysis reveals that many portfolios are not admissible under the Solvency II standard formula
- In a nutshell: The standard approach is unable to distinguish portfolios according to their risk-return profiles and does hence not produce economically sensible results

  ➡ Major concerns:

- The standard formula of Solvency II gives strong incentives to invest in widely undiversified portfolios with low returns and volatilities – insurance practice: Strong investments in government bonds
- Negative effects for main stakeholders: Low asset returns, risk situation within the industry may be much higher than expected

Thank you very much for your attention!

Back Up

Back-up: “True” ruin probability