Saving for Retirement in a Low Interest Rate Environment: Are Life Insurance Products Good Investments?

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1. Introduction

• Background

- EUT can not explain preferences for investment guarantee products

- CPT however can only explain the demand for simple guarantee products

- Hence: Why clique-style interest rate guarantees? (typical guarantee form in participating life insurance contracts for instance in Germany, France, Switzerland)

- Possible reason: Policyholder may care about interim value changes (and not just final payoffs)


- Result: Under MCPT complex guarantees outperform more simple ones (and products without guarantees)
1. Introduction

- **What we do**
  - Focusing on the savings part
  - Two asset Merton model with stochastic assets and interest rates
  - Introducing default risk in respect to embedded investment guarantees
  - Average death and surrender probabilities are taken into account
  - Comparison of three product forms: 1) direct investment (Merton solution), 2) Merton portfolio with point-to-point guarantee and 3) with cliquet-style option

- **Main finding**
  - In general not even MCPT preferences can explain the demand for cliquet-style options
2. Model framework

- **Investment portfolio**

  - Two asset model: Risky equity index evolves according to a GBM; interest rates evolve according to a one-factor Vasicek model

  - Evolution of the investment portfolio for \( t = 1, \ldots, T \) and initial condition \( A_0 \)

\[
A_t^- = A_{t-1}^+ \cdot \exp \left\{ \gamma \int_{t-1}^{t} r_s \, ds + (1 - \gamma) \left( \mu_A \cdot \Delta t - 0.5 \sigma_A^2 \cdot \Delta t \cdot (1 - \gamma) + \sigma_A \left( \rho \cdot \Delta Z_t^\mathbb{P} + \sqrt{1 - \rho^2} \cdot \Delta W_t^\mathbb{P} \right) \right) \right\}
\]

- **Point-to-point guarantee** with initial premium \( P_0 \)

\[
-L_{t}^{p,p} = L_{t-1}^{p,p} \cdot \exp \left( g^{p,p} \cdot \Delta t \right) + \alpha^{p,p} \cdot \max \left( \pi_T \cdot A_T^+ - L_T^{p,p}, 0 \right) \cdot 1_{\{T=t\}}
\]
2. Model framework

• Cliquet-style guarantee

\[-L_t^{cs} = +L_{t-1}^{cs} \cdot \left[ 1 + \max\left(g^{cs}, \alpha^{cs}(A_t^-/A_{t-1}^+ - 1)\right) \right]\]

• Policyholder account adjustment for regular premium payments as well as death and surrender probabilities

\[+L_t = -L_t \cdot (1 - p_t^{d,s}) + P_r \cdot \prod_{i=1}^{t} (1 - p_i^{d,s})\]

- Maintaining the balance between the asset and liability side (adjustment of the investment portfolio)

\[A_t^+ = A_t^- - L_t \cdot p_t^{d,s} + P_r \cdot \prod_{i=1}^{t} (1 - p_i^{d,s})\]
2. Model framework

- Development of the policyholder’s share in the total assets over time

\[
\pi_t = \frac{\pi_{t-1} \cdot A_t^- - p_i^{d,s} \cdot L_t + P_r \cdot \prod_{i=1}^{t} (1 - p_i^{d,s})}{A_t^+}
\]

- with

\[
\pi_0 = \frac{P_0}{P_0 + E_0}
\]

- Insolvencies can occur in $t = 1, \ldots, T$

- Early payouts before $T$ are invested in the money market

- **Policyholder’s payoff** in $T$

\[
P_T = L_T \cdot \mathbb{1}_{\{T>T\}} + M_T + D_T
\]
2. Model framework

- Embedded investments guarantees are valued under the risk-neutral measure Q

- Hence, all contracts are “fairly priced” (i.e., contracts possess a net present value of zero for both stakeholder groups)

- Policyholder uses MCPT for contract valuation (cf. Russ / Schelling (2017))

\[ V_{\text{comb}}(P) = s \cdot \left( \sum_{t=1}^{T-1} V_t^{\text{int}}(P_t) \right) + (1 - s) \cdot V_{\text{term}}(P_T) \]

- Certainty equivalent concept is used to represent the outcomes of the analyses

- Numerical examples are provided via Monte Carlo simulation

- Parameters are motivated in detail in the paper
### 3. Model parametrization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
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</thead>
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<tr>
<td><strong>Contract</strong></td>
<td></td>
<td></td>
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<tr>
<td>Policyholder single up-front premium</td>
<td>$P_0$</td>
<td>1.0 (C.U.)</td>
</tr>
<tr>
<td>Policyholder regular premium</td>
<td>$P_r$</td>
<td>1.0 (C.U.)</td>
</tr>
<tr>
<td>Contract duration</td>
<td>$T$</td>
<td>30 (years)</td>
</tr>
<tr>
<td>Length of a time interval</td>
<td>$[t_i, t_{i+1}]$</td>
<td>1 (year)</td>
</tr>
<tr>
<td><strong>Interest Rate Process</strong></td>
<td></td>
<td></td>
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<tr>
<td>Initial interest rate level</td>
<td>$r_0$</td>
<td>$-0.01%$</td>
</tr>
<tr>
<td>Volatility of the short rate dynamics</td>
<td>$\sigma_r$</td>
<td>0.6%</td>
</tr>
<tr>
<td>Long term average interest rate</td>
<td>$\theta$</td>
<td>2.4%</td>
</tr>
<tr>
<td>Mean reversion speed</td>
<td>$\kappa$</td>
<td>0.08</td>
</tr>
<tr>
<td>Market price of risk of the short rate dynamics</td>
<td>$\lambda_{MP}$</td>
<td>$-0.18$</td>
</tr>
<tr>
<td><strong>Asset Process</strong></td>
<td></td>
<td></td>
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<tr>
<td>Drift of the geometric Brownian Motion</td>
<td>$\mu_A$</td>
<td>9.0%</td>
</tr>
<tr>
<td>Volatility of the geometric Brownian Motion</td>
<td>$\sigma_A$</td>
<td>19.2%</td>
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<td>Correlation of asset and interest rate processes</td>
<td>$\rho$</td>
<td>7.7%</td>
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<td><strong>Solvency regulation</strong></td>
<td></td>
<td></td>
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<tr>
<td>Upper bound of the ruin probability for $[0, T]$</td>
<td>$\varepsilon$</td>
<td>0.5%</td>
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<tr>
<td><strong>Biometric factors</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of death or early surrender</td>
<td>$p_{d,s}$</td>
<td>4.2%</td>
</tr>
</tbody>
</table>
4. Numerical results

• Concept:

  - We aim to find contract parameters that maximize policyholder's utility among all admissible parameter combinations

  - E.g., for a given guarantee level, we find the participation rate and the asset allocation that maximize utility given the restriction explained before

• Cases

  - 1) Maximum CE levels for a product with point-to-point guarantee versus the CE of a direct investment

  - 2) Maximum CE levels for a product with cliquet-style guarantee versus the CE of a direct investment
4. Numerical results

Case 1): point-to-point

Scenario 1: $\phi=0.7$, $\lambda=2.25$, $\phi=0.4$

Scenario 2: $\phi=0.7$, $\lambda=2.25$, $\phi=0.8$
4. Numerical results

Case 1): point-to-point

Scenario 3: $\phi=0.7$, $\lambda=3.0$, $\psi=0.4$

Scenario 4: $\phi=0.7$, $\lambda=3.0$, $\psi=0.8$
4. Numerical results

Case 2): cliquet-style

Scenario 1: $\phi=0.7$, $\lambda=2.25$, $\phi=0.4$

Scenario 2: $\phi=0.7$, $\lambda=2.25$, $\phi=0.8$
4. Numerical results

Case 2): cliquet-style

Scenario 3: $\phi=0.7$, $\lambda=3.0$, $\psi=0.4$

Scenario 4: $\phi=0.7$, $\lambda=3.0$, $\psi=0.8$
5. Findings and outlook

- **CPT:** Direct investment offers higher utility when taking default risk of the investment guarantee into account

- **MCPT:** Only in rather extreme cases with very large values for the interim weight investment guarantees can posses an additional value for policyholders

  - Parameter regulation and product standardization in general reduces policyholder’s utility

  - Why do our results differ from Russ / Schelling (2017)?

  - Even if policyholder’s utility is reduces via investment guarantees, there could be other reasons why such options should be part of old-age provisions

  - But: Why cliquet-style options?