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The Risk of Model Misspecification and its Impact on Solvency Measurement in the Insurance Sector

joint paper with Caroline Siegel and Joël Wagner

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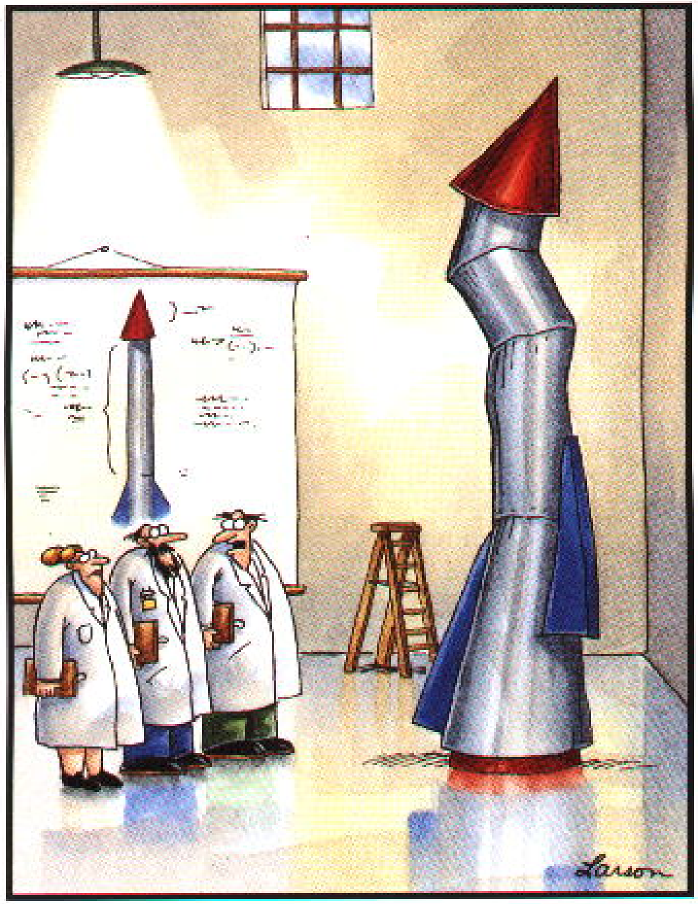

"It's time we face reality, my friends. ... We're not exactly rocket scientists."

Background

- The recent financial crisis has shown that quantitative solvency models can give insurers, regulators, and policyholders a false sense of security
- Most solvency frameworks incorporate linear correlation but neglect non-linear dependencies and stochastic jumps and might perform rather poor in extreme scenarios
- "Model risk" should be analyzed in more details in order to understand the explanatory power of stochastic solvency models

Contribution

- Analysis of model risk in combination with parameter risk of solvency models for property-liability insurers
- Examination of the effects of introducing stochastic jumps, linear, or non-linear dependencies into a basic solvency model on capital requirements and shortfall risk measures
- Consideration of reducing the model uncertainty of solvency models by means of mandatory interim financial reports

Aim

- Examination of the impact of changes to the specification of a solvency model on capital requirements, the shortfall probability and the expected policyholder deficit of a property-liability insurer
- Estimation of the derivations in capital requirements, shortfall probabilities and expected policyholder deficit when using a standard solvency model (Swiss Solvency Test SST and Solvency II) in the case the "real world" behaves differently (we refer to these derivations as "model risk")

Related Literature

- Literature regarding parameter risk (risk of errors in the parameter estimation within a specific model framework)
- Model risk (in general more important than parameter risk)

Concept

- One a) standard model framework in the sense of the SST / Solvency II is defined and its explanatory power is analyzed if b) the underlying stochastic process or c) the dependency structure differs from what had been assumed in a)

Model setting (I)

a) Basic setting

- Standard solvency model as it might be specified by a European regulatory authority
- Usage of independent Brownian motions for the modeling of two asset classes and liabilities
- Calculation of capital requirements as required by the proposed Solvency II framework and the Swiss Solvency Test

$$A_t = A_{1,t} + A_{2,t}.$$

$$dA_{i,t} = \mu_{A_i} A_{i,t} dt + \sigma_{A_i} A_{i,t} dW^{\mathbb{P}}_{A_i,t},$$

$$dL_t = \mu_L L_t dt + \sigma_L L_t dW^{\mathbb{P}}_{L,t},$$

$$A_{i,t} = A_{i,0} \cdot \exp\left((\mu_{A_i} - \sigma_{A_i}^2 / 2) t + \sigma_{A_i} W^{\mathbb{P}}_{A_i,t} \right)$$

$$L_t = L_0 \cdot \exp\left((\mu_L - \sigma_L^2 / 2) t + \sigma_L W^{\mathbb{P}}_{L,t} \right).$$

- One year horizon t = 0, 1; assets $A_{i,t}$, are divided in a high-risk (i = 1) and a low-risk investment (i = 2)

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Model setting (II)

- Available economic capital AEC and solvency capital SC

$$AEC_t = A_t - L_t.$$

$$AEC_{t-1} = A_{t-1} - L_{t-1} \geq SC_t.$$

- Solvency II requirement (value at risk approach with confidence level αVaR = 0,5%) and SST approach (tail value at risk approach with confidence level αTVaR = 1%)

$$X_t = AEC_t/(1+r_f) - AEC_{t-1}.$$

$$SC_t^{\mathrm{VaR}} = -\mathrm{VaR}_{\alpha_{\mathrm{VaR}}}(X_t).$$

$$SC_t^{\mathrm{TVaR}} = \mathrm{TVaR}_{\alpha_{\mathrm{TVaR}}} = -\mathrm{E}(X_t | X_t \leq \mathrm{VaR}_{\alpha_{\mathrm{TVaR}}}),$$

- Risk measurers

$$SP_t = P\left(A_t < L_t\right).$$

$$EPD_t = \mathrm{E}(\max(L_t - A_t, 0))/(1+r_f).$$

Model setting (III)

b) Uncertainty in modeling the development of assets and liabilities

- Introduction of stochastic jumps into the liability process (cf. Merton 1976)

c) Uncertainty in modeling dependencies

- Linear dependencies (pairwise linear correlation measured with Pearson's correlation coefficient)

$$dW^{\mathbb{P}}_{A_1,t}dW^{\mathbb{P}}_{A_2,t} = \rho(A_{1,t}, A_{2,t})dt,\ dW^{\mathbb{P}}_{A_1,t}dW^{\mathbb{P}}_{L,t} = \rho(A_{1,t}, L_t)dt,\ \text{and}\ dW^{\mathbb{P}}_{A_2,t}dW^{\mathbb{P}}_{L,t} = \rho(A_{2,t}, L_t)dt.$$

- Non-linear dependencies (three-dimensional non-exchangeable Clayton copula (cf. McNeil et al. (2005))

Concept

- Measurement of the deviations in risk measures when comparing the three modified solvency models to the basic setting via a risk measure ratio:

$$\text{risk measure ratio} = \frac{\text{risk measure}^{BS}}{\text{risk measure}^{RE}}.$$

- Quantification of the reduction in model uncertainty due to requiring interim financial reports in addition to the annually disclosed financial statements

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Stylized graphs to illustrate the updating mechanism when considering mandatory interim reports

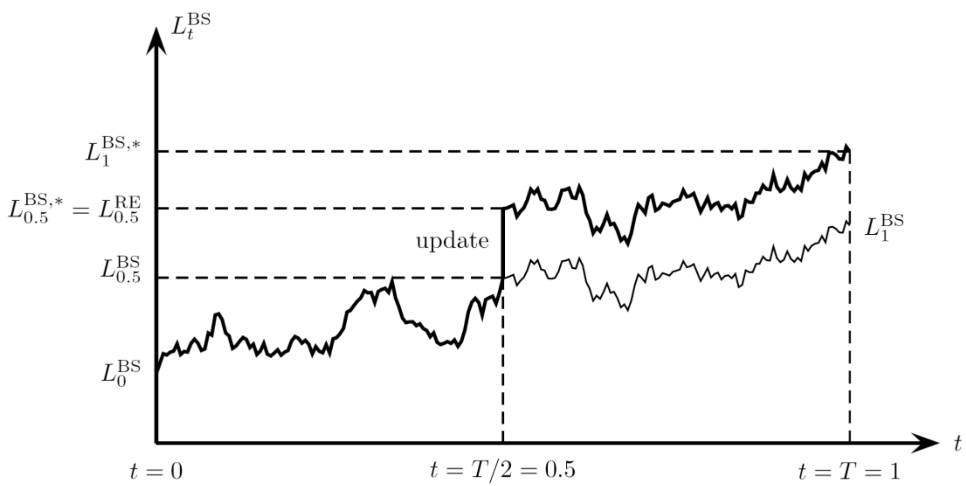

Illustration of a semi-annual update of the liability process from the basic setting (BS) with values from a more realistic framework (RE). The starred values correspond to the values obtained after updating at time t=0.5.

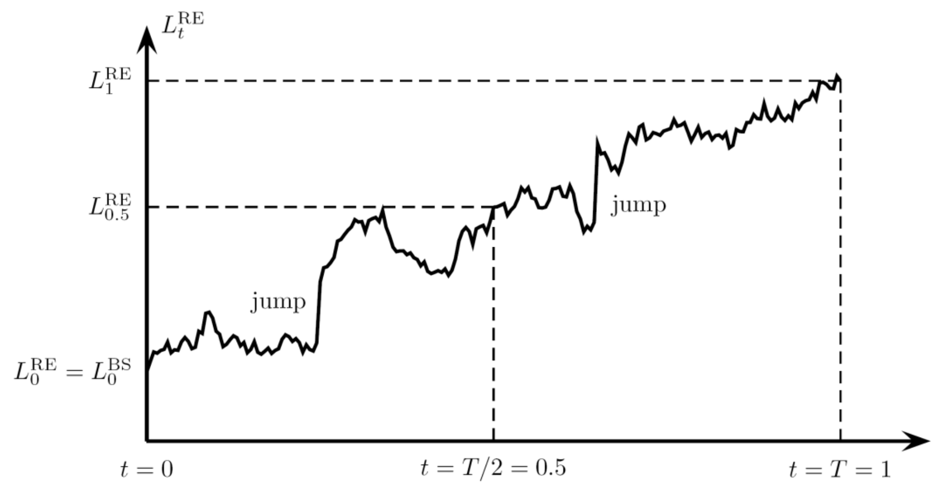

Illustration of the development of liabilities in the model framework including a jump component (RE), displayed by two jumps in the considered period T=1.

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Numerical analyses

- Reference case a) + case b) and c)
- Sensitivity analysis
- Examples based on a Monte Carlo simulation
- For reasonable accuracy, 5'000'000 iterations (for a)) and 10'000'000 iterations (for b) and c)) are calculated

Input parameters (I)

- Asset class i = 1: Proxy SMI 1988-2009; Asset class i = 2: Proxy SBI Domestic Government 1997-2009
- Asset Allocation: Stocks 5%; Bonds 95%
- Liabilities with jump => E(Y) is set to 1.05, σ(Y) = 5% (cf. Gatzert / Schmeiser, IME 2008)

$$J_t = \sum_{j=1}^{N_t} (Y_j - 1) .$$

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Input parameters (II)

Parameter	Symbol	Initial value at $t = 0$
Time horizon	T	1
Size of time steps within T	dT	$\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{12}\}$
Market value of asset class 1 (high-risk investments)	$A_{1,0}$	CU 0.5 billion
Market value of asset class 2 (low-risk investments)	$A_{2,0}$	CU 9.5 billion
Market value of liabilities	L_0	CU 8 billion
Risk-free rate of return	r_f	0.02
Drift of the geometric Brownian motion of asset class 1	μ_{A_1}	0.08
Drift of the geometric Brownian motion of asset class 2	μ_{A_2}	0.04
Drift of the geometric Brownian motion of liabilities	μ_L	0.03
Volatility of the geometric Brownian motion of asset class 1	σ_{A_1}	0.2
Volatility of the geometric Brownian motion of asset class 2	σ_{A_2}	0.08
Volatility of the geometric Brownian motion of liabilities	σ_L	0.05

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Input parameters (III)

Pearson's correlation coefficient between asset class 1 and asset class 2	$\rho(A_1, A_2)$	$[0, 1]$
Pearson's correlation coefficient between asset class 1 and liabilities	$\rho(A_1, L)$	0
Pearson's correlation coefficient between asset class 2 and liabilities	$\rho(A_2, L)$	0
Copula parameter modeling the dependence between asset class 1 and 2	θ_1	$[0, 8]$
Copula parameter modeling the dependence between assets and liabilities	θ_2	0
Expected value of Y_j	$E(Y_j)$	1.05
Volatility of Y_j	$\sigma(Y_j)$	0.05
Average number of jumps over time horizon (intensity)	λ	$[0, 0.5]$
Parameter for the log-normal distribution of Y_j	a	0.048
Parameter for the log-normal distribution of Y_j	b^2	0.023
Exceedance probability of value at risk	α	0.005

Overview and Interpretation

Risk measure	Basic Model	Model with jumps ($\lambda = 0.5$)	Model with linear correlation ($\rho = 1$)	Model with Clayton copula ($\theta_1 = 8$)
SC^{VaR}	$13.4 \cdot 10^8$ (0.0%)	$23.3 \cdot 10^8$ (73.9%)	$14.8 \cdot 10^8$ (10.4%)	$14.7 \cdot 10^8$ (9.7%)
SC^{TVaR}	$13.9 \cdot 10^8$ (0.0%)	$24.7 \cdot 10^8$ (77.7%)	$15.3 \cdot 10^8$ (10.1%)	$15.3 \cdot 10^8$ (10.1%)
SP	0.0001 (0.0%)	0.0106 (10'500%)	0.0003 (200%)	0.0003 (200%)
EPD	$15.3 \cdot 10^3$ (0.0%)	$47.6 \cdot 10^5$ (31'011.1%)	$53.4 \cdot 10^3$ (249.0%)	$49,7 \cdot 10^3$ (224.8%)

Table 2: Values of the four risk measures in the basic setting, the jump component model (for $\lambda = 0.5$), the model including linear correlation between $A_{1,t}$ and $A_{2,t}$ (with $\rho = 1$) and the model including non-linear dependence between $A_{1,t}$ and $A_{2,t}$ via a Clayton copula (with $\theta = 8$). Values in brackets denote the percentage increase compared to the value of the basic setting.

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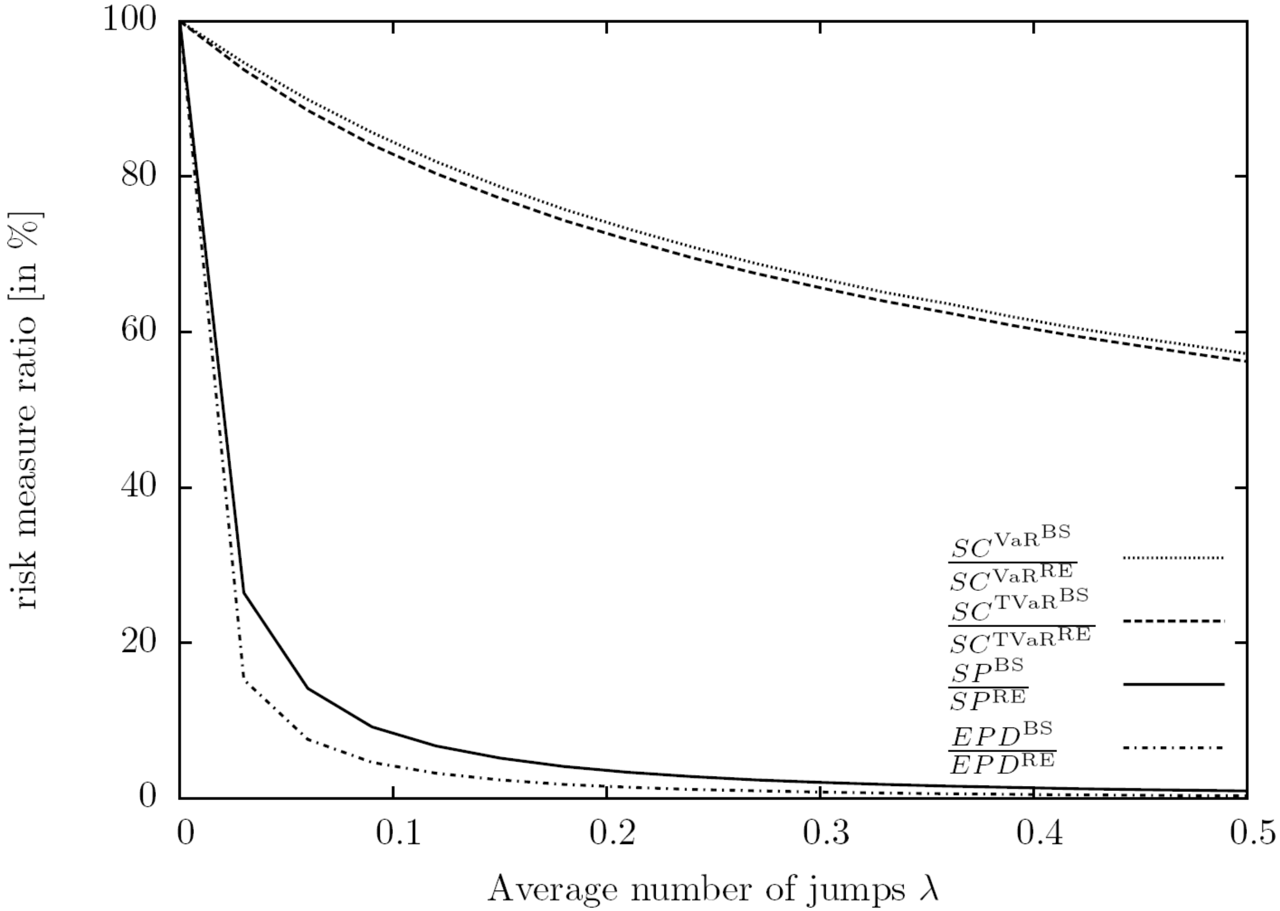
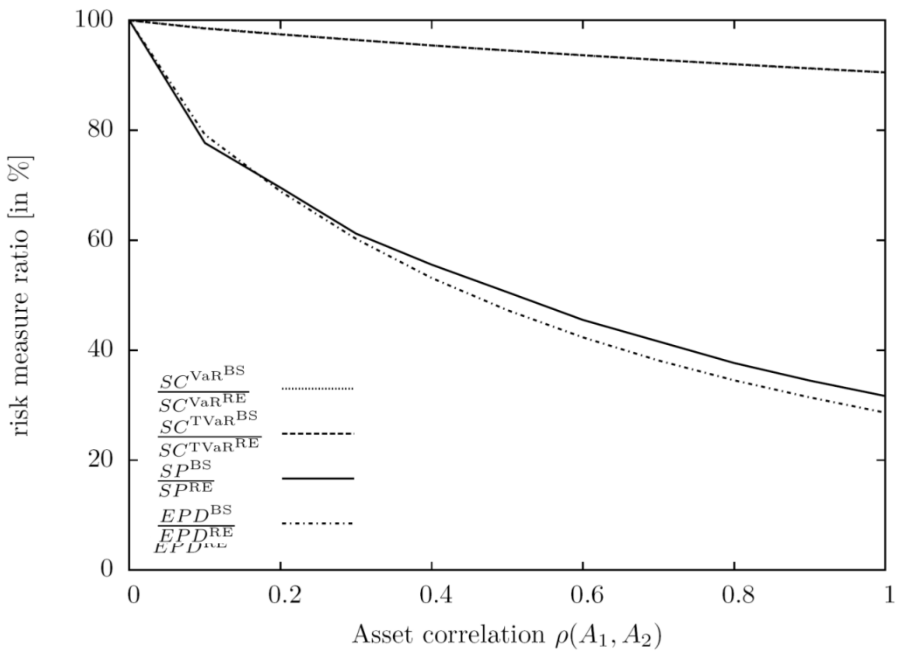
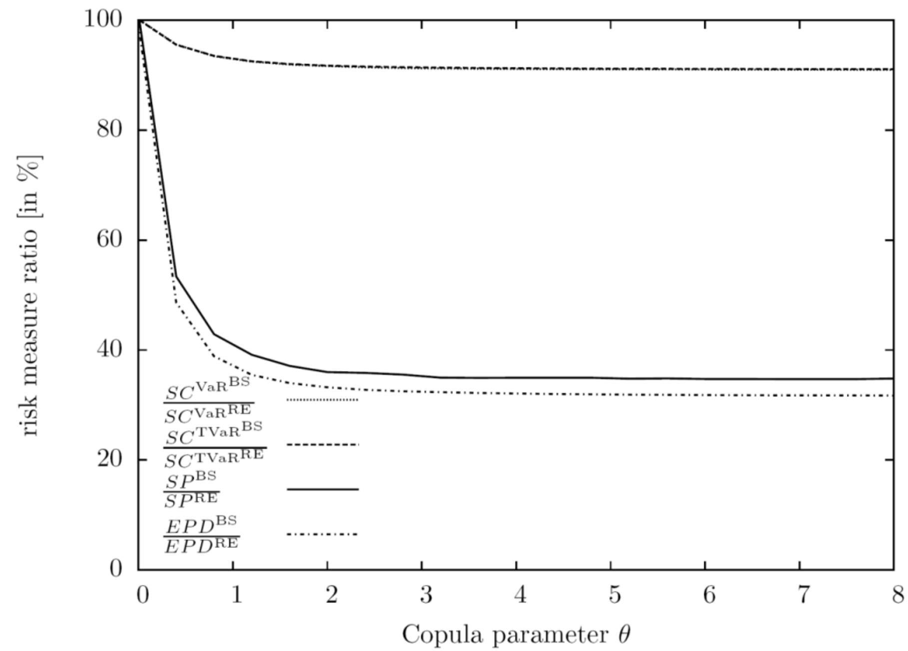
Risk measure ratios in case of stochastic jumps for different numbers of jumps per year (λ ∈ [0,0.5])



Risk measure ratios in case of linear correlation between the two asset classes for $\rho(A_1,A_2) \in [0,1]$, and in case of non-linear dependence introducing a Clayton copula function for $\theta \in [0,8]$





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Number of updates (n) on financial information per year and its impact on the four risk measure ratios

(a) Impact of n on risk measure ratios when including stochastic jumps with $\lambda = 0.2$.

n	$\frac{SC^{\text{VaR}^{\text{BS}}}}{SC^{\text{VaR}^{\text{RE}}}}$	$\frac{SC^{\text{TVaR}^{\text{BS}}}}{SC^{\text{TVaR}^{\text{RE}}}}$	$\frac{SP^{\text{BS}}}{SP^{\text{RE}}}$	$\frac{EPD^{\text{BS}}}{EPD^{\text{RE}}}$
1	74.0%	72.7%	3.7%	1.5%
2	87.7%	87.7%	42.5%	37.2%
4	94.4%	94.2%	70.1%	64.8%
12	98.0%	98.1%	88.8%	87.1%

(b) Impact of n on risk measure ratios when including linear correlation with $\rho = 0.2$.

n	$\frac{SC^{\text{VaR}^{\text{BS}}}}{SC^{\text{VaR}^{\text{RE}}}}$	$\frac{SC^{\text{TVaR}^{\text{BS}}}}{SC^{\text{TVaR}^{\text{RE}}}}$	$\frac{SP^{\text{BS}}}{SP^{\text{RE}}}$	$\frac{EPD^{\text{BS}}}{EPD^{\text{RE}}}$
1	97.5%	97.4%	69.6%	68.9%
2	98.7%	98.7%	86.4%	83.1%
4	99.3%	99.3%	89.8%	87.4%
12	99.8%	99.8%	99.6%	99.1%

(c) Impact of n on risk measure ratios when including a Clayton copula with $\theta = 0.5$.

n	$\frac{SC^{\text{VaR}^{\text{BS}}}}{SC^{\text{VaR}^{\text{RE}}}}$	$\frac{SC^{\text{TVaR}^{\text{BS}}}}{SC^{\text{TVaR}^{\text{RE}}}}$	$\frac{SP^{\text{BS}}}{SP^{\text{RE}}}$	$\frac{EPD^{\text{BS}}}{EPD^{\text{RE}}}$
1	94.6%	94.7%	48.4%	44.1%
2	97.7%	97.8%	78.0%	73.7%
4	98.9%	98.9%	88.2%	89.3%
12	99.8%	99.7%	98.4%	98.9%

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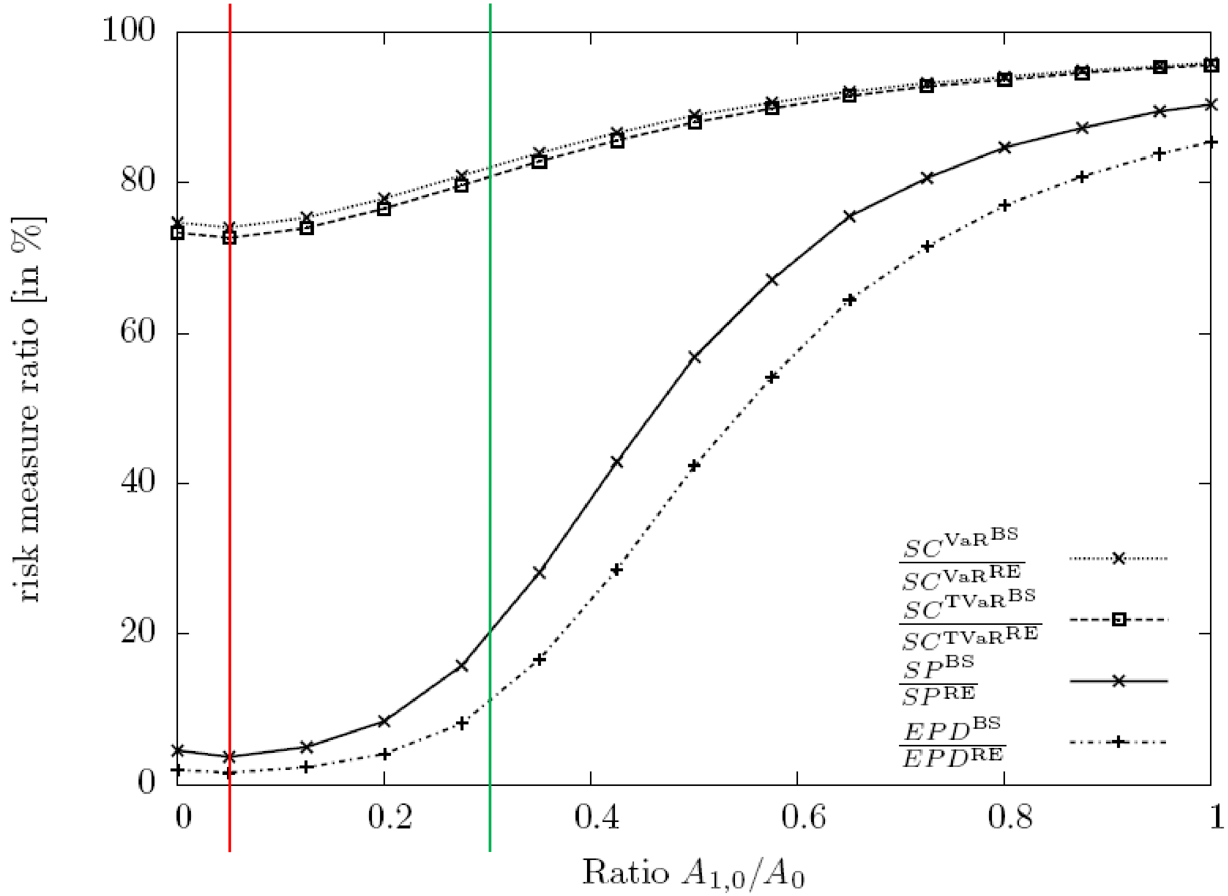
Sensitivity analysis (I)

- Basis setting a) and jump model b) with λ = 0.2; variation of the asset allocation



(a) Risk measure deviations depending on the $A_{1,0}/A_0$ ratio when comparing the basic setting with the model framework of Section 3.1 with $\lambda = 0.2$

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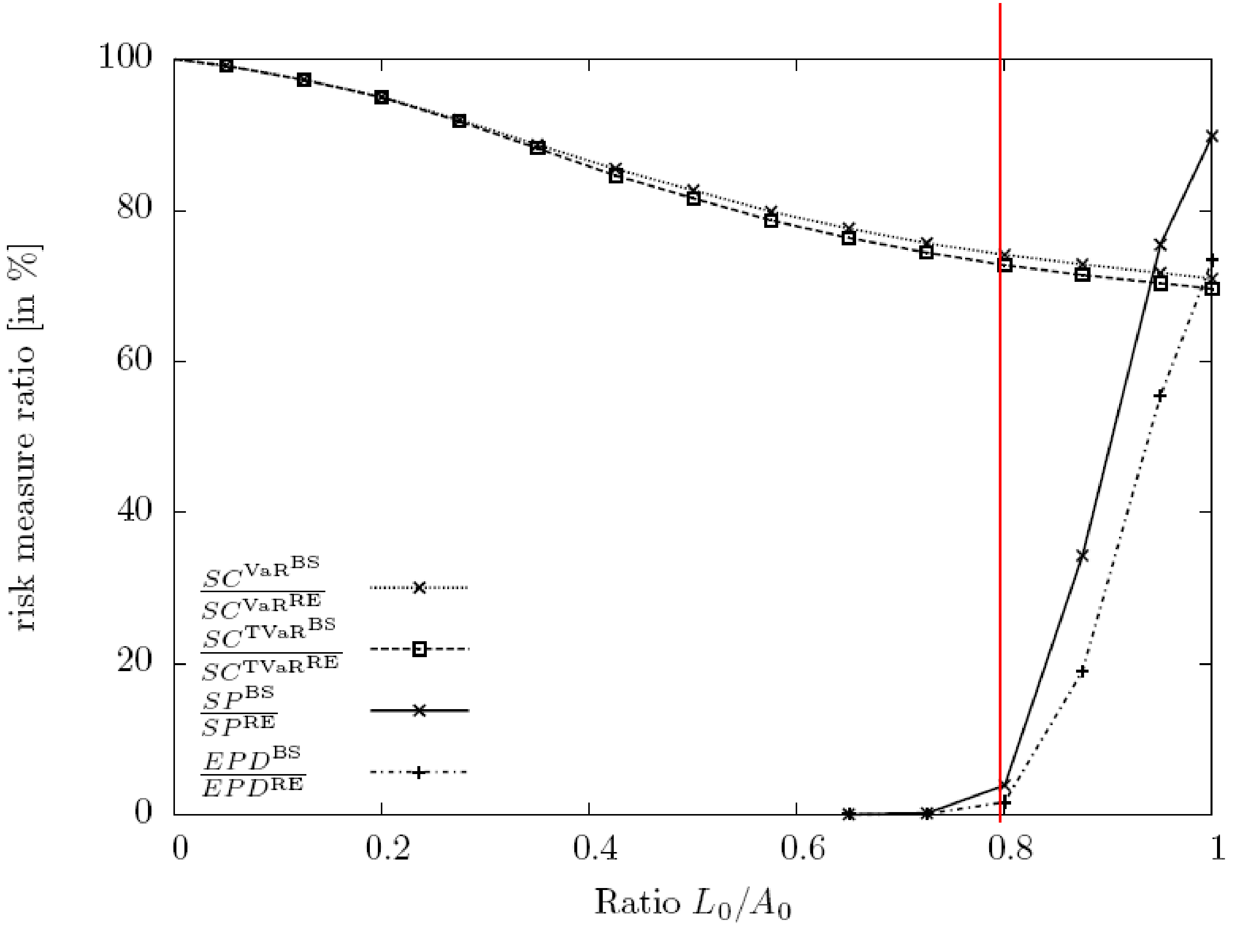
Sensitivity analysis (II)

- Basis setting a) and jump model b) with $\lambda = 0.2$; variation of the asset to liability ratio



(b) Risk measure deviations depending on the L_0/A_0 ratio when comparing the basic setting with the model framework of Section 3.1 with $\lambda = 0.2$

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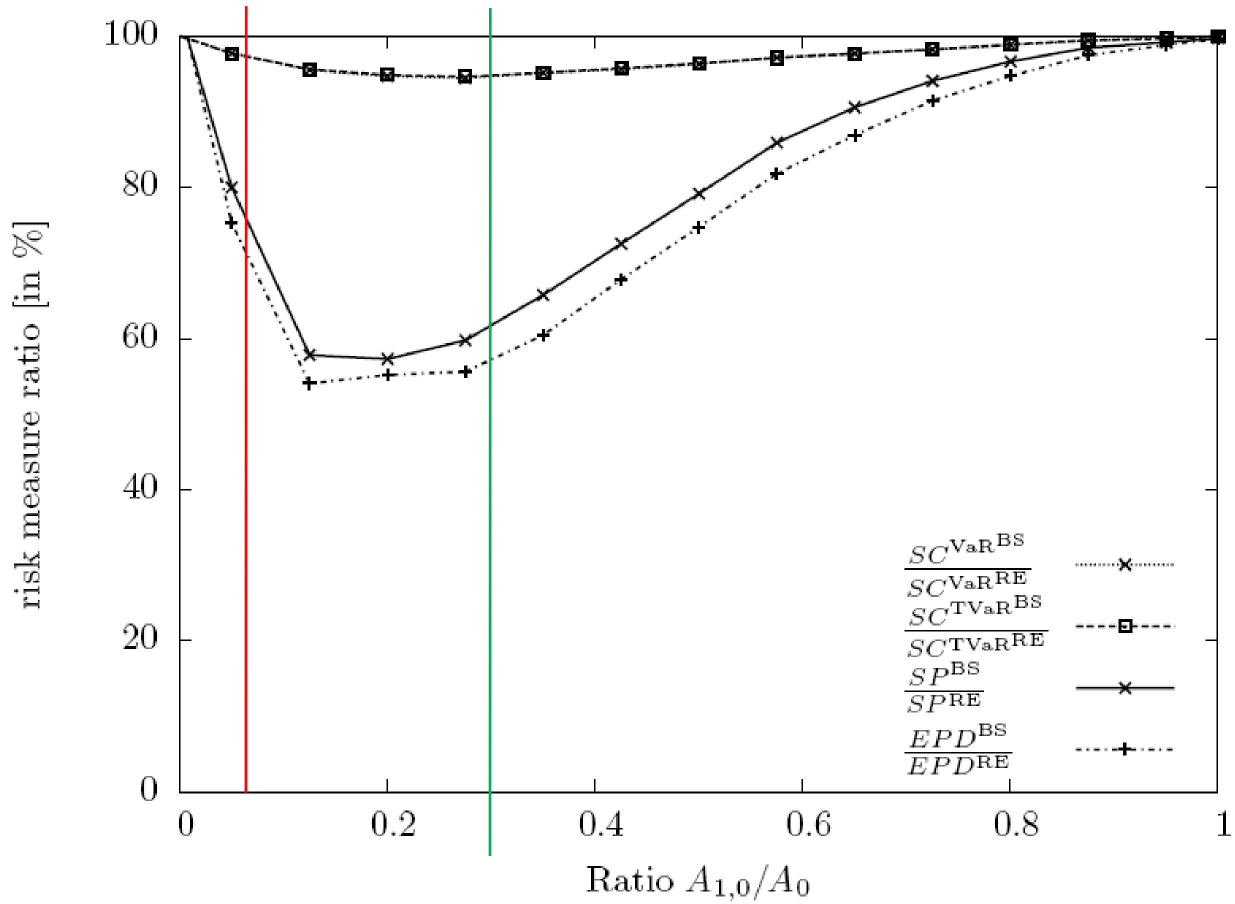
Sensitivity analysis (III)

- Basis setting a) and dependency model c) with $p(A_1, A_2) = 0.2$; variation of the asset allocation



(a) Risk measure deviations for different $A_{1,0}/A_0$ ratios in model with linear correlation $\rho(A_1, A_2) = 0.2$.

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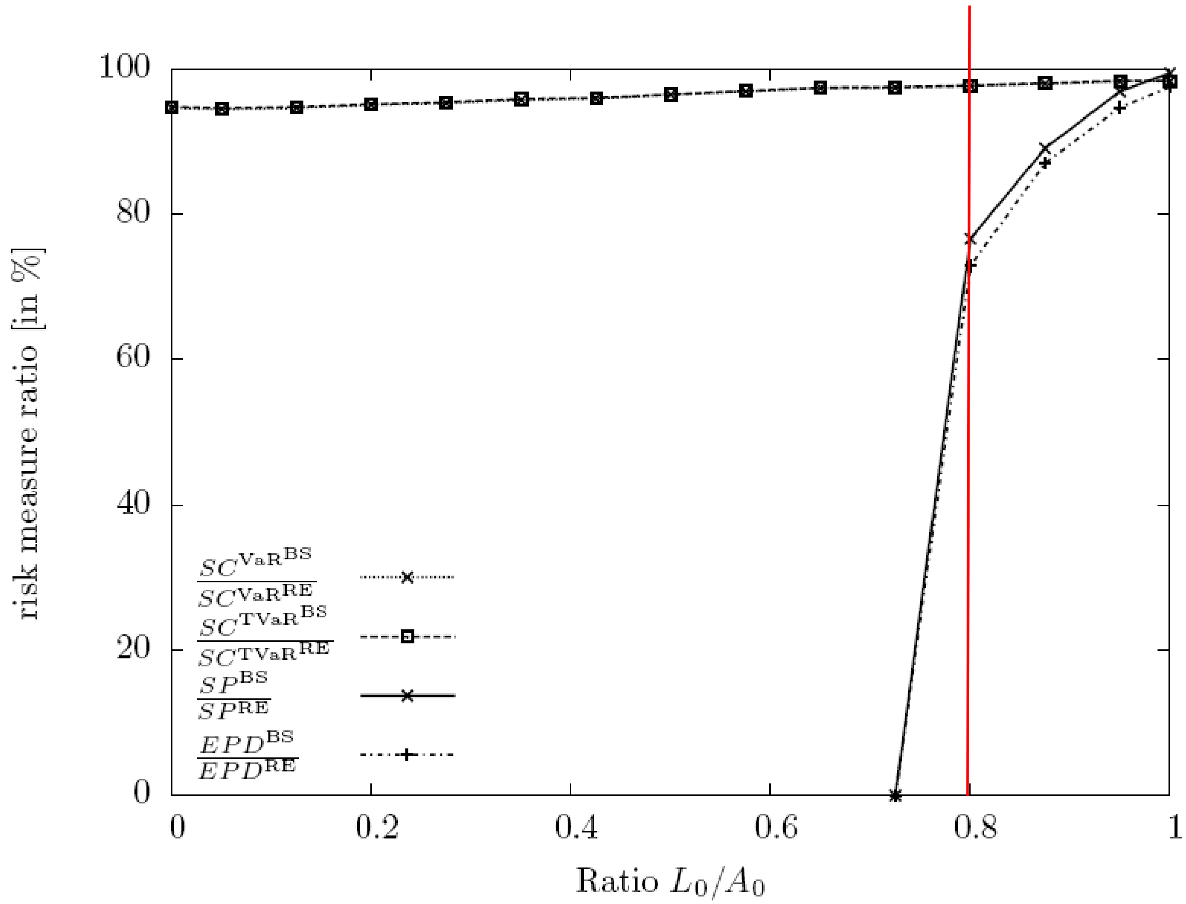
Sensitivity analysis (IV)

- Basis setting a) and dependency model c) with p(A_1, A_2) = 0.2; variation of the asset to liability ratio



(b) Risk measure deviations for different L_0/A_0 ratios in model with linear correlation $\rho(A_1, A_2) = 0.2$.

4. Implications and Conclusion

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- Stochastic jumps and non-linear dependencies seem to have a greater impact on capital requirements and shortfall risk measures than linear correlation
- The extensive discussion which risk measure (value at risk vs. tail value at risk) should be used in the solvency context seems not to be too important from the point of view of our paper
- Shortfall risk measures react much more sensitive to modifications in the solvency framework than capital requirements. Therefore, the (sole) consideration of the sensitivity of capital requirements as risk measures might underestimate the actual model risk an insurance company is exposed to
- Hence, the focus on the sensitivity of the capital requirements as it is currently done in the SST (and planned in Solvency II) is rather misleading. For the policyholders – and in the end maybe for taxpayers too –, the expected policyholder deficit is the important figure
- However, the expected policyholder deficit reacts extremely sensitive to modifications in the model setup. The resilience of these kind of models is in this respect questionable
- Mandatory interim reports on the solvency and financial situation of the property-liability insurer could be a useful way to reduce model risk, from a regulatory point of view

Policy implications

- The focus on the sensitivity of the capital requirements - as it is currently done in the SST (and planned in Solvency II) – gives a wrong impression on model stability
- The EPD – the main relevant figure from the policyholders' point of view – is extremely unstable regarding even very small model derivations
- If we believe the risk-incentive problem to be true in the insurance sector, we should regulate the EPD-ratio (too) – by taking different scenarios / assumption into account
- As the calibration of more complex models is very difficult in insurance praxis (and the "true" development of the underlying is unknown anyway), an easy way to reduce risk can be obtained by increasing the updating points in time. Hence, instead of using more complex models with doubtful parameterization, the regular may rather want to ask for mandatory interim reports

Literature

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