The Merits of Pooling Claims Revisited

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1. Introduction

- Risk pooling – "product law" of insurance
  - Conveys the impression that risk pooling does generate an additional value for the policyholders
  - More precisely, premiums decrease / safety level increases
  - Aim is to extend and combine previous work by focusing on the merits of pooling from the policyholder's perspective
- Result
2. Pooling Claims: The Base Case

- Total claim amount

\[ S = \sum_{i=1}^{n} X_i \]

- Premium of risk \( i \) (with safety loading \( c > 0 \))

\[ \pi_i = E(X_i) + c \]

- Collective premium

\[ \pi = \sum_{i=1}^{n} E(X_i) + nc = E(S) + nc = n\pi_i \]
**Case A: Fixed ruin probability**

\[ R = P(S > \pi) = \varepsilon \quad \iff \quad P(S > E(S) + n\mathcal{c}(n)) = \varepsilon \]

\[ R = 1 - N\left(\frac{E(S) + n\mathcal{c}(n) - E(S)}{\sigma(S)}\right) = 1 - N\left(\frac{\mathcal{c}(n)}{\sigma(X_i) \cdot \sqrt{n}}\right) = \varepsilon \]

\[ \frac{\mathcal{c}(n)}{\sigma(X_i) \cdot \sqrt{n}} = z_{1-\varepsilon} \quad \iff \quad \mathcal{c}(n) = \frac{z_{1-\varepsilon} \cdot \sigma(X_i)}{\sqrt{n}} \]

- Interpretation: Fixed safety level, premiums decrease
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- **Case B: Fixed premium**

\[
R = 1 - N \left( \frac{E(S) + nc - E(S)}{\sigma(S)} \right) = 1 - N \left( \frac{c}{\sigma(X_i)} \cdot \sqrt{n} \right) \xrightarrow{n \to \infty} 0
\]

- Interpretation: Fixed premium, safety level increases

- Summary (at first glance):

  Policyholder seem to benefit from pooling claims

  Premiums seem to play a role
3. Policyholder’s Point of View

- Wealth position (with $r = 0$) with / without risk pooling

\[
W_i = A_i - X_i + \pi_i + I_i + E_i
\]

1) Frictionless and efficient market

- Debtholder position

\[
PV(I_i) = PV(X_i) - \frac{1}{n} PV\left(\max[S - \pi, 0]\right)
\]

- Equityholder position

\[
PV(E_i) = \frac{1}{n} PV\left(\max[\pi - S, 0]\right)
\]
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- Fair premium calculation

\[
PV(I_i + E_i) = PV(I_i) + PV(E_i) \\
= PV(X_i) - \frac{1}{n} PV(max[S - \pi, 0]) + \frac{1}{n} PV(max[\pi - S, 0]) \\
= PV(X_i) + \frac{1}{n} PV(\pi - S) \\
= PV(X_i) + \pi_i - \frac{1}{n} \sum_{i=1}^{n} PV(X_i) \\
= \pi_i.
\]

- Fulfilled for all premium principles

\[
\pi_i = E(X_i) + c \quad c \in \mathbb{R}
\]

- Hence, whether risk pooling in the sense of Case A or B is fulfilled or not is of no importance. No additional value can be created from the diversification of unsystematic risk
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- Same holds true for risk-neutral policyholders

- Example (with $E(X_i) = 30$, $\sigma(X_i) = 10$, $r = 0$, risk-neutral market)

Table 1: Premiums $\pi_i$ and present values of payouts $PV(I_i)$ and $PV(E_i)$ for pooling claims for a given ruin probability of 1% (Case A – fixed ruin probability)

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>1000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_i$</td>
<td>53.2635</td>
<td>37.3566</td>
<td>33.2900</td>
<td>32.3263</td>
<td>30.7357</td>
<td>30.2326</td>
</tr>
<tr>
<td>$c(n)$</td>
<td>23.2635</td>
<td>7.3566</td>
<td>3.2900</td>
<td>2.3263</td>
<td>0.7357</td>
<td>0.2326</td>
</tr>
<tr>
<td>$PV(I_i)$</td>
<td>29.9661</td>
<td>29.9893</td>
<td>29.9952</td>
<td>29.9966</td>
<td>29.9990</td>
<td>29.9996</td>
</tr>
<tr>
<td>$PV(E_i)$</td>
<td>23.2974</td>
<td>7.3673</td>
<td>3.2947</td>
<td>2.3297</td>
<td>0.7367</td>
<td>0.2330</td>
</tr>
<tr>
<td>$PV(I_i + E_i)$</td>
<td>53.2635</td>
<td>37.3566</td>
<td>33.2900</td>
<td>32.3263</td>
<td>30.7357</td>
<td>30.2326</td>
</tr>
</tbody>
</table>
Merits of Pooling Claims Revisited

- Examples for fixed premiums

**Table 2**: Ruin probability of the pool $R$, present values of payouts $PV(I_i)$ $PV(E_i)$ for the case of pooling claims for a given safety loading $c = 0.5$ (Case B – fixed premium)

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>1000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_i$</td>
<td>30.5000</td>
<td>30.5000</td>
<td>30.5000</td>
<td>30.5000</td>
<td>30.5000</td>
<td>30.5000</td>
</tr>
<tr>
<td>$c$</td>
<td>0.5000</td>
<td>0.5000</td>
<td>0.5000</td>
<td>0.5000</td>
<td>0.5000</td>
<td>0.5000</td>
</tr>
<tr>
<td>$R = P(S &gt; \pi)$</td>
<td>48.0061 %</td>
<td>43.7184 %</td>
<td>36.1837 %</td>
<td>30.8538 %</td>
<td>5.6923 %</td>
<td>0.0000 %</td>
</tr>
<tr>
<td>$PV(I_i)$</td>
<td>26.2556</td>
<td>28.9727</td>
<td>29.6509</td>
<td>29.8022</td>
<td>29.9923</td>
<td>30.0000</td>
</tr>
<tr>
<td>$PV(E_i)$</td>
<td>4.2444</td>
<td>1.5273</td>
<td>0.8491</td>
<td>0.6978</td>
<td>0.5077</td>
<td>5.0000</td>
</tr>
<tr>
<td>$PV(I_i + E_i)$</td>
<td>30.5000</td>
<td>30.5000</td>
<td>30.5000</td>
<td>30.5000</td>
<td>30.5000</td>
<td>30.5000</td>
</tr>
</tbody>
</table>

**Table 3**: Premiums $\pi_i$ and payouts $(I_i + E_i)$ for the case of pooling claims for a fixed premium level per of $\pi_i = 29.00$ (Case B – fixed premium)

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>1000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_i$</td>
<td>29.0000</td>
<td>29.0000</td>
<td>29.0000</td>
<td>29.0000</td>
<td>29.0000</td>
<td>29.0000</td>
</tr>
<tr>
<td>$c$</td>
<td>-1.0000</td>
<td>-1.0000</td>
<td>-1.0000</td>
<td>-1.0000</td>
<td>-1.0000</td>
<td>-1.0000</td>
</tr>
<tr>
<td>$R = P(S &gt; \pi)$</td>
<td>53.9828 %</td>
<td>62.4085 %</td>
<td>76.0250 %</td>
<td>84.1345 %</td>
<td>99.9217 %</td>
<td>100.0000 %</td>
</tr>
<tr>
<td>$PV(I_i)$</td>
<td>25.4906</td>
<td>28.1759</td>
<td>28.8004</td>
<td>28.9167</td>
<td>28.9999</td>
<td>29.0000</td>
</tr>
<tr>
<td>$PV(E_i)$</td>
<td>4.5094</td>
<td>0.8241</td>
<td>0.1996</td>
<td>0.0833</td>
<td>0.0001</td>
<td>0.0000</td>
</tr>
<tr>
<td>$PV(I_i + E_i)$</td>
<td>29.0000</td>
<td>29.0000</td>
<td>29.0000</td>
<td>29.0000</td>
<td>29.0000</td>
<td>29.0000</td>
</tr>
</tbody>
</table>
2) Risk-averse policyholder

- Hybrid model with $a > 0$

$$\Phi = E(W_i) - \frac{a}{2} \cdot \sigma^2(W_i)$$

- Expected wealth independent of $c$ and $n$

- Variance independent of $c$, but depending on $n$

$$\sigma^2(W_i) = \sigma^2(-X_i + I_i + E_i) = \ldots = \frac{1}{n} \sigma^2(X_i)$$

- Utility increases c.p. with an increasing number of pool members
4. Summary

• Merits of risk pooling in the definition used in chapter 2 under Case A and Case B give no hint whether risk pooling is beneficial for policyholders or not

• However, situations in which risk pooling is beneficial for policyholders are easy to define (and well known)

• In any case: Valuation of both stakes – debtholder and equityholder position – is necessary to get a clear picture

• A sound economic interpretation of the classical “merits of pooling” definitions of Case A and B is not clear to us