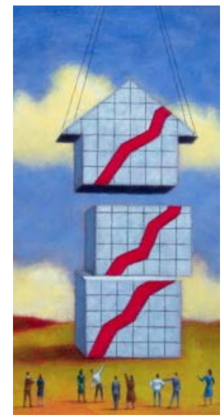
Institute of Insurance Economics

University of St.Gallen

The Merits of Pooling Claims Revisited





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1. Introduction

- Risk pooling – "product law" of insurance

- Conveys the impression that risk pooling does generate an additional value for the policyholders

- More precisely, premiums decrease / safety level increa-ses

- Aim is to extend and combine previous work by focusing on the merits of pooling from the policyholder's per-spective

- Result

2. Pooling Claims: The Base Case

- Total claim amount

$$S = \sum_{i=1}^{n} X_i$$

- Premium of risk i (with safety loading c > 0)

$$\pi_i = E\left(X_i\right) + c$$

- Collective premium

$$\pi = \sum_{i=1}^{n} E\left(X_i\right) + nc = E\left(S\right) + nc = n\pi_i$$

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- <u>Case A</u>: Fixed ruin probability

$$R = P(S > \pi) \overset{!}{=} \varepsilon \quad \Leftrightarrow \quad P(S > E(S) + nc(n)) \overset{!}{=} \varepsilon$$

$$R = 1 - N\left(\frac{E(S) + nc(n) - E(S)}{\sigma(S)}\right) = 1 - N\left(\frac{c(n)}{\sigma(X_i)} \cdot \sqrt{n}\right) \overset{!}{=} \varepsilon$$

$$\frac{c(n)}{\sigma(X_i)} \cdot \sqrt{n} = z_{1-\varepsilon} \quad \Leftrightarrow \quad c(n) = \frac{z_{1-\varepsilon} \cdot \sigma(X_i)}{\sqrt{n}}$$

- Interpretation: Fixed safety level, premiums decrease

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- <u>Case B</u>: Fixed premium

$$R = 1 - N\left(\frac{E(S) + nc - E(S)}{\sigma(S)}\right) = 1 - N\left(\frac{c}{\sigma(X_i)} \cdot \sqrt{n}\right) \underset{n \to \infty}{\rightarrow} 0$$

- Interpretation: Fixed premium, safety level increases

- Summary (at first glance):

Policyholder seem to benefit from pooling claims

Premiums seem to play a role

3. Policyholder's Point of View

- Wealth position (with *r* = 0) with / without risk pooling

$$W_i = A_i - X_i + \pi_i + I_i + E_i$$

1) Frictionless and efficient market

- Debtholder position

$$PV(I_i) = PV(X_i) - \frac{1}{n} PV(\max[S - \pi, 0])$$

- Equityholder position

$$PV(E_i) = \frac{1}{n} PV(\max[\pi - S, 0])$$

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- Fair premium calculation

$$
\begin{aligned}
PV(I_i + E_i) &= PV(I_i) + PV(E_i) \\
&= PV(X_i) - \frac{1}{n} PV(\max[S - \pi, 0]) + \frac{1}{n} PV(\max[\pi - S, 0]) \\
&= PV(X_i) + \frac{1}{n} PV(\pi - S) \\
&= PV(X_i) + \pi_i - \frac{1}{n}\sum_{i=1}^{n} PV(X_i) \\
&= \pi_i .
\end{aligned}
$$

- Fulfilled for all premium principles

$$\pi_i = E(X_i) + c \qquad c \in \mathbb{R}$$

- Hence, whether risk pooling in the sense of Case A or B is fulfilled or not is of no importance. No additional value can be created from the diversification of unsystematic risk

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- Same holds true for risk-neutral policyholders

- Example (with $E(X_i) = 30$, $\sigma(X_i) = 10$, $r = 0$, risk-neutral market)

Table 1: Premiums π_i and present values of payouts $PV(I_i)$ and $PV(E_i)$ for pooling claims for a given ruin probability of 1 % (*Case A – fixed ruin probability*)

n	1	10	50	100	1000	10000
π_i	53.2635	37.3566	33.2900	32.3263	30.7357	30.2326
$c(n)$	23.2635	7.3566	3.2900	2.3263	0.7357	0.2326
$PV(I_i)$	29.9661	29.9893	29.9952	29.9966	29.9990	29.9996
$PV(E_i)$	23.2974	7.3673	3.2947	2.3297	0.7367	0.2330
$PV(I_i + E_i)$	53.2635	37.3566	33.2900	32.3263	30.7357	30.2326

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- Examples for fixed premiums

Table 2: Ruin probability of the pool *R*, present values of payouts $PV(I_i)$ $PV(E_i)$ for the case of pooling claims for a given safety loading *c* = 0.5 (*Case B – fixed premium*)

n	1	10	50	100	1000	10000
π_i	30.5000	30.5000	30.5000	30.5000	30.5000	30.5000
c	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
$R = P(S > \pi)$	48.0061 %	43.7184 %	36.1837 %	30.8538 %	5.6923 %	0.0000 %
$PV(I_i)$	26.2556	28.9727	29.6509	29.8022	29.9923	30.0000
$PV(E_i)$	4.2444	1.5273	0.8491	0.6978	0.5077	5.0000
$PV(I_i + E_i)$	30.5000	30.5000	30.5000	30.5000	30.5000	30.5000

Table 3: Premiums π_i and payouts $(I_i + E_i)$ for the case of pooling claims for a fixed premium level per of π_i = 29.00 (*Case B – fixed premium*)

n	1	10	50	100	1000	10000
π_i	29.0000	29.0000	29.0000	29.0000	29.0000	29.0000
c	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
$R = P(S > \pi)$	53.9828 %	62.4085 %	76.0250 %	84.1345 %	99.9217 %	100.0000 %
$PV(I_i)$	25.4906	28.1759	28.8004	28.9167	28.9999	29.0000
$PV(E_i)$	4.5094	0.8241	0.1996	0.0833	0.0001	0.0000
$PV(I_i + E_i)$	29.0000	29.0000	29.0000	29.0000	29.0000	29.0000

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2) Risk-averse policyholder

- Hybrid model with a > 0

$$\Phi = E(W_i) - \frac{a}{2} \cdot \sigma^2 (W_i)$$

- Expected wealth independent of c and n

- Variance independent of c, but depending on n

$$\sigma^2 (W_i) = \sigma^2 (-X_i + I_i + E_i) = ... = \frac{1}{n} \sigma^2 (X_i)$$

- Utility increases c.p. with an increasing number of pool members

4. Summary

- Merits of risk pooling in the definition used in chapter 2 under Case A and Case B give no hint whether risk pooling is beneficial for policyholders or not

- However, situations in which risk pooling is beneficial for policyholders are easy to define (and well known)

- In any case: Valuation of both stakes – debtholder and equityholder position – is necessary to get a clear picture

- A sound economic interpretation of the classical “merits of pooling” definitions of Case A and B is not clear to us