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HOW SENSITIVE IS THE PRICING OF LOOK-BACK AND INTEREST RATE GUARANTEES WHEN CHANGING THE MODELLING ASSUMPTIONS?

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Abstract

This paper aims to give detailed insights into the price sensitivity of embedded investment guarantees provided by unit-linked life insurance products. Particularly, it analyzes the model and parameter risk from the provider's perspective. We compare two different forms of investment guarantees: Interest Rate Guarantees (IRG) and Lookback Guarantees (LBG). Via Monte Carlo simulation, the prices of the embedded investment guarantees are estimated assuming the underlying to evolve according to a normal or double-exponential jump-diffusion model. The input parameters are derived from empirical data for the different asset classes. In a first step, the parameters of the IRG and the LBG are adjusted such that the prices of these two guarantees are equal. In a second step, a detailed comparison is made between the price sensitivities of both guarantee forms when the initial modelling parameters are changed. Finally, we investigate how parameter changes affect the investor payoff under the different guarantee forms and model assumptions used for the dynamic of the underlying.

Keywords: Unit-Linked life insurance, Interest Rate Guarantee, Lookback Guarantee, Risk neutral Valuation, Sensitivity Analysis.

1. INTRODUCTION

Subsequent to the last financial crisis, the modelling assumptions frequently used for pricing and risk management in the financial services industry have come into focus. The dynamics of the financial market requires to consider the unpredictable variations of the risk model hypotheses when pricing different forms of investment guarantees typically embedded in equity-linked insurance products. In practice, equity-linked products are mostly priced under the geometric Brownian motion assumptions. However, when the historical prices of the underlying asset are analyzed and considered as a basis to set the pricing assumptions with respect to the risk model, it can be recognized that asset returns rarely fit the standard Brownian motion. Instead, they rather tend to follow a jump-diffusion model (cf. Ramezani & Zeng, 2007). Moreover, the corresponding parameter assumptions rarely remain constant over time. Consequently, the quantification of the economic impact that model and parameter uncertainty involve, becomes crucial. Particularly, the guarantees' price sensitivity to changes in the modelling assumptions emerges as an essential element for practitioners to acquire a better comprehension of the functionality of the different guarantee forms and the implied risk. It is economically important for the providers acknowledge these sensitivities to accurately assess the risk involved in the interest of adopting the appropriate risk management strategies.

For several years, great effort has been devoted to the study of option pricing, including equity-linked life insurance products with guarantees. Brennan & Schwartz (1976) were among the first to investigate equity-linked life insurance with an asset value guarantee. The authors formulated the insured benefit as the sum of an insured amount and an immediate exercisable call option. Later, Bacinello & Ortu (1993) discussed the pricing when the benefit has an endogenous guarantee, i.e., it depends on the premium payments. More recent research in this field includes an article by Lee (2003). The author studied the pricing of different types of guarantees embedded with path-dependent options, covering the interest rate and lookback guarantees. Gatzert & Schmeiser (2009) compared the pricing and performance of Lookback and Interest Rate Guarantees (point-to-point) via a numerical analysis using a Monte Carlo simulation. In this context, they focused on different investment strategies by assuming constant drift and volatility of the asset price process over time. Specific research into the sensitivity of the option price for products with guarantees were initially conducted by Beenstock & Brasse (1986). Iseger and Oldenkamp concentrated their work on the computation of the option Greeks, given name of the price derivatives that indicate the sensitivity of the option price, for products with investment guarantees (cf. Iseger & Oldenkamp, 2005, 2006a,b).

The pricing of equity-linked life insurance products with guarantees has been largely studied under the assumptions of the Black-Scholes option pricing model (cf. Black & Scholes, 1973). However, it has been demonstrated that the historical developments of different risky asset returns often include jumps, stochastic interest rates and stochastic volatility. This functioning involves a number of risks and uncertainties that are not completely taken into consideration by modelling the underlying asset with the traditional geometric Brownian motion. Consequently, various jump diffusion models have been proposed by different authors, with two of the most popular being the Merton (1976) and Kou (2002) models. The inclusion of the jump risk component "destroys" the market completeness assumed in the model context within the geometric Brownian motion, hence multiple risk-neutral measures can be derived (cf., e.g., Cont & Tankov, 2003). Merton proposes to ignore the risk premium for jumps, and then a specific choice of risk-neutral measure can be made. In this context, this paper contributes to the literature by comparing the prices and sensitivities of Interest Rate Guarantees (IRG) and Lookback Guarantees (LBG) under jump diffusion models.

The aim of this paper is to analyze and compare the option prices and parameter sensitivities of the savings part of unit-linked life insurance products with different investment guarantees when the underlying asset follows jump-diffusion processes. Particularly, we assume that the jump follows a normal distribution, Merton jump-diffusion process (MJD)(cf. Merton, 1976), or a double-exponential distribution, Kou jump-diffusion process (KJD)(cf. Kou, 2002). We compare two specific guarantees embedded in the savings part of the unit-linked life insurance contract: an Interest Rate Guarantee (IRG) and a Lookback Guarantee (LBG). The first guarantee form (IRG) assures a minimum interest rate on the premiums paid into the contract. The LBG provides a maturity payoff defined as the number of units acquired over the contract multiplied by the highest value of the unit price attained during the term of the policy.

We perform a numerical analysis for different premium payment modalities (single or monthly premium), policy terms and underlying asset volatility. In a first stage, the parameters for the Merton and Kou jump-diffusion models are estimated using the Expectation-Maximization algorithm (cf. Dempster et al., 1977). As closed form solutions are not available for most of the observed cases, numerical analysis is provided by means of a Monte Carlo simulation. For each one of the underlying asset frameworks, we calibrate the option prices of both guarantees to be equal using risk-neutral valuation technique. We also estimate their corresponding expected maturity payoffs under the empirical distribution. We found that under Merton and Kou frameworks have the expected payoff is the same even if their payoff distributions are different. In the second part of our research, we are interested in determining which of the two guarantees has a higher sensitivity to changes in parameters. IRG proved to be in general more sensitive to changes in the risk-free rate than LBG. Further, IRG products with single premiums are more affected by changes in the risk-free rate than products with periodic premiums. On the contrary, the LBG products showed greater sensitivity than the IRG products with regard to the volatility parameter. Finally, the inclusion of jumps in our modelling assumptions requires an extension of the price sensitivity analysis regarding the additional parameters that describe the frequency and intensity of the jumps. The sensitivities of the option price concerning changes in the frequency of the jumps and the volatility of the jump were higher for LBG than for IRG products, while jump intensity seemed not to have a great impact on the option prices. Our research seeks to give important hints to regulators and providers of unit-linked life insurance products regarding the model and parameter risk when offering IRGs and LBGs.

This paper is organized as follows: Section 2 introduces the model frameworks for the interest rate and lookback guarantees and the underlying asset dynamics. Section 3 contains an estimation of the empirical parameters as well as the numerical calculations of option prices for both guarantees. Section 3.3 comprises an option price sensitivity analysis. Finally, some economic implications and conclusions are stated in Section 4.

2. MODEL FRAMEWORK

2.1 LOOKBACK AND INTEREST RATE GUARANTEE

The unit-linked life insurance products with embedded investment guarantees have a contract duration term of T years. We focus on single premium and level monthly premium payments. The analysis concentrates on the savings component of unit-linked life insurance policies and it does not consider the share of the premium used to cover the term life insurance and the transaction costs. Moreover, we ignore surrender and paid-up options; we neglect the investors' mortality risk within the duration of the contract as well.

The model that describes the dynamics of the mutual fund is adopted from Gatzert & Schmeiser (2009). Let $t_0 = 0, t_1, ...t_{N-1}, t_N = T$ denote the sequence of times at which the premium P is paid. The underlying mutual fund is typically split into units, where S_{t_i} denotes the unit price at time t_i. Thus the number of units acquired by the investor at time t_i is

$$n_{t_i} = \frac{P}{S_{t_i}} \qquad i \in \{0, ..., N-1\}.$$

The accumulated number of units at the end of period t_i is given by

$$N_{t_i} = \sum_{j=0}^{i-1} n_{t_i} \qquad i \in \{0, ..., N-1\}$$

Particularly, for single premium products, the investor can only purchase units once at t=0, i.e., $N_{t_0} = N_{T_N} = P/S_0$. The fund value at time t_i is defined as

$$F_{t_i} = (F_{t_{i-1}} + P)\frac{S_{t_i}}{S_{t_{i-1}}}$$

The fund value at time T is

$$F_T = N_T \cdot S_T = P \cdot \sum_{j=0}^{N-1} \frac{S_T}{S_{t_j}} \tag{1}$$

Let G_T be the guarantee value and L_T the corresponding payoff received by the investor at maturity T. The payoff L_T of the investor at T can be expressed as the fund value at maturity F_T plus a put option on this value with strike G_T.

$$L_T = max\,(F_T, G_T)\; = F_T + max\,(G_T - F_T, 0) \tag{2}$$

The particular form of G_T^g for the IRG corresponds to a minimum guaranteed interest rate g earned on the premiums paid and it can be written as

$$G_T^g = P \cdot \sum_{j=0}^{N-1} e^{g(T-t_j)} \tag{3}$$

The LBG G_T^{LBG} is equal to the highest value of the index H_T that has been realized during the policy term; multiplied by the total number of units acquired at time T it can be written as

$$G_T^{LBG} = N_T \cdot H_T, \tag{4}$$

with H_T defined as

$$H_T = \max_{j \in \{0,\dots,N-1\}} S_{t_j} \tag{5}$$

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2.2 DYNAMICS OF THE UNDERLYING ASSET

The inclusion of a jump component to the geometric Brownian motion (GBM) seeks to capture occasional discontinuous breaks on the asset returns as a response to the arrival of outside news in the financial markets (cf. Trautmann & Beinert, 1995). We assume that the price of the underlying asset (S_t) evolves according to a jump-diffusion process with independent increments, which consists of a GBM overlapped by a compound Poisson process. We assume a frictionless market in which the completeness property fails as a consequence of adding the jump risk (cf., e.g. Cont & Tankov, 2003). Therefore, multiple *risk-neutral* measures can be adopted such that the discounted asset price is a martingale. Merton (1976) proposes to choose the *risk-neutral* measure $\mathbb{Q}$, equivalent to the physical probability measure $\mathbb{P}$, by changing the drift of the diffusion part and leaving the jump part unchanged. We assume then that under the physical probability measure $\mathbb{P}$, the development of the unit price S_t can be described by

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW(t) + d\left(\sum_{i=1}^{N_t} (V_i - 1)\right) \tag{6}$$

where, $\{W(t); t \geq 0\}$ is a Wiener Process and $\{N_t; t \geq 0\}$ is a Poisson process with intensity λ. μ and $\sigma > 0$ are the drift and volatility of the diffusion part, and $\{V_i > 0; i = 1, 2, ...\}$ is a sequence of independent identically distributed and non-negative random variables such that $Y_i = ln(V_i)$. We assume that the processes $\{W(t); t \geq 0\}$, $\{N_t; t \geq 0\}$ and the random variables $\{Y_i; i = 1, 2, ...\}$ are stochastically independent. The GBM therefore corresponds to a special case of the jump-diffusion model when $\lambda = 0$. Applying Ito's lemma to solve equation (6), the log-return $R_{t_j} = ln\left(S_{t_j}/S_{t_{j-1}}\right)$ for the model with jumps is given by

$$\begin{aligned} R_{t_j} &= \left(\mu - \frac{1}{2}\sigma^2\right)(t_j - t_{j-1}) + \sigma\sqrt{t_j - t_{j-1}}(W_{t_j} - W_{t_{j-1}}) + \left(\sum_{i=1}^{N_{t_j}} ln(V_i) - \sum_{i=1}^{N_{t_{j-1}}} ln(V_i)\right) \\ &= \left(\mu - \frac{1}{2}\sigma^2\right)(t_j - t_{j-1}) + \sigma\sqrt{t_j - t_{j-1}}(W_{t_j} - W_{t_{j-1}}) + \left(\sum_{i=1}^{N_{t_j}} Y_i - \sum_{i=1}^{N_{t_{j-1}}} Y_i\right) \\ &= \left(\mu - \frac{1}{2}\sigma^2\right) \cdot \Delta t_j + \sigma\sqrt{\Delta t_j}(W_{t_j} - W_{t_{j-1}}) + \sum_{i=1}^{N_{\Delta t_j}} Y_i. \end{aligned} \tag{7}$$

with $\Delta t_j = t_j - t_{j-1}$ and $N_{\Delta t_j} = N_{t_j} - N_{t_{j-1}}$ denoting the independent increments of the Poisson process. If Δt_j is small, as it is the case of daily returns ($\Delta t_j = 1/360$), $N_{\Delta t_j}$ can be approximated to a binomial random variable B such that $Pr(B = 0) = 1 - \lambda\Delta t_j$ and $Pr(B = 1) = \lambda\Delta t_j$. From this it follows that R_{t_j} can be re-written as

$$R_{t_j} = \left(\mu - \frac{1}{2}\sigma^2\right) \cdot \Delta t_j + \sigma\sqrt{\Delta t_j}\xi_{t_j} + BY \tag{8}$$

The return for the period t_j under the *risk-neutral* measure $\mathbb{Q}$ takes the form

$$R^{\mathbb{Q}}_{t_j} = \left(r_f - \frac{1}{2}\sigma^2 - \lambda k\right)\Delta t_j + \sigma\sqrt{\Delta t_j}\xi_{t_j} + BY \tag{9}$$

where $\left\{\xi_{t_j}; t \geq 0\right\}$ is a standard Wiener process, r_f the risk-free rate of return, and $k = E[V] - 1$. In the Merton (1976) model, $Y = ln(V)$ has a normal distribution with density function

$$f_Y(y) = \frac{1}{\sqrt{2\pi}s} e^{-\frac{(y-\kappa)^2}{2s^2}} \tag{10}$$

where κ is the mean and s is the standard deviation of Y. $E[V]$ is given by

$$E[V] = e^{\kappa + \frac{s^2}{2}} \tag{11}$$

The corresponding expected value $\mu'[R]$ and variance $\sigma'^2[R]$ of the daily return R can be written as

$$\mu'[R] = \mu' + \lambda' \cdot \kappa \tag{12}$$

$$\sigma'^2[R] = \sigma'^2 + \lambda'[s^2 + (1 - \lambda')\kappa^2] \tag{13}$$

Here $\mu' = \mu\Delta t$, $\sigma' = \sigma\Delta t$ and $\lambda' = \lambda\Delta t$. In the case of the model framework proposed by Kou (2002), $Y = ln(V)$ has an asymmetric double-exponential distribution with density function defined by

$$f_Y(y) = p \cdot \eta_u \cdot e^{-\eta_u y}\mathbf{1}_{\{y \geq 0\}} + q \cdot \eta_d \cdot e^{\eta_d y}\mathbf{1}_{\{y<0\}}, \quad \eta_u > 1,\ \eta_d > 0 \tag{14}$$

where $p, q \geq 0$ represent the probability of upward and downward jumps and $p+q = 1$. The $\eta_u > 1$ restriction assures that $E[V] < \infty$ and $E[S_t] < \infty$. Thus $E[V]$ is given by

$$E[V] = p\frac{\eta_u}{\eta_u - 1} + q\frac{\eta_d}{\eta_d + 1}, \quad \eta_u > 1,\ \eta_d > 0 \tag{15}$$

In this case, the expected value $\mu'[R]$ and variance $\sigma'^2[R]$ of the daily return R can be formulated as

$$\mu'[R] = \mu' + \lambda' \left(\frac{p}{\eta_u} - \frac{q}{\eta_d}\right) \tag{16}$$

$$\sigma'^2[R] = \sigma'^2 + \lambda' \left[pq\left(\frac{1}{\eta_u} + \frac{1}{\eta_d}\right)^2 + \left(\frac{p}{\eta_u^2} + \frac{q}{\eta_d^2}\right)\right] + \left(\frac{p}{\eta_u} - \frac{q}{\eta_d}\right)^2 \lambda'(1 - \lambda') \tag{17}$$

2.3 VALUATION OF THE INVESTMENT GUARANTEE

In order to obtain the option guarantee coverage, the investor is asked to pay an arbitrage free premium π_0 at t=0 in addition to the regular insurance premium payment. π_0 is given by

$$\pi_0 = e^{-r_f T} \cdot E^{\mathbb{Q}}\left[max(G_T - F_T, 0)\right] \tag{18}$$

where $E^{\mathbb{Q}}$ denotes the expected value under the equivalent *risk-neutral* measure $\mathbb{Q}$.

3. NUMERICAL CALCULATIONS

3.1 PARAMETERS ESTIMATES

The underlying asset parameters for the Merton and Kou frameworks were calibrated for two different weighted portfolios of stocks, real estate, government and corporate bonds. Each asset class is represented by one market index. Stocks (E), real estate (REIT), government bonds (GB) and corporate bonds (CB) market indices are given by STOXX 50, Euronext IEIF REIT Europe, Barclays Euro Government Bond 10-year term and Barclays Euro-Aggregate Corporate, respectively. Additionally, a money market (MM) index, the three-month Euribor, was included as a proxy for the risk-free rate of return r_f and we rounded its volatility to zero. For each index, we extracted the historical daily returns between January 2004 and December 2013 from Bloomberg. The mean μ_I and standard deviation σ_I of each asset class index are summarized in Table 1. The associated correlation matrix can be found in Table 9 of Appendix C.

Table 1: Mean μ_I and standard deviation σ_I of the annualized return for selected indices

Asset Class	**Abbrev.**	**Index**	$\mu_I(\%)$	$\sigma_I(\%)$
Money Market	(MM)	EURIBOR 3-Month	2.05	0.078
Real Estate	(REIT)	Euronext IEIF REIT Europe	5.31	27.55
Stocks	(E)	STOXX 50	7.19	27.17
Government Bonds	(GB)	Barclays Euro Government Bond	7.65	5.70
Corporate Bonds	(CB)	Barclays Euro-Aggregate Corporate	6.25	3.36

This table contains the annualized mean μ_I and standard deviation σ_I of the selected indices. The historical data were extracted on a daily basis for all indices for the period January 2004 to December 2013.

Table 2 details the percentage share of each asset class used to constitute the two reference portfolios and the corresponding mean μ_P and volatility σ_P of each portfolio. Portfolio A has an annual volatility (σ_P) of 7% and Portfolio B has an annual volatility of 3.5%.

The parameters of the Merton model and the Kou model for each portfolio were estimated through the Expectation-Maximization algorithm (EM algorithm) for incomplete data (cf. Dempster et al., 1977). The EM algorithm estimates the optimum parameters by iteratively minimizing the likelihood function (see details of the EM algorithm in the Appendix A and B). Table 3 and Table

Table 2: Summary of portfolio composition and the corresponding annual mean μ_P and standard deviation σ_P

	$\mu_P(\%)$	$\sigma_P(\%)$	**Share (REIT) (%)**	**Share (E) (%)**	**Share (GB) (%)**	**Share (CB) (%)**
Portfolio A	7.10	7.00	14.00	16.00	51.25	18.75
Portfolio B	6.36	3.50	5.00	5.00	5.65	84.35

4 contain the EM-estimated parameters of the two reference Portfolios A and B for the Merton and Kou models, respectively. The annual returns μ_P and the total volatility σ_P of the reference portfolios can be calculated by replacing the corresponding Merton parameters estimates from Table 3 in equation 12 and equation 13, as well as the Kou parameter estimates from Table 4 in equation 16 and equation 17.

Table 3: EM parameter estimates of the Merton jump-diffusion model for the two reference portfolios

	$\sigma_P(\%)^{a}$	$\mu(\%)$	$\sigma(\%)$	λ	$\kappa(\%)$	$s(\%)$
Portfolio A						
Estimate	7.00	14.40	4.69	74.07	-0.10	0.597
Std. Err	-	(0.15)	(0.01)	(0.90)	(0.00)	(0.00)
Portfolio B						
Estimate	3.50	10.84	2.65	48.29	-0.09	0.318
Std. Err	-	(0.07)	(0.01)	(0.9071)	(0.00)	(0.00)

[a] Total annual portfolio volatility. Std. Err = Standard Error of the parameter estimate. Method: Bootstrap with 300 replications.

Table 4: EM parameter estimates of the Kou jump-diffusion model for the two reference portfolios

	$\sigma_P(\%)^{a}$	$\mu(\%)$	$\sigma(\%)$	λ	$p(\%)$	η_u	η_d
Portfolio A							
Estimate	7.00	15.93	4.06	168.36	44.84	347.18	303.45
Std. Err	-	(0.40)	(0.02)	(2.45)	(0.51)	(2.49)	(2.00)
Portfolio B							
Estimate	3.50	7.59	2.32	164.25	60.88	922.80	532.33
Std. Err	-	(0.25)	(0.01)	(3.02)	(0.57)	(6.08)	(3.09)

[a] Total annual portfolio volatility. Std. Err = Standard Error of the parameter estimate. Method: Bootstrap with 300 replications.

Figure 1 compares the empirical density functions of the daily returns of Portfolios A and B, and

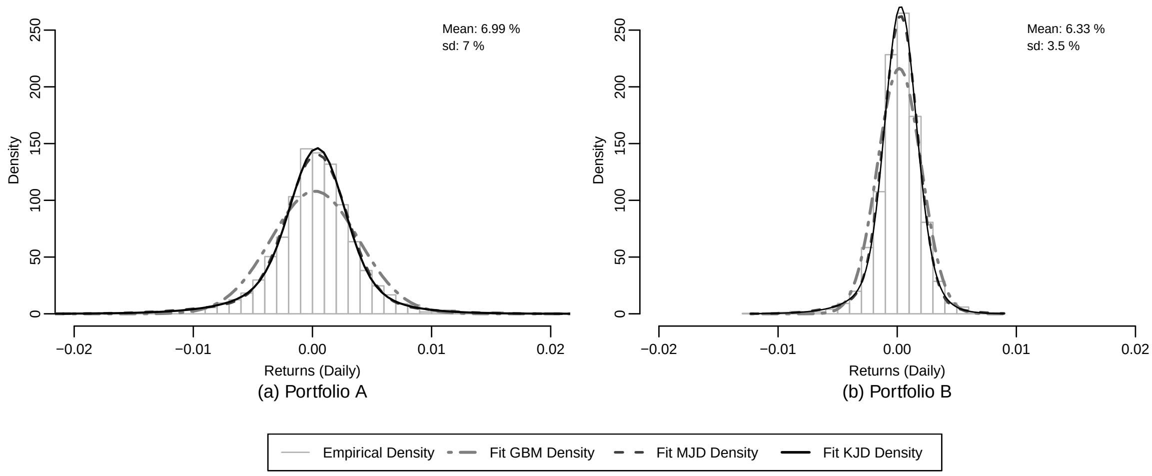


Figure 1: Fitted and empirical density distributions of daily returns for the two selected portfolios. The parameters of the geometric Brownian motion (GBM) fitted distribution are the mean and standard deviation of each portfolio (right corner of plot (a) and (b)). The parameters for the Merton model (MJD) and Kou model (KJD) are presented in Table 3 and Table 4, respectively.

the corresponding density functions of the GBM, MJD and KJD. It can be observed that the density functions of the MJD and the KJD approximate the peakedness of the empirical density functions of the daily returns for both portfolios far more precisely than the GBM.

We highlight the model selection by calculating the widely used Bayesian Information Criterion (BIC) proposed by Schwarz (1978). This criterion states that the "*best fit*" model corresponds to the one that has the largest BIC value. The BIC value is given by

$$BIC = -2log\left[L(\mathbf{R}, \hat{\theta})\right] + nlog(m) \tag{19}$$

where $L(\mathbf{R}, \hat{\theta})$ is the maximum likelihood function evaluated at $\hat{\theta}$, which denotes the vector of the estimated parameters, n the number of independent parameters, i.e., dimension of $\hat{\theta}$, and m the number of observations used for the estimation. The vector of parameters θ for the Merton model is $\theta = (\mu, \sigma, \lambda, \kappa, s)$, and the corresponding vector of parameters for the Kou model is $\theta = (\mu, \sigma, \lambda, p, \eta_u, \eta_d)$. Table 5 compares the BIC values of each selected portfolio under the two jump-diffusion frameworks studied in this paper and the GBM. The results confirm the general statement that the financial returns are better explained by jump-diffusion models instead of the widely used GBM.

Table 5: BIC values of the underlying asset for geometric Brownian motion, Merton and Kou models

	Brownian Motion	**Merton Model**	**Kou Model**
Portfolio A	-21052.77	**-21420.65**	-21405.15
Portfolio B	-24543.25	-24839.27	**-24841.56**

The parameter estimates use to calculate the BIC of Portfolio A and Portfolio B for the geometric Brownian motion are: drift = μ_P, mean return of the portfolio and volatility = σ_P, standard deviation of the portfolio return. The BICs for the Merton and Kou models are calculated using the parameter estimates presented in Table 3 and Table 4, respectively.

3.2 OPTION PRICE ESTIMATES

The definition of the IRG and LBG and the case of regular premium payments imply asset path dependence of the fund values. Hence, in general an explicit formula for the option prices cannot be derived. As a consequence, we estimate the guarantee prices via Monte Carlo simulation with $n = 500,000$ paths. Common random numbers together with antithetic variates were used to reduce the variance of the estimates (cf. Boyle, 1977; Boyle et al., 1997). We assumed that the log-returns of the underlying asset satisfy equation (8)for the empirical development, and equation (9) for the development under the risk-neutral measure. The jumps are normally (cf. equation 10) or double-exponentially distributed (cf. equation 14). The Merton and Kou parameters correspond to those that were estimated in Section 3.1 for Portfolio A ($\sigma_P = 7\%$) and Portfolio B ($\sigma_P = 3.5\%$). The risk-free interest rate r_f used for the price estimation under the *risk-neutral* measure is 2.5%, which coincides with the mean value of the money market (MM) index. The maturity terms of the policies correspond to $T_1 = 5$ and $T_2 = 10$, T given in *years*. The two payment modalities are: single premium with $P_1 = 6000$ and $P_2 = 12000$, for T_1 and T_2 respectively, and monthly level premiums $P^{(m)} = 100$ for both terms (cf. Gatzert & Schmeiser, 2009). The maximum price value of the underlying asset for the LBG is locked every three months. The underlying price simulation was performed on a daily basis. Finally, we independently calibrated for the Merton and Kou models the minimum guaranteed interest rate g, such that π_0^{IRG}, the premium of the

IRG, and π_0^{LBG}, the premium of the LBG, are equal. The corresponding g was assessed via the secant method. The equality in the initial premiums permits to compare the sensitivity of the two guarantee forms to changes in the initial parameters hypotheses.

Table 6: Descriptive statistics and performance of the interest rate and lookback guarantees for the Merton and Kou models.

	$T = 5$				$T = 10$			
	Portfolio A ($\sigma_P = 7\%$)		Portfolio B ($\sigma_P = 3.5\%$)		Portfolio A ($\sigma_P = 7\%$)		Portfolio B ($\sigma_P = 3.5\%$)	
	IRG	LBG	IRG	LBG	IRG	LBG	IRG	LBG
Merton Model[a]								
Single Premium								
Guarantee (G_T)	6673.06	6000.00	6461.89	6000.00	14502.95	12000.00	13896.22	12000.00
g (%)	2.13	0.00	1.48	0.00	1.89	0.00	1.47	0.00
Price	386.47	386.47	112.59	112.59	959.50	959.50	243.12	243.12
Std. Err	0.34	0.27	0.17	0.11	0.88	0.67	0.42	0.25
$E(L_T)$	8640.61	8763.58	8257.80	8281.04	24748.61	25166.28	22729.36	22793.57
$\sigma(L_T)$	289.20	287.43	109.21	114.17	1166.45	1217.16	444.53	454.55
$\gamma_1(L_T)$	1.76	1.09	0.11	0.18	1.68 1.31	0.23	0.26	
Monthly Premiums								
Guarantee	6632.17	6000.00	6331.44	6000.00	14027.88	12000.00	13174.25	12000.00
g (%)	3.88	0.00	2.10	0.00	3.02	0.00	1.82	0.00
Price	383.82	383.82	108.92	108.92	915.57	915.57	223.33	223.33
Std. Err	0.17	0.28	0.11	0.11	0.46	0.69	0.28	0.23
$E(L_T)$	7312.12	7377.98	7088.61	7107.85	17702.66	17969.38	16842.32	16889.89
$\sigma(L_T)$	182.02	177.66	56.61	62.80	532.43	562.18	197.91	209.97
$\gamma_1(L_T)$	1.61	1.36	0.15	0.39	1.79	1.24	0.10	0.23
Kou Model[b]								
Single Premium								
Guarantee	6673.26	6000.00	6461.91	6000.00	14500.74	12000.00	13895.49	12000.00
g (%)	2.13	0.00	1.48	0.00	1.89	0.00	1.47	0.00
Price	386.52	386.52	112.88	112.88	959.92	959.92	243.68	243.68
Std. Err	0.36	0.28	0.17	0.12	0.93	0.71	0.42	0.25
$E(L_T)$	8639.15	8762.07	8257.85	8281.36	24745.11	25163.10	22729.40	22794.21
$\sigma(L_T)$	450.88	446.73	188.04	188.42	1871.04	1884.19	741.47	743.07
$\gamma_1(L_T)$	0.37	0.31	0.04	0.05	0.47	0.45	0.10	0.11
Monthly Premiums								
Guarantee	6632.20	6000.00	6331.84	6000.00	14027.12	12000.00	13174.12	12000.00
g (%)	3.88	0.00	2.10	0.00	3.02	0.00	1.82	0.00
Price	383.88	383.88	109.22	109.22	916.17	916.17	223.87	223.87
Std. Err	0.22	0.30	0.11	0.11	0.56	0.73	0.28	0.24
$E(L_T)$	7311.54	7377.32	7088.53	7107.96	17702.12	17969.02	16842.20	16890.23
$\sigma(L_T)$	235.61	232.87	97.39	97.92	830.13	830.43	339.78	340.93
$\gamma_1(L_T)$	0.48	0.42	0.00	0.05	0.40	0.38	0.04	0.06

IRG= Interest Rate Guarantee; LBG=Lookback Guarantee; Std. Err=standard error of the option price estimate; g = minimum rate of return; L_T = contract's payoff at maturity T. The payoff L_T estimates were calculated under the empirical measure $\mathbb{P}$. The Monte Carlo simulation is obtained by calculating the prices of the underlying asset on a daily basis and 500,000 simulation paths. The risk-free interest rate is $r_f = 2.05\%$ for both models.

[a] The parameters for the Merton model are: Portfolio A $\mu = 14.40\%$, $\sigma = 4.69\%$, $\lambda = 74.07$, $\kappa = -0.10\%$, $s = 0.597\%$; and Portfolio B $\mu = 10.84\%$, $\sigma = 2.65\%$, $\lambda = 48.29$, $\kappa = -0.09\%$, $s = 0.318\%$.

[b] The parameters for the Kou model are: Portfolio A $\mu = 15.93\%$, $\sigma = 4.06\%$, $\lambda = 168.36$, $p = 44.84\%$, $\eta_u = 347.18$, $\eta_d = 303.45$; and Portfolio B $\mu = 7.59\%$, $\sigma = 2.32\%$, $\lambda = 164.25$, $p = 60.88$, $\eta_u = 922.80$, $\eta_d = 532.33$.

Table 6 details the estimated option prices and maturity payouts obtained when the underlying asset follows Merton and Kou jump-diffusion models. Several points from Table 6 can be discussed. Firstly, the option prices do not show differences when they are modelled under the two different underlying frameworks. Secondly, the standard errors of the price estimates with the Kou model are slightly greater than Merton model standard errors. Thirdly, the expected values of the maturity payoff are approximately equal for both frameworks. Even though, the standard deviation for the payouts modelled under the Kou model are 1.3 to 1.7 times greater than the standard deviation of the payout modelled with the Merton approach.

3.3 OPTION PRICE SENSITIVITY ANALYSIS

In this section, we discuss how the option prices of the IRG and LBG react as the underlying assumptions of the model parameters change. Our analysis focuses on the drift μ and volatility σ parameters of the diffusion part of the two jump-diffusion frameworks. Subsequently, the sensitivities of the option price to changes in the jumps' parameters are analyzed separately for the Merton and Kou models. The sensitivity of financial options prices to changes in the parameters is usually assessed by computing the first degree partial derivatives of the option price, also-called "*Greeks*". Nevertheless, some difficulties in the comparison of the different price partial derivatives might arise from the various scales of the parameters in jump-diffusion models. We intend to avoid this issue by measuring the price sensitivities through price partial elasticities. The *partial elasticity* measures the percentage by which the option price π_0 changes when a particular parameter θ_i changes by 1% percent, where θ_i corresponds to the i-th component of the vector of parameters θ in the model of the underlying asset. The *partial elasticity* of the guarantee price π_0 with respect to the parameter θ_i is denoted by

$$e_{\theta_i} = \frac{\partial \pi_0}{\partial \theta_i} \frac{\theta_i}{\pi_0} \tag{20}$$

where $\frac{\partial \pi_0}{\partial \theta_i}$ is the partial derivative of the option price π_0 with respect to the component θ_i. The partial derivative of the option price with respect to the parameter θ_i is estimated via the indirect re-simulation method (cf. Broadie & Glasserman, 1996) and it is defined as

$$\frac{\partial \pi_0}{\partial \theta_i} \approx \frac{\pi_0(\theta_1, ..., \theta_{i-1}, (1+\epsilon)\theta_i, \theta_{i+1}, ..., \theta_n) - \pi_0(\theta_1, ..., \theta_{i-1}, (1-\epsilon)\theta_i, \theta_{i+1}, ..., \theta_n)}{2\epsilon\theta_i} \quad \epsilon \to 0 \tag{21}$$

where $\epsilon = 0.0001\%$ for all parameters except for p (probability of upward jump) in the Kou model, and the jump frequency parameter λ for the Merton and Kou models. In these two particular cases, we use $\epsilon = 1\%$ given that p and λ are described by discrete distributions which generate instability in the partial derivative estimator when ϵ is too close to zero.

Table 7 contains the estimated partial elasticities of the option price for the IRG and LBG. The partial derivatives of the option price can be found in Table 10 of Appendix D. The table includes two main panels which report the price sensitivities obtained for the Merton and Kou models, respectively. Each panel presents the option price partial elasticities calculated for the IRG and the LBG. The sensitivities were calculated for single premium and monthly premium modalities, the two different maturity terms and the two different selected portfolios. In general, LBGs showed higher elasticity values, thus higher sensitivity to changes in model parameters than IRGs, except for the risk-free rate r_f.

The results show that the overall responses of the IRGs to changes in the risk-free rate r_f were greater than in the cases of the LBGs for both underlying asset models. The sensitivities of the IRG are over 70% higher than the LBG products for single premiums. These differences between the elasticities of the IRGs and LBGs are reduced for monthly premiums. We notice that the IRG sensitivities are about 40% higher when the payment modality is single premium, compared to sensitivities for the monthly premium modality. On the contrary, the LBG elasticities show that single premium products are less sensitive to changes in the risk-free parameter than monthly premiums. In addition, it can be recognized that as the overall volatility of the portfolio is reduced by half, the risk-free sensitivities of both guarantees increase by a factor of 2 to 2.7. Lastly, we see that IRGs raise their sensitivities by a higher percentage (on average 50%) than LBGs (an average an increase of 18%) for longer-term maturities.

Table 7: Parameter elasticities of the option price π_0 of the interest rate and lookback guarantees for Merton and Kou models.

	$T = 5$				$T = 10$			
	Portfolio A ($\sigma_P = 7\%$)		Portfolio B ($\sigma_P = 3.5\%$)		Portfolio A ($\sigma_P = 7\%$)		Portfolio B ($\sigma_P = 3.5\%$)	
	IRG	LBG	IRG	LBG	IRG	LBG	IRG	LBG
Merton Model[a]								
Single Premium								
e_{r_f}	-0.760	-0.445	-1.876	-0.957	-1.096	-0.523	-2.827	-1.048
e_σ	0.435	0.697	0.875	1.167	0.487	0.777	1.045	1.274
e_λ	0.266	0.425	0.335	0.446	0.296	0.474	0.395	0.475
e_κ	0.011	0.019	0.046	0.066	0.013	0.021	0.056	0.072
e_s	0.522	0.837	0.612	0.820	0.587	0.933	0.732	0.899
Monthly Premiums								
e_{r_f}	-0.531	-0.501	-1.381	-1.016	-0.783	-0.643	-2.140	-1.167
e_σ	0.223	0.720	0.554	1.182	0.268	0.811	0.710	1.294
e_λ	0.136	0.440	0.209	0.452	0.160	0.495	0.266	0.483
e_κ	0.006	0.020	0.029	0.067	0.007	0.023	0.037	0.073
e_s	0.267	0.866	0.384	0.831	0.321	0.975	0.497	0.913
Kou Model[b]								
Single Premium								
e_{r_f}	-0.764	-0.445	-1.869	-0.953	-1.102	-0.524	-2.820	-1.048
e_σ	0.327	0.524	0.668	0.893	0.366	0.584	0.799	0.975
e_λ	0.142	0.228	0.125	0.168	0.159	0.250	0.146	0.184
e_p	-0.022	-0.025	-0.272	-0.381	-0.031	-0.032	-0.324	-0.407
e_{η_u}	-0.256	-0.407	-0.297	-0.387	-0.289	-0.454	-0.355	-0.424
e_{η_d}	-0.385	-0.624	-0.566	-0.770	-0.433	-0.693	-0.677	-0.845
Monthly Premiums								
e_{r_f}	-0.531	-0.502	-1.379	-1.013	-0.782	-0.642	-2.133	-1.166
e_σ	0.168	0.541	0.424	0.904	0.201	0.609	0.544	0.991
e_λ	0.072	0.236	0.082	0.171	0.086	0.263	0.101	0.187
e_p	-0.015	-0.027	-0.167	-0.374	-0.021	-0.035	-0.199	-0.382
e_{η_u}	-0.133	-0.420	-0.190	-0.392	-0.160	-0.473	-0.242	-0.430
e_{η_d}	-0.195	-0.646	-0.352	-0.781	-0.236	-0.726	-0.457	-0.859

IRG = interest rate guarantee; LBG=lookback g uarantee. The Monte Carlo simulation is obtained by calculating the prices of the underlying asset on a daily basis and 500,000 simulation paths. The risk-free interest rate is $r_f = 2.05\%$ for both models.

[a] The parameters for the Merton model are: Portfolio A $\mu = 14.40\%$, $\sigma = 4.69\%$, $\lambda = 74.07$, $\kappa = -0.10\%$, $s = 0.597\%$; and Portfolio B $\mu = 10.84\%$, $\sigma = 2.65\%$, $\lambda = 48.29$, $\kappa = -0.09\%$, $s = 0.318\%$.

[b] The parameters for the Kou model are: Portfolio A $\mu = 15.93\%$, $\sigma = 4.06\%$, $\lambda = 168.36$, $p = 44.84\%$, $\eta_u = 347.18$, $\eta_d = 303.45$; and Portfolio B $\mu = 7.59\%$, $\sigma = 2.32\%$, $\lambda = 164.25$, $p = 60.88$, $\eta_u = 922.80$, $\eta_d = 532.33$.

The re-simulation estimates were calculated for $\epsilon = 0.0001\%$ except for p and λ where $\epsilon = 1\%$ is used.

Regarding the volatility parameter σ, we observe that for Merton and Kou models the IRG is less sensitive to changes in the volatility parameter than the LBG. The σ-elasticities for the IRG are between -70% and -17% smaller than the σ-elasticities observed in LBGs. The IRG shows that the effect of changes in the initial parameter σ is less significant for the monthly premium products than for single premiums. The sensitivities when the product has periodic premiums are on average 70% lower than the sensitivities observed for single premiums. In contrast, the LBG elasticities do not show large differences, differences are on average 2%, between the sensitivities for the two premium payment modalities. By comparing the sensitivity results of Portfolios A and B, we observed that the IRG's sensitivities double when the overall volatility is reduced, while the LBG's sensitivities increase by only 60%. Finally, both guarantees increased their sensitivity as the maturity term of the policy increases.

The discussed results seem to confirm our expectations. As the IRG promises a minimum return over the investment, changes in the risk-free rate r_f will mainly affect this product rather than changes in the volatility parameter. On the contrary, LBG products are affected to a large extent by volatility parameters, which highly influences the maximum price the underlying asset can reach.

The sensitivity analysis with respect to jump parameters was performed separately for the Merton and Kou models. On the one hand, the elasticities of the three parameters related to the jumps ($lambda, \kappa, s$) for the Merton model (Table 7: upper panel) indicate that LBG products have on average 45% greater sensitivity than IRGs in the numerical cases observed. The LBG does not show important differences in the price sensitivities for single and monthly premiums. On the contrary, the IRG seems to importantly reduce its sensitivities to changes in the jump parameters when the product has periodic premium payments. The comparison between the sensitivities of the two guarantees when the overall volatility of the portfolio is reduced, shows that the elasticity values are greater for Portfolio B (with the lowest volatility $\sigma_P = 3.5\%$) than for Portfolio A (with the highest volatility $\sigma_P = 7\%$). Moreover, the elasticities of the jump size parameter κ show an increase of around 70% when the overall volatility of the portfolio is reduced.

On the other hand, the sensitivities of the Kou model jump parameters $(\lambda, p, \eta_u, \eta_d)$ (Table 7: lower panel) demonstrate that the LBG is about 40% more sensitive to changes in jump parameters than the IRG. As it was observed before in the Merton model, single premium products have 66% higher sensitivity than monthly premium products in the case of IRG, while the LBG sensitivities remain barely affected by changes in the premium payment frequency. Additionally, a reduction in the overall volatility of the reference portfolio results in an increase in the jump sensitivities of the IRG price. In particular, a significant increase in the sensitivity of the upward jump probability parameter p can be observed. The elasticities of this parameter for Portfolio B (the portfolio with the lowest volatility) are approximately 9 to 12 times the p-elasticities of Portfolio A. The elasticities of the other jump parameters λ, η_u and η_d increase on average 20%, 35% and 70%, respectively. Similarly, the p-sensitivities of the LBG increase between 10 and 15 times when the overall volatility is reduced. The sensitivities of the jump frequency λ and upward jump parameter η_u decrease by 27% and 7%, respectively, while the sensitivities of the parameter of negative jump η_d increase by 21%.

Finally, the option price sensitivity to changes in the total portfolio volatility σ_P are displayed in Figure 2. We present the results for the Merton model, since the Merton and Kou models did not reveal substantial differences with respect to the option price (cf. Figure 4 in Appendix D for the Kou model). Portfolio A with annual volatility $\sigma_P = 7\%$ was taken as the base case and we analyze fluctuations between -50% and 100%; therefore, the total portfolio volatility $\sigma_P \in [3.50\%, 14\%]$. The change in the total volatility of the portfolio σ_P requires the re-estimation of the parameters for the Merton and Kou models. The fixed and unchanged parameters are the risk-free interest rate $r_f = 2.05\%$ and the minimum guaranteed interest rate g. The minimum guaranteed interest rates g are those that were calculated when the option prices were estimated for a portfolio volatility of $\sigma = 7.00\%$. For the contracts with a maturity term T=5, the guaranteed interest rates are g=2.13% for the contract with a single premium and 3.88% for the contracts with monthly premiums. For maturity term T=10, the guaranteed rates are g=1.89% for single premium contracts and 3.02% for monthly premiums.

Figure 2 indicates that in general IRG products are less sensitive to changes in parameters than LBG products. On the one hand, the IRG prices increase almost linearly when the overall volatility of the portfolio σ_P changes. It can be seen from Figure 2(a) and Figure 2(c) that a reduction of 50% in the volatility of portfolio A to $\sigma_P = 3.50\%$ reduces the price of the IRG by half. Likewise, if the total portfolio volatility is twice the volatility of the base case, i.e., $\sigma_P = 14.00\%$,

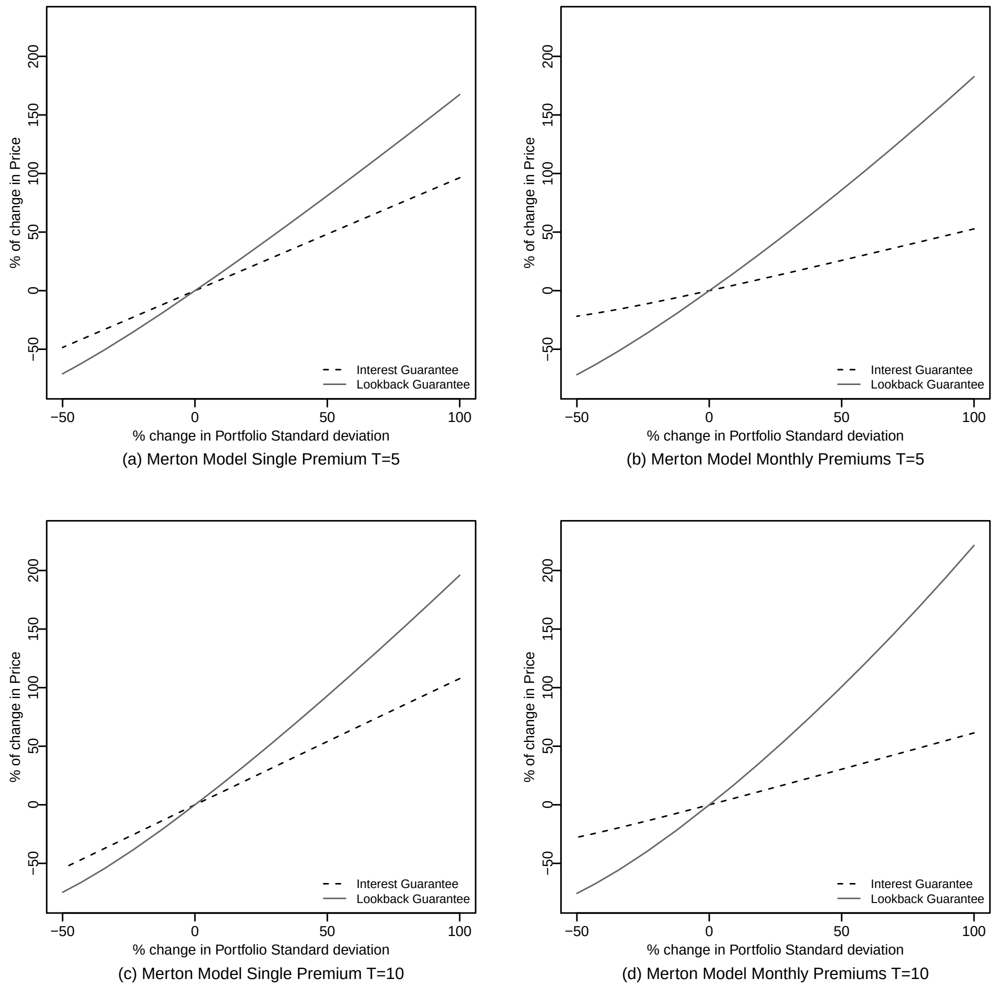


Figure 2: Sensitivity of the option price π_0. Percentage of changes of the interest rate and lookback guarantee prices when the total volatility of Portfolio A ($\sigma_P = 7\%$) changes. The underlying asset is modelled using the assumptions of the Merton model. The risk-free interest rate is $r_f = 2.05\%$. (a) Single premium contract, maturity term T=5, minimum guaranteed interest rate g=2.13% and a price $\pi_0 = 386.47$ in the base case. (b) Monthly premium contract, maturity term T=5, minimum guaranteed interest rate g=3.88% and a price $\pi_0 = 383.82$ in the base case. (c) Single premium contract, maturity term T=10, minimum guaranteed interest rate g=3.02% and a price $\pi_0 = 959.50$ in the base case. (d) Monthly premium contract, maturity term T=10, minimum guaranteed interest rate g=1.89% and a price $\pi_0 = 915.57$ in the base case.

the option price nearly doubles in value. A smaller sensitivity can be observed in the monthly premium contracts illustrated in Figure 2(b) and Figure 2(d). When the portfolio volatility σ_P is reduced by half, the option price decreases by 25%, and if σ_P doubles, then the prices of the guarantee increase by approximately 50%. On the other hand, the option price of the LBG did not show substantial differences between price sensitivities of the LBG for single and monthly premiums. The price of the guarantee decreases by over 50% when the portfolio volatility σ_P drops by half and if the portfolio doubles its volatility, the guarantee price increases between 1.6 and 2 times.

3.4 MATURITY PAYOFF SENSITIVITY

With regard to the sensitivity analysis of the maturity payoff, Table 8 shows the elasticities of the expected maturity payoff ($E[L_T]$) organized into two main panels, which report the values obtained for the Merton and Kou models, respectively. Each panel presents the maturity payoff partial elasticities for the IRG and the LBG. The elasticities were calculated for single premium and monthly premium contracts with two different maturity terms and reference portfolios offering two different volatilities ($\sigma_P = 3.50\%$ or $\sigma_P = 7.00\%$). The partial derivatives of the payoff L_T can be found in Table 11 of Appendix E. In general, the comparison of the sensitivities of the IRGs and the LBGs does not show large differences, except for the volatility parameters. On average the differences in the sensitivity values of both guarantees are between -1% and +12% for the Merton and the Kou models. The IRGs showed to be on average 70% less sensitive to changes in the parameters σ and s than LBGs for the Merton model. Similarly, for the Kou model the IRGs have about 80% less sensitivity to changes in the volatility parameter σ than the LBGs. However, it can be seen that changes in the volatility parameter σ do not lead to substantial changes in the expected payoff for both guarantees. Single premium products for both guarantees are two times more sensitive than products under the periodic premium modality. As expected, a decrease of 50% in the portfolio volatility (comparing sensitivities of Portfolio A and B) means less risk taken by the investor, and smaller sensitivities of the payoff to changes in the overall parameters. Additionally, as the term of the contract is longer, we observe an average increase of 100% in the sensitivity for all parameters except for σ.

Lastly, Figure 3 shows the sensitivity of the maturity payoff to changes in the total volatility of the portfolio. We only present the results for the Merton model, as the Merton and Kou models only show small differences regarding the expected payoff at maturity $E[L_T]$ (see Figure 5 in Appendix

Table 8: Parameter elasticities of the expected maturity payoff $E[L_T]$ of the interest rate and lookback guarantee for Merton and Kou models.

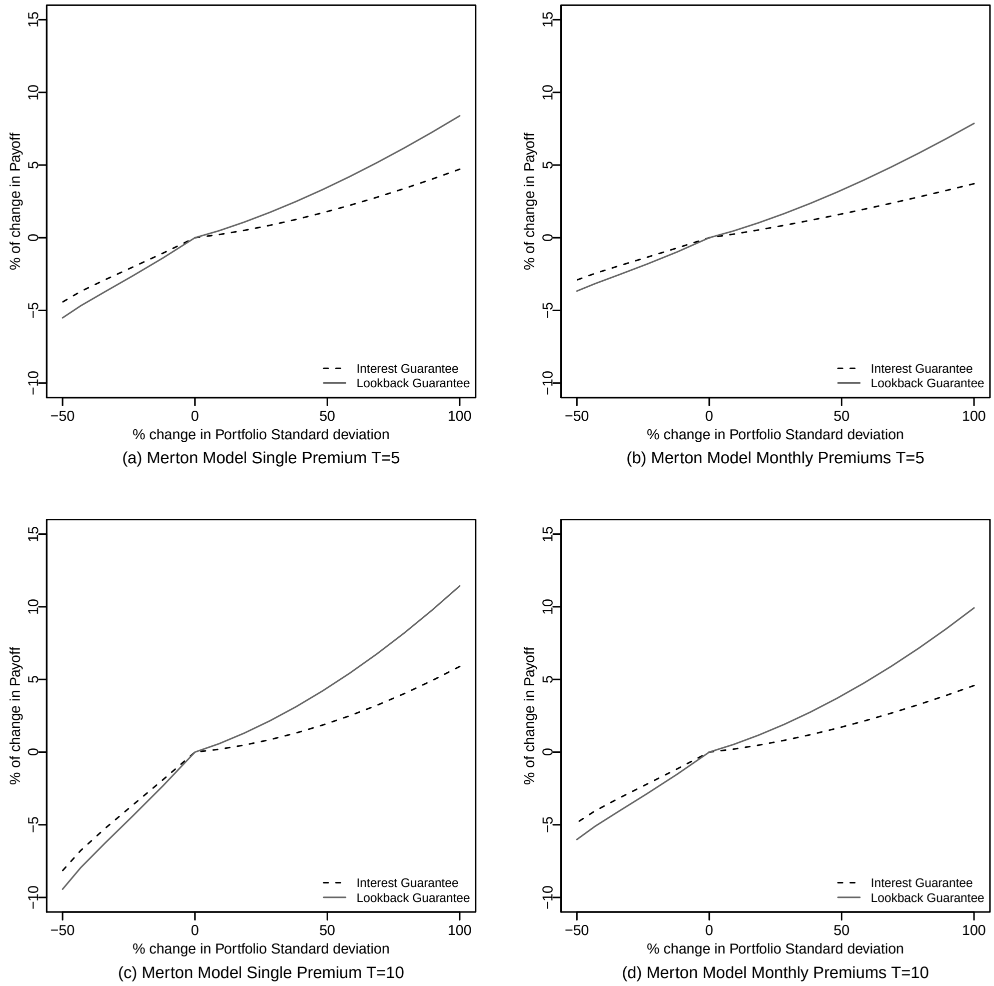
	$T=5$				$T=10$			
	Portfolio A ($\sigma_P = 7\%$)		Portfolio B ($\sigma_P = 3.5\%$)		Portfolio A ($\sigma_P = 7\%$)		Portfolio B ($\sigma_P = 3.5\%$)	
	IRG	LBG	IRG	LBG	IRG	LBG	IRG	LBG
Merton Model[a]								
Single Premium								
$e^{L_T}_{\mu}$	0.686	0.664	0.542	0.531	1.431	1.382	1.084	1.073
$e^{L_T}_{\sigma}$	0.007	0.020	0.000	0.005	0.002	0.020	0.000	0.005
$e^{L_T}_{\lambda}$	-0.337	-0.318	-0.223	-0.216	-0.711	-0.675	-0.446	-0.439
$e^{L_T}_{\kappa}$	-0.347	-0.336	-0.224	-0.219	-0.725	-0.700	-0.448	-0.443
$e^{L_T}_{s}$	0.020	0.036	0.003	0.006	0.028	0.050	0.005	0.009
Monthly Premiums								
$e^{L_T}_{\mu}$	0.323	0.330	0.542	0.531	0.778	0.753	1.084	1.073
$e^{L_T}_{\sigma}$	0.010	0.021	0.001	0.005	0.005	0.021	0.000	0.005
$e^{L_T}_{\lambda}$	-0.155	-0.151	-0.118	-0.113	-0.384	-0.362	-0.248	-0.242
$e^{L_T}_{\kappa}$	-0.164	-0.166	-0.119	-0.115	-0.394	-0.381	-0.249	-0.244
$e^{L_T}_{s}$	0.018	0.031	0.002	0.005	0.021	0.039	0.003	0.007
Kou Model								
Single Premium								
$e^{L_T}_{\mu}$	0.759	0.735	0.379	0.371	1.583	1.529	0.759	0.751
$e^{L_T}_{\sigma}$	0.005	0.015	0.000	0.004	0.001	0.015	0.000	0.004
$e^{L_T}_{\lambda}$	-0.411	-0.387	-0.060	-0.056	-0.863	-0.820	-0.121	-0.117
$e^{L_T}_{p}$	2.223	2.151	1.478	1.446	4.634	4.477	2.959	2.926
$e^{L_T}_{\eta_u}$	-1.047	-1.020	-0.543	-0.533	-2.175	-2.112	-1.086	-1.076
$e^{L_T}_{\eta_d}$	1.444	1.384	0.601	0.585	3.020	2.900	1.202	1.186
Monthly Premiums								
$e^{L_T}_{\mu}$	0.358	0.365	0.201	0.195	0.861	0.833	0.423	0.415
$e^{L_T}_{\sigma}$	0.008	0.015	0.000	0.004	0.004	0.016	0.000	0.004
$e^{L_T}_{\lambda}$	-0.188	-0.184	-0.032	-0.028	-0.465	-0.439	-0.067	-0.063
$e^{L_T}_{p}$	1.047	1.067	0.784	0.758	2.518	2.437	1.648	1.615
$e^{L_T}_{\eta_u}$	-0.497	-0.513	-0.288	-0.281	-1.185	-1.156	-0.605	-0.595
$e^{L_T}_{\eta_d}$	0.674	0.677	0.318	0.305	1.638	1.571	0.670	0.653

IRG = Interest Rate Guarantee; LBG=Lookback Guarantee. The payoff L_T estimates were calculated under the empirical measure $\mathbb{P}$. The Monte Carlo simulation is obtained by calculating the prices of the underlying asset on a daily basis and 500,000 simulation paths. The risk-free interest rate is $r_f = 2.05\%$ for both models.

[a] The parameters for the Merton model are: Portfolio A $\mu = 14.40\%$, $\sigma = 4.69\%$, $\lambda = 74.07$, $\kappa = -0.10\%$, $s = 0.597\%$; and Portfolio B $\mu = 10.84\%$, $\sigma = 2.65\%$, $\lambda = 48.29$, $\kappa = -0.09\%$, $s = 0.318\%$.

[b] The parameters for the Kou model are: Portfolio A $\mu = 15.93\%$, $\sigma = 4.06\%$, $\lambda = 168.36$, $p = 44.84\%$, $\eta_u = 347.18$, $\eta_d = 303.45$; and Portfolio B $\mu = 7.59\%$, $\sigma = 2.32\%$, $\lambda = 164.25$, $p = 60.88$, $\eta_u = 922.80$, $\eta_d = 532.33$.

The re-simulation estimates were calculated for $\epsilon = 0.0001\%$ except for p and λ where $\epsilon = 1\%$ is used.



(a) Merton Model Single Premium T=5

(b) Merton Model Monthly Premiums T=5

(c) Merton Model Single Premium T=10

(d) Merton Model Monthly Premiums T=10

Figure 3: Sensitivity of the expected payoff $E[L_T]$ at maturity T. Percentage of changes in the interest rate and lookback guarantees prices when the total volatility of Portfolio A ($\sigma_P = 7\%$) changes. The underlying asset is modelled using the assumptions of the Merton model. The risk-free interest rate is $r_f = 2.05\%$. (a) Single premium contract, maturity term T=5, minimum guaranteed interest rate g=2.13% and a price $E[L_T] = 8640.61$ in the base case. (b) Monthly premium contract, maturity term T=5, minimum guaranteed interest rate g=3.88% and a price $E[L_T] = 7312.12$ in the base case. (c) Single premium contract, maturity term T=10, minimum guaranteed interest rate g=3.02% and a price $E[L_T] = 24748.61$ in the base case. (d) Monthly premium contract, maturity term T=10, minimum guaranteed interest rate g=1.89% and a price $E[L_T] = 17702.66$ in the base case.

E for the Kou model). As stated in Section 3.3, the base portfolio is Portfolio A with annual volatility $\sigma_P = 7\%$ which fluctuates between -50% and 100%, i.e., the total portfolio volatility

$\sigma_P \in [3.50\%, 14\%]$. The parameters of the Merton and Kou models are re-estimated for each σ_P in the interval of variation, while the risk-free interest rate $r_f = 2.05\%$ and the minimum guaranteed interest rate g remain invariant. The minimum guaranteed interest rates g are those that were calculated when the option prices were estimated for a portfolio volatility of $\sigma = 7.00\%$. The guaranteed interest rates for the contracts with a maturity term T=5 are g=2.13% for the contract with a single premium and 3.88% for the contracts with a monthly premium. For T=10 the corresponding guaranteed rates are g=1.89% for single premium contracts and 3.02% for monthly premiums.

In general, the expected payoff of both guarantees shows a change of around -10% and 10% when the portfolio volatility σ_P has a variation range between [-0.5, 1]. Marginal differences can be observed between the changes in the expected payoff of the IRG and LBG when the portfolio volatility σ_P decreases. However, the differences are more pronounced when the portfolio volatility σ_P increases. In this case, the LBG expected payoff increases between 7% and 10% when the portfolio volatility doubles, while the payoff of the IRG has an increase of less than 6% in all cases.

4. SUMMARY AND CONCLUSIONS

The valuation of the option price and its sensitivities for equity-linked products with guarantees is of both theoretical interest and practical importance. Using empirical data, the paper provides a comparison of IRGs and LBGs when the pricing risk model includes jumps. In addition, we analyze the price and maturity payout sensitivity to changes in the initial modelling assumptions. In summary, the Merton and Kou jump-diffusion models lead to similar option prices as well as maturity payoffs with nearly the same expected value but different standard deviations. In respect to the sensitivity of the price of the guarantee, our analysis reveals that from the point of view of the providers the LBGs usually have higher parameter uncertainty than the IRGs, except for the sensitivity of the risk-free interest rate. Furthermore, longer maturities increase the sensitivity of both products. In most of the cases an increase in the periodicity of the premium payments leads to a sensitivity reduction of the option price for the IRG, while an opposite effect can be observed for the LBG. These differences in the parameter uncertainty may be reflected in a higher loading factor in the fair option price of LBGs, in order to properly account for these risks.

With regard to the LBG contracts, single premium and monthly premium contracts did not show significant differences with respect to the risk that arises from the parameter uncertainty, so similar risk management measures can be adopted by the provider for the different payment modalities. However, the distinction between the premium payment modality seems to be important when the IRG is offered. The monthly premium contracts may have a smaller loading factor on the option price than single premium contracts. This sensitivity situation poses a substantial additional risk to the provider of such forms of guarantee. Finally, for the maturity payoff, which is mainly on the interest for the investor, we observed that small changes in the initial modelling parameters do not substantially affect its return.

APPENDIX

A. EM Algorithm - Mixture of Gaussian Distributions for the Merton Model

One of the most common examples of the application of the EM algorithm is the parameter estimation of a mixture of Gaussian distributions (cf. for the following Dempster et al., 1977; McLachlan et al., 2004). Let $\theta' = (\mu', \sigma', \lambda', \kappa, s)$ be the vector of daily parameters for the Merton jump-diffusion model, e.g. $\mu' = \mu\triangle t$, $\sigma'^2 = \sigma^2\triangle t$, $\lambda' = \lambda\triangle t$ and $\triangle t = 1/360$, and R the daily returns random variable defined in Equation (8). The variable R can be interpreted as a random variable taking into account the case when the return does not exhibit a jump $Prob(R = X) = 1 - \lambda'$, and in the case where we face a jump in the return $Prob(R = X + Y) = \lambda'$. The random variable X describes the diffusion part of the return is normally distributed with mean $\mu_1 = \mu' - 0.5\sigma'^2$ and variance ${\sigma_1}^2 = \sigma'^2$. The random variable Y is also normally distributed and it denotes the jump component of the return process. Thus X+Y is a random variable with mean $\mu_2 = \mu_1 + \kappa$ and variance ${\sigma_2}^2 = \sigma'^2 + s^2$. The density function $f_R(r)$ is then defined by

$$f_R(r) = \alpha_1 \cdot \mathscr{N}(r \mid \mu_1, \sigma_1^2) + \alpha_2 \cdot \mathscr{N}(r \mid \mu_2, {\sigma_2}^2) \tag{22}$$

where $\mathscr{N}(\cdot)$ denotes the density function of the normal distribution, $\alpha_1 = 1 - \lambda'$, $\alpha_2 = \lambda'$, $\mu_1 = \mu' - 0.5 \cdot \sigma'^2$, ${\sigma_1}^2 = \sigma'^2$, $\mu_2 = \mu_1 + \kappa$ and ${\sigma_2}^2 = \sigma'^2 + s^2$. Let $\{r_1, r_2, ..., r_N\}$ be a set of N daily market data observations of the return random variable R, then the log-likelihood function for R is given by

$$ln\ L(\mathbf{R}, \theta) = \sum_{n=1}^{N} ln f_R(r_n) \tag{23}$$

The goal is to maximize the likelihood function with respect to the parameters of θ'. In a first step, an initial $\theta'^{(0)}$ is set, and the initial value of the log-likelihood function is calculated. In the *expectation* step, or E-step, we use the current values of the parameters $\theta'^{(m)}$ to evaluate the posterior probabilities, or responsibilities $w_{n,k}$ that the k-th component takes in explaining r_n in the distribution function given by equation (22). $w_{n,k}$ results from

$$w_{n,k} = \frac{\alpha_k \mathcal{N}(r_n \mid \mu_k^{(m)}, \sigma_k^{2(m)})}{f_R(r_n \mid \theta^{(m)})} \quad k = 1, 2 \text{ and } n = 1, .., N \tag{24}$$

In the *maximization* step, or M step, we use the probabilities calculated by equation (24) in order to re-estimate the means μ_k, variances σ_k, and mixing coefficients α_k for $k = 1, 2$. Let N_k denote the sum over all $n = 1, .., N$ probabilities for the k-th component:

$$N_k = \sum_{n=1}^{N} w_{n,k}$$

The re-estimated parameters of equation 22 for the $(m+1)$-th iteration are then given by

$$\hat{\mu}_k^{(m+1)} = \frac{1}{N_k} \sum_{n=1}^{N} w_{n,k} r_n \tag{25}$$

$$\hat{\sigma}_k^{(m+1)} = \sqrt{\frac{1}{N_k} \sum_{n=1}^{N} w_{n,k} \left(r_n - \mu_k^{(m+1)} \right)^2} \tag{26}$$

$$\hat{\alpha}_k^{(m+1)} = \frac{N_k}{N} \tag{27}$$

The re-estimated components of parameter estimator $\hat{\theta}^{(m+1)}$ for the Merton model are

$$\begin{aligned}
\hat{\mu}^{(m+1)} &= \hat{\mu}_1^{(m+1)} + \frac{\hat{\sigma}_1^{2(m+1)}}{2} \\
\hat{\sigma}^{2(m+1)} &= \hat{\sigma}_1^{2(m+1)} \\
\hat{\lambda}^{(m+1)} &= \hat{\alpha}_1^{(m+1)} \\
\hat{\kappa}^{(m+1)} &= \hat{\mu}_2^{(m+1)} - \hat{\mu}_1^{(m+1)} \\
\hat{s}^{2(m+1)} &= \hat{\sigma}_2^{2(m+1)} - \hat{\sigma}_1^{2(m+1)}
\end{aligned}$$

In practice, the algorithm is expected to converge when the change in the log likelihood function, or alternatively in the parameters, falls below some threshold. In particular, the stopping criterion is then defined by $\left\| \hat{\theta}^{(m+1)} - \hat{\theta}^{(m)} \right\| < \epsilon$ with $\epsilon = 10^{-6}$.

B. EM Algorithm - Double Exponential Distribution of Jumps for the Kou Model

The EM algorithm applied to the double exponential jump diffusion process has been widely analyzed in the literature. However, the maximization of the likelihood function requires the derivative of an integral which makes the direct application of the EM algorithm difficult and time consuming (cf. Ramezani & Zeng, 2007). As an alternative, we estimate the parameters that maximize the likelihood function as those that are calculated from the method of moments. The method of moments gives an estimation of the parameters with an acceptable accuracy (cf. for the following Dempster et al., 1977; McLachlan et al., 2004; Lindström, 2012).

Let be $\theta' = (\mu', \sigma', \lambda', p, \eta_u, \eta_d)$ the vector of daily parameters for the Kou jump-diffusion model, e.g., $\mu' = \mu\triangle t$, $\sigma'^2 = \sigma^2\triangle t$, $\lambda' = \lambda\triangle t$ and $\triangle t = 1/360$, and R the daily returns random variable defined in equation (8). The variable R can be interpreted as a random variable such that $Prob[R = X] = 1 - \lambda'$ whenever the return does not present a jump, and $Prob[R = X + Y] = \lambda'$ when the return presents a jump. The random variable X is normally distributed with mean $\mu_1 = \mu' - 0.5\sigma'^2$ and variance ${\sigma_1}^2 = \sigma'^2$, and it represents the diffusion part of the return. The random variable Y exhibits a double-exponential distribution and it describes the jump component of the return process. Y corresponds to a random variable that is composed of two components: a positive jump $Prob[Y = Y_u] = p$ and a negative jump $Prob[Y = Y_d] = 1 - p$. Y_u and Y_d are exponentially distributed with mean $1/\eta_u$ and $-1/\eta_d$, respectively. The density function $f_R(r)$ is given by

$$f_R(r) = \alpha_1 \mathscr{N}(r \mid \mu_1, {\sigma_1}^2) + \alpha_2 EMG_2(r \mid \mu_1, {\sigma_1}^2, \eta_u) + \alpha_3 EMG_3(r \mid \mu_1, {\sigma_1}^2, \eta_d) \quad (28)$$

$$EMG_2(r \mid \mu_1, \sigma_1^2, \eta_u) = \int_{-\infty}^{r} N(x \mid \mu_1, {\sigma_1}^2)\eta_u e^{-\eta_u(r-x)} dx$$

$$EMG_3(r \mid \mu_1, {\sigma_1}^2, \eta_d) = \int_{r}^{\infty} N(x \mid \mu_1, {\sigma_1}^2)\eta_d e^{\eta_d(r-x)} dx$$

with $\alpha_1 = 1-\lambda'$, $\alpha_2 = \lambda' \cdot p$ and $\alpha_3 = \lambda' \cdot (1-p)$. $\mathscr{N}(\cdot)$ is the density function of the normal distribution, and $EMG(\cdot)$ is the density function of an exponentially modified Gaussian distribution, which is the sum of an exponential random variable and a normal random variable. Let $\{r_1, r_2, ..., r_N\}$ be a set of N daily market data observations of the return random variable R, then the log-likelihood function for R is given by

$$ln\ L(\mathbf{R}, \theta) = \sum_{n=1}^{N} ln f_R(r_n)$$

The goal is to maximize the likelihood function with respect to the parameters of θ'. In a first step, an initial $\theta'^{(0)}$ is set, and the initial value of the log-likelihood function is calculated. In the *expectation* step, or E-step, we use the current values of the parameters $\theta'^{(m)}, m = 0, 1, ...$ to evaluate the posterior probabilities, or responsibilities $w_{n,k}$ that the k-th component takes for explaining the return r_n in the distribution function given by equation (22). The $w_{n,k}$ is given by

$$w_{n,1} = \frac{\alpha_1^{(m)} N(r_n \mid \mu_1^{(m)}, {\sigma_1}^{2(m)})}{f_R(r_n \mid \theta^{(m)})} \quad n = 1, .., N \tag{29}$$

$$w_{n,k} = \frac{\alpha_k^{(m)} EMG_k(r_n \mid \mu_1^{(m)}, {\sigma_1}^{2(m)}, \eta_J^{(m)})}{f_R(r_n \mid \theta^{(m)})} \quad k = 2, 3 \quad J = u, d \text{ and } n = 1, .., N \tag{30}$$

In the *maximization* step, or M step, we use the probabilities calculated by equations (29) and (30) in order to re-estimate the mixed coefficients α_k and first three central moments $M_{1,k} = \mu_k$ (mean), $M_{2,k} = \sigma_k$ (variance) and $M_{3,k} = \gamma_k$ (skewness) of the k-th component $(k = 1, 2, 3)$ of the density function in equation (28). Let N_k be defined as the sum over all $n = 1, .., N$ of probabilities for the k-th component, it can be written as

$$N_k = \sum_{n=1}^{N} w_{n,k} \quad k = 1, 2, 3$$

The mixed coefficients α_k and the first three central moments $M_{1,k} = \mu_k$ (mean), $M_{2,k} = \sigma_k$ (variance) and $M_{3,k} = \gamma_k$ (skewness) of the k-th component of the density function in equation (28) for the $(m + 1)$-th iteration are given by

$$\hat{\mu}_k^{(m+1)} = \frac{1}{N_k}\sum_{n=1}^{N} w_{n,1} r_n \tag{31}$$

$$\hat{\sigma}_k^{(m+1)} = \sqrt{\frac{1}{N_k}\sum_{n=1}^{N} w_{n,k}\left(r_n - \mu_k^{(m+1)}\right)^2} \tag{32}$$

$$\hat{\gamma}_k^{(m+1)} = \frac{\frac{1}{N_k}\sum_{n=1}^{N} w_{n,k}\left(r_n - \mu_k^{(m+1)}\right)^3}{\left(\sigma_k^{(m+1)}\right)^3} \tag{33}$$

$$\hat{\alpha}_k^{(m+1)} = \frac{N_k}{N} \tag{34}$$

The re-estimated components of parameter estimator $\theta^{(m+1)}$ for the Kou model are

$$\begin{aligned}
\frac{1}{\hat{\eta}_u^{(m+1)}} &= \hat{\sigma}_2{}^{(m+1)}\left(\frac{\hat{\gamma}_2^{2(m+1)}}{2}\right)^{\frac{1}{3}} \\
\frac{1}{\hat{\eta}_d^{(m+1)}} &= -\hat{\sigma}_3{}^{(m+1)}\left(\frac{\hat{\gamma}_3^{2(m+1)}}{2}\right)^{\frac{1}{3}} \\
\hat{\lambda'}^{(m+1)} &= 1 - \hat{\alpha}_1^{(m+1)} \\
\hat{p}^{(m+1)} &= \frac{\hat{\alpha}_2^{(m+1)}}{\hat{\lambda'}^{(m+1)}} \\
\hat{q}^{(m+1)} &= 1 - \hat{p}^{(m+1)} \\
\hat{\sigma'}^{2(m+1)} &= S^2 - \lambda'^{(m+1)}\left[p^{(m+1)}q^{(m+1)}\left(\frac{1}{\eta_u^{(m+1)}} + \frac{1}{\eta_d^{(m+1)}}\right)^2 + \left(\frac{p^{(m+1)}}{\eta_u^{2(m+1)}} + \frac{q^{(m+1)}}{\eta_d^{2(m+1)}}\right)\right] \\
&\quad - \left(\frac{p^{(m+1)}}{\eta_u^{(m+1)}} - \frac{q^{(m+1)}}{\eta_d^{(m+1)}}\right)^2 \lambda'^{(m+1)}(1 - \lambda'^{(m+1)}) \\
\hat{\mu'}^{(m+1)} &= \overline{R} - \hat{\alpha}_2^{(m+1)}\frac{1}{\hat{\eta}_u^{(m+1)}} + \hat{\alpha}_3^{(m+1)}\frac{1}{\hat{\eta}_d^{(m+1)}} + \frac{1}{2}\hat{\sigma'}^{2(m+1)}
\end{aligned}$$

Here $\overline{R}$ and S^2 denote the corresponding mean and variance of the sample:

$$\overline{R} = \frac{1}{N}\sum_{n=1}^{N} r_n$$

$$S^2 = \frac{1}{N-1}\sum_{n=1}^{N}\left(r_n - \overline{R}\right)^2$$

In practice, the choice of the initial parameters ${\theta'}^{(0)}$ plays an important role in order to assure the convergence of the algorithm. Typically, the starting vector is chosen close to the real parameter values. As the EM algorithm for the Gaussian mixture, the algorithm is expected to converge when the change in the log-likelihood function, or alternatively in the parameters, falls below some threshold. The stopping criterion is then defined by $\left\|\hat{\theta}^{(m+1)} - \hat{\theta}^{(m)}\right\| < \epsilon$ and $\epsilon = 10^{-6}$.

C. CORRELATION MATRIX FOR THE SELECTED INDICES

Table 9: Selected Indices correlation matrix

Index	GB	REIT	E	CB
GB	1.0000	-0.2915	-0.4239	0.6619
REIT	-0.2915	1.0000	0.6890	-0.1316
E	-0.4239	0.6890	1.0000	-0.2308
CB	0.6619	-0.1316	-0.2308	1.0000

GB=Government Bond Index, Barclays Euro Government Bond 10-year term. REIT=Real Estate Index, Euronext IEIF REIT Europe. E= Stocks Index, STOXX 50. CB=Corporate Bond Index. Barclays Euro-Aggregate Corporate Bond 10-year term. Historical daily returns of all indices were extracted for the period from January 2004 to December 2013.

D. SENSITIVITY OF THE OPTION PRICE

Table 10: Partial derivatives of the option price π_0 with respect to changes in parameters of the interest rate and lookback guarantee for the Merton and Kou models.

	$T = 5$				$T = 10$			
	Portfolio A ($\sigma_P = 7\%$)		Portfolio B ($\sigma_P = 3.5\%$)		Portfolio A ($\sigma_P = 7\%$)		Portfolio B ($\sigma_P = 3.5\%$)	
	IRG	LBG	IRG	LBG	IRG	LBG	IRG	LBG
Merton Model								
Single Premium								
$\partial\pi_0/\partial r_f$	-14288	-8375	-10299	-5249	-51274	-24464	-33508	-12419
$\partial\pi_0/\partial\sigma$	3583	5746	3721	4966	9975	15912	9604	11707
$\partial\pi_0/\partial\lambda$	1	2	1	1	4	6	2	2
$\partial\pi_0/\partial\kappa$	-4443	-7524	-5605	-8001	-12604	-20889	-14673	-18786
$\partial\pi_0/\partial s$	33766	54165	21669	29024	94230	149872	55947	68703
Monthly Premiums								
$\partial\pi_0/\partial r_f$	-9942	-9378	-7332	-5394	-34941	-28695	-23298	-12703
$\partial\pi_0/\partial\sigma$	1825	5897	2282	4865	5235	15849	5994	10921
$\partial\pi_0/\partial\lambda$	1	2	0.4	1	2	6	1	2
$\partial\pi_0/\partial\kappa$	-2187	-7847	-3350	-7807	-6470	-20941	-8994	-17536
$\partial\pi_0/\partial s$	17134	55623	13159	28467	49144	149405	34864	64121
Kou Model								
Single Premium								
$\partial\pi_0/\partial r_f$	-14365	-8360	-10284	-5244	-51434	-24445	-33500	-12446
$\partial\pi_0/\partial\sigma$	3110	4979	3253	4347	8646	13781	8404	10254
$\partial\pi_0/\partial\lambda$	1	1	0.2	0.3	2	3	1	1
$\partial\pi_0/\partial p$	-19	-21	-50	-71	-67	-69	-130	-163
$\partial\pi_0/\partial\eta_u$	-0.3	-0.5	-0	-0.1	-1	-1	-0.1	-0.1
$\partial\pi_0/\partial\eta_d$	-0.5	-0.8	-0.1	-0.2	-1.4	-2.2	-0.3	-0.4
Monthly Premiums								
$\partial\pi_0/\partial r_f$	-9942	-9395	-7342	-5391	-34912	-28694	-23280	-12729
$\partial\pi_0/\partial\sigma$	1585	5110	1998	4259	4522	13726	5252	9568
$\partial\pi_0/\partial\lambda$	0.4	1.2	0.2	0.4	1.1	3.3	0.5	0.9
$\partial\pi_0/\partial p$	-13	-23	-31	-70	-46	-75	-80	-153
$\partial\pi_0/\partial\eta_u$	-0.2	-0.5	-0.0	-0.1	-0.4	-1.3	-0.7	-0.1
$\partial\pi_0/\partial\eta_d$	-0.3	-0.8	-0.1	-0.2	-0.7	-2.2	-0.2	-0.4

IRG = Interest Rate Guarantee; LBG=Lookback Guarantee. The payoff L_T estimates were calculated under the empirical measure $\mathbb{P}$. The Monte Carlo simulation is obtained by calculating the prices of the underlying asset on a daily basis and 500,000 simulation paths. The risk-free interest rate is $r_f = 2.05\%$ for both models.

[a] The parameters for the Merton model are: Portfolio A $\mu = 14.40\%$, $\sigma = 4.69\%$, $\lambda = 74.07$, $\kappa = -0.10\%$, $s = 0.597\%$; and Portfolio B $\mu = 10.84\%$, $\sigma = 2.65\%$, $\lambda = 48.29$, $\kappa = -0.09\%$, $s = 0.318\%$.

[b] The parameters for the Merton model are: Portfolio A $\mu = 15.93\%$, $\sigma = 4.06\%$, $\lambda = 168.36$, $p = 44.84\%$, $\eta_u = 347.18$, $\eta_d = 303.45$; and Portfolio B $\mu = 7.59\%$, $\sigma = 2.32\%$, $\lambda = 164.25$, $p = 60.88$, $\eta_u = 922.80$, $\eta_d = 532.33$.

The re-simulation estimates were calculated for $\epsilon = 0.0001\%$ except for p and λ where $\epsilon = 1\%$ is used.

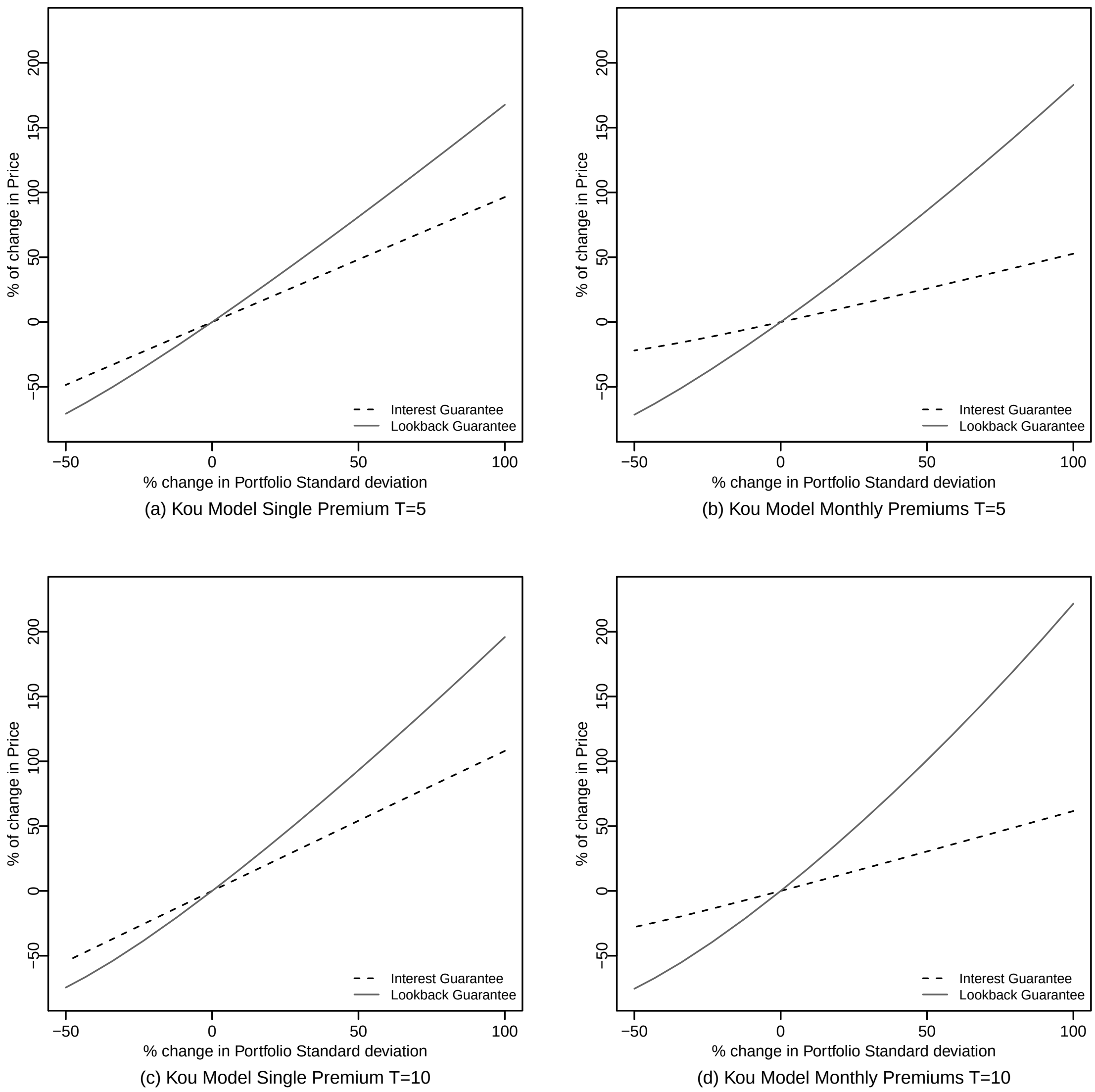


Figure 4: Sensitivity of the option price π_0. Percentage of change in the interest rate and lookback guarantee prices when the total volatility of Portfolio A ($\sigma_P = 7\%$) changes. The underlying asset is modelled using the assumptions of the Kou model. The risk-free interest rate is $r_f = 2.05\%$. (a) Single premium contract, maturity term T=5, minimum guaranteed interest rate g=2.13% and a price $\pi_0 = 386.52$ in the base case. (b) Monthly premium contract, maturity term T=5, minimum guaranteed interest rate g=3.88% and a price $\pi_0 = 383.88$ in the base case. (c) Single premium contract, maturity term T=10, minimum guaranteed interest rate g=3.02% and a price $\pi_0 = 959.92$ in the base case. (d) Monthly premium contract, maturity term T=10, minimum guaranteed interest rate g=1.89% and a price $\pi_0 = 916.17$ in the base case.

E. SENSITIVITY OF THE DERIVATIVES OF THE EXPECTED PAYOFF

Table 11: Partial derivatives of the expected payoff L_T with respect to changes in parameters of the interest rate and lookback guarantees for the Merton and Kou models.

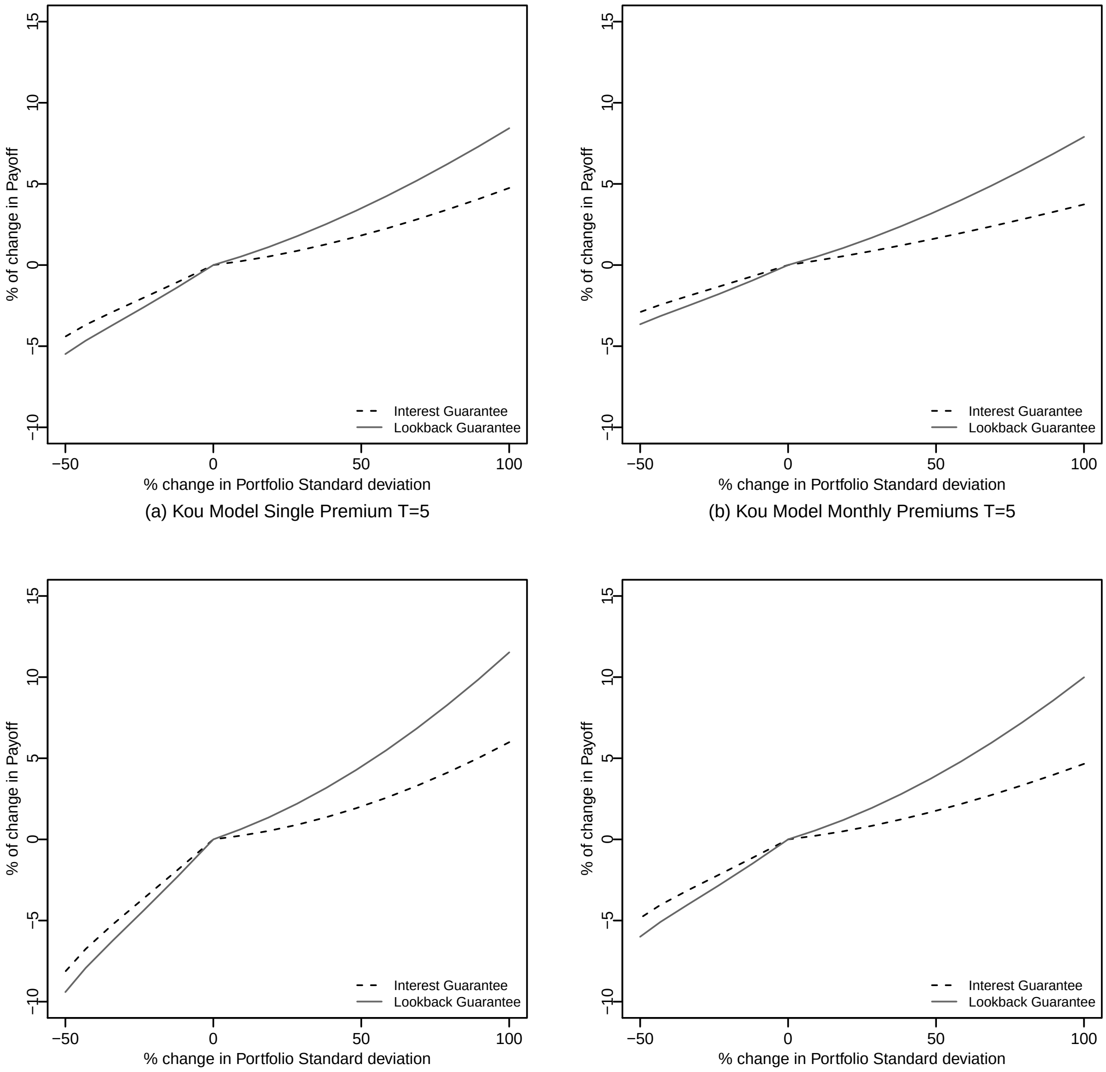
	$T = 5$				$T = 10$			
	Portfolio A ($\sigma_P = 7\%$)		Portfolio B ($\sigma_P = 3.5\%$)		Portfolio A ($\sigma_P = 7\%$)		Portfolio B ($\sigma_P = 3.5\%$)	
	IRG	LBG	IRG	LBG	IRG	LBG	IRG	LBG
Merton Model								
Single Premium								
$\partial L_T/\partial \mu$	41178	40408	41255	40532	245902	241591	227293	225537
$\partial L_T/\partial \sigma$	1206	3731	34	1660	928	10942	-46	4509
$\partial L_T/\partial \lambda$	-14171	-13552	-38	-37	-85503	-82604	-210	-207
$\partial L_T/\partial \kappa$	3045812	2984815	1990355	1952362	18198311	17863663	10966022	10872599
$\partial L_T/\partial s$	29632	53393	6545	16249	117516	210933	34811	62204
Monthly Premiums								
$\partial L_T/\partial \mu$	16422	16886	18810	18256	95678	93977	93805	92302
$\partial L_T/\partial \sigma$	1612	3251	153	1434	2046	7958	-3	3366
$\partial L_T/\partial \lambda$	-5493	-5425	-17	-17	-33029	-31606	-87	-85
$\partial L_T/\partial \kappa$	1213132	1244665	907240	878090	7078389	6943086	4525698	4446766
$\partial L_T/\partial s$	22510	38406	3805	11470	61610	117241	14475	34695
Kou Model								
Single Premium								
$\partial L_T/\partial \mu$	41173	40407	41254	40526	245857	241548	227292	225538
$\partial L_T/\partial \sigma$	1037	3226	34	1457	782	9485	-31	3938
$\partial L_T/\partial \lambda$	-7586	-7259	-3	-3	-45670	-44138	-17	-16
$\partial L_T/\partial p$	42819	42028	20049	19672	255739	251238	110471	109545
$\partial L_T/\partial \eta_u$	-26	-26	-5	-5	-155	-153	-27	-27
$\partial L_T/\partial \eta_d$	41	40	9	9	246	240	51	51
Monthly Premiums								
$\partial L_T/\partial \mu$	16422	16889	18801	18250	95664	93960	93801	92298
$\partial L_T/\partial \sigma$	1394	2813	141	1259	1756	6909	2	2937
$\partial L_T/\partial \lambda$	-2938	-2905	-1	-1	-17617	-16871	-7	-7
$\partial L_T/\partial p$	17078	17554	9133	8847	99408	97657	45582	44798
$\partial L_T/\partial \eta_u$	-10	-11	-2	-2	-60	-60	-11	-11
$\partial L_T/\partial \eta_d$	16	16	4	4	96	93	21	21

IRG = Interest Rate Guarantee; LBG=Lookback Guarantee. The payoff L_T estimates were calculated under the empirical measure $\mathbb{P}$. The Monte Carlo simulation is obtained by calculating the prices of the underlying asset on a daily basis and 500,000 simulation paths. The risk-free interest rate is $r_f = 2.05\%$ for both models.

[a] The parameters for the Merton model are: Portfolio A $\mu = 14.40\%$, $\sigma = 4.69\%$, $\lambda = 74.07$, $\kappa = -0.10\%$, $s = 0.597\%$; and Portfolio B $\mu = 10.84\%$, $\sigma = 2.65\%$, $\lambda = 48.29$, $\kappa = -0.09\%$, $s = 0.318\%$.

[b] The parameters for the Merton model are: Portfolio A $\mu = 15.93\%$, $\sigma = 4.06\%$, $\lambda = 168.36$, $p = 44.84\%$, $\eta_u = 347.18$, $\eta_d = 303.45$; and Portfolio B $\mu = 7.59\%$, $\sigma = 2.32\%$, $\lambda = 164.25$, $p = 60.88$, $\eta_u = 922.80$, $\eta_d = 532.33$.

The re-simulation estimates were calculated for $\epsilon = 0.0001\%$ except for p and λ where $\epsilon = 1\%$ is used.



(a) Kou Model Single Premium T=5

(b) Kou Model Monthly Premiums T=5

(c) Kou Model Single Premium T=10

(d) Kou Model Monthly Premiums T=10

Figure 5: Sensitivity of the expected payoff $E[L_T]$ at maturity T. Percentage of change in the interest rate and lookback guarantees prices when the total volatility of Portfolio A ($\sigma_P = 7\%$) changes. The underlying asset is modelled using the assumptions of the Kou model. The risk-free interest rate is $r_f = 2.05\%$. (a) Single premium contract, maturity term T=5, minimum guaranteed interest rate g=2.13% and a price $E[L_T] = 8639.15$ in the base case. (b) Monthly premium contract, maturity term T=5, minimum guaranteed interest rate g=3.88% and a price $E[L_T] = 7311.54$ in the base case. (c) Single premium contract, maturity term T=10, minimum guaranteed interest rate g=3.02% and a price $E[L_T] = 24745.11$ in the base case. (d) Monthly premium contract, maturity term T=10, minimum guaranteed interest rate g=1.89% and a price $E[L_T] = 17702.12$ in the base case.

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