CONSUMPTION-BASED ASSET PRICING IN INSURANCE MARKETS: YET ANOTHER PUZZLE?

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Abstract
We fit the classical consumption-based asset pricing model with power utility to historical property-casualty insurance market data. In doing so, we consider two alternatives for the estimation of the relative risk aversion coefficient. First, we apply an extended version of Stein’s Lemma introduced by Söderlind (2009a), which builds on a bivariate mixture normal distribution and thus allows for skewed and leptokurtic asset returns, given the log stochastic discount factor is Gaussian. Second, we follow Hansen and Singleton (1983) in assuming that consumption growth and asset returns are jointly lognormally distributed as well as homoskedastic. Both approaches are complemented by Hansen and Jagannathan (1991) volatility bounds. Based on aggregate annual premiums and claims for Australia, Germany, Italy, Japan, Netherlands, and the United States, we are able to provide evidence of yet another asset pricing anomaly. More specifically, the consumption-based model implies even larger relative risk aversion coefficients in the insurance sectors than in the equity markets of the aforementioned countries. To solve this insurance premium puzzle, we draw on the loss aversion and narrow framing approach by Barberis et al. (2001) as well as the second-degree expectation dependence framework by Dionne et al. (2015), with encouraging results.

Keywords: Consumption-Based Asset Pricing · Equity Premium Puzzle · Insurance Premium Puzzle · Domain-Specific Risk Aversion · Prospect Theory · Higher-Order Risk Preferences

JEL classification: G02; G12; G22

1 Introduction
The classical consumption-based model, which was established through the work of Rubinstein (1976), Lucas (1978), Breeden (1979), Grossman and Shiller (1981), as well as Hansen and Singleton (1983), is one of the most influential accomplishments in modern asset pricing theory. Utility-maximizing individuals in a representative-agent economy face the intertemporal choice of either consuming their wealth or investing it in a risky asset. The first-order condition for this tradeoff leads to the central pricing equation of the model: the demand for the asset is adjusted until the loss in utility suffered due to a slightly lower consumption level today equals the gain in expected utility achieved by being able to consume a little more of the future payoff of the asset. Therefore, equilibrium asset prices are expectations of payoffs discounted at the representative investors’ marginal rate of substitution. Based on this simple idea, it is possible to derive risk corrections that rely on the covariance of payoffs or returns with marginal utility and thus, ultimately, consumption. Assets that perform well when the investor is able to consume
abundantly, but pay little in times when his consumption is restrained, are perceived to be risky and
sell for prices below their expected payoff discounted at the risk-free interest rate. Insurance policies, in
contrast, indemnify their holder after the occurrence of a loss in wealth, thus reducing the volatility of
consumption. Hence, individuals are prepared to accept a negative expected return on such contracts.

Despite its theoretical appeal, the consumption-based model repeatedly failed in empirical applica-
tions. Its most famous shortcoming is the inability to explain risk premiums observed in postwar stock
market data with a reasonable degree of risk aversion. This is the famous equity premium puzzle, which
was described by Mehra and Prescott (1985) for the United States and, since then, has received a lot
of scholarly attention. Wheatley (1988) as well as Campbell (2003) found evidence for the puzzle in
many developed economies, while Donadelli and Prosperi (2012) revealed its existence in a number
of emerging markets. Early on, Kandel and Stambaugh (1991) suggested that it might be necessary to
contemplate higher values for the risk aversion coefficient. This, however, leads to the emergence
of another well-known asset pricing anomaly: the risk-free rate puzzle as constituted by Weil (1989). Given
extreme risk aversion, power utility agents are extraordinarily reluctant to engage in intertemporal sub-
stitution. This implies that the empirically-observed low and stable risk-free interest rates can only be
explained by the consumption-based model, if investors exhibit a subjective time discount factor greater
than one. Although such a negative time preference is theoretically possible, it is not very plausible as
individuals are typically impatient, favoring earlier over later consumption (see, e.g., Kocherlakota, 1996).

Thus, ever since the discovery of the equity premium puzzle, economics and finance researchers have
targeted more meaningful risk aversion levels. The dominant strand of literature in this regard centers on
model refinements by means of separated time and risk preferences (see, e.g., Epstein and Zin, 1989, 1991),
habit formation (see, e.g., Abel, 1990; Constantinides, 1990; Ferson and Constantinides, 1991; Campbell
and Cochrane, 1999), idiosyncratic consumption shocks (see, e.g., Mankiw, 1986; Weil, 1992; Heaton
and Lucas, 1996; Constantinides and Duffie, 1996; Gomes and Michaelides, 2008), and rare economic disasters
(see, e.g., Rietz, 1988; Barro, 2006, 2009; Gabaix, 2008, 2012; Wachter, 2013). However, none of these
solutions is considered to be fully satisfactory (see, e.g., Mehra and Prescott, 2003). Other research
efforts have focused on long-run persistence in consumption and dividend growth (see, e.g., Bansal
and Yaron, 2004; Bansal et al., 2010; Kojjen et al., 2010), loss aversion (see, e.g., Benartzi and Thaler, 1995;
Barberis et al., 2001; Barberis and Huang, 2001, 2009), disappointment aversion (see, e.g., Routledge
and Zin, 2010), ambiguity aversion (see, e.g., Chen and Epstein, 2002; Gollier, 2011; Rieger and Wang, 2012),
and, most recently, higher-order risk preferences (see Dionne et al., 2015). In addition, there have been
attempts to improve the model’s estimation basis by relying on stockholder samples (see, e.g., Vissing-
Jorgensen and Attanasio, 2003), long-run consumption changes (see, e.g., Parker and Julliard, 2005), as
well as forward-looking survey and option data (see Söderlind, 2009b). Finally, some authors explored
whether factors such as transaction costs (He and Modest, 1995; Luttmer, 1996; Bansal and Coleman,
1996), borrowing constraints (see, e.g., Constantinides et al., 2002), and taxation (see, e.g., McGrattan
and Prescott, 2003) drive the equity risk premium.
Apart from equities, the consumption-based model has also been applied to fixed income (see, e.g., Backus et al., 1989; Wachter, 2006), stock options (see, e.g., Liu et al., 2005; Backus et al., 2011), currencies (see Verdelhan, 2010) and even catastrophe bonds (see Dieckmann, 2011). Somewhat surprisingly, however, the empirical finance pricing literature has not yet tested its suitability for insurance contracts, although they are the typical textbook example for an asset that is negatively correlated with consumption and therefore positively correlated with marginal utility.\(^1\) We fill this gap by fitting the classical consumption-based model with power utility to historical property-casualty insurance market data. In doing so, we consider two alternatives for the estimation of the relative risk aversion (RRA) coefficient. First, we apply an extended version of Stein’s Lemma introduced by Söderlind (2009a), which builds on a bivariate mixture normal distribution and thus allows for skewed and leptokurtic asset returns, given the log stochastic discount factor (SDF) is Gaussian. Second, we follow Hansen and Singleton (1983) in assuming that consumption growth and asset returns are jointly lognormally distributed as well as homoskedastic. Both approaches are complemented by Hansen and Jagannathan (1991) volatility bounds. Based on aggregate annual premiums and claims for Australia, Germany, Italy, Japan, the Netherlands, and the United States, we are able to provide evidence of yet another asset pricing anomaly. More specifically, the consumption-based model implies even larger RRA coefficients in the insurance sectors than in the equity markets of the aforementioned countries. To solve this insurance premium puzzle, we draw on the loss aversion and narrow framing approach by Barberis et al. (2001) as well as the second-degree expectation dependence framework by Dionne et al. (2015), with encouraging results.

The rest of the paper is organized as follows. In the next section, we revisit the classical consumption-based model and derive the two procedures that will be employed for its empirical application. Furthermore, in the third section, we describe our data set, conduct the main empirical analysis for the stock and insurance markets of six countries, and present the RRA estimates that give rise to an insurance premium puzzle. In the penultimate section, we then discuss selected modifications of the consumption-based model and implement the two most promising ones, relying on loss aversion and narrow framing as well as higher-order risk preferences of the representative investors. Finally, in the last section, we summarize our findings and draw our conclusion.

### 2 The Consumption-Based Model Revisited

#### 2.1 The Basic Pricing Equation

We focus on the common one-period discrete-time version of the consumption-based model, which can be easily extended to derive the price of any stream of risky future cash flows. The notation used in the following is based on Campbell (2003) and Cochrane (2005). Consider a random payoff \( \tilde{X}_{t+1} \) at time

\(^1\)Cochrane (2005), e.g., puts it as follows: “If you buy an asset whose payoff covaries negatively with consumption, it helps to smooth consumption and so is more valuable than its expected payoff might indicate. Insurance is an extreme example. Insurance pays off exactly when wealth and consumption would otherwise be low — you get a check when your house burns down. For this reason, you are happy to hold insurance, even though you expect to lose money — even though the price of insurance is greater than its expected payoff discounted at the risk-free rate.”
In order to determine what $\tilde{X}_{t+1}$ is worth to a representative investor at time $t$, we draw on the time-separable utility $U(\cdot)$ that he derives from his deterministic current and stochastic future levels of consumption, denoted $C_t$ and $\tilde{C}_{t+1}$:

$$U(C_t, \tilde{C}_{t+1}) = u(C_t) + \beta E_t \left[ u(\tilde{C}_{t+1}) \right].$$

(1)

$E_t[\cdot]$ is the conditional expectation, given all information available at time $t$. The intra-period utility function $u(\cdot)$ is increasing ($u'(\cdot) > 0$) and concave ($u''(\cdot) < 0$). Thus, it reflects a rational desire for more consumption in combination with decreasing marginal utility. The curvature of $u(\cdot)$ also governs aversion to risk and to intertemporal substitution: the more stable consumption is across states of the economy and over time, the better. Furthermore, impatience is captured by the subjective time discount factor $\beta$ ($< 1$): people want to consume earlier rather than later. Given the investor possesses the endowments $W_t$ and $\tilde{W}_{t+1}$ and has complete flexibility in buying or selling an amount $\xi$ of the asset at a price $P_t$, he faces the following optimization problem:

$$\max_{\xi} \quad u(C_t) + \beta E_t \left[ u(\tilde{C}_{t+1}) \right] \quad \text{s.t.} \quad C_t = W_t - P_t \xi, \quad \tilde{C}_{t+1} = \tilde{W}_{t+1} + \tilde{X}_{t+1} \xi.$$ 

(2)

We now insert the constraints into the target function and form the first-order derivative:

$$\frac{\partial U(C_t, \tilde{C}_{t+1})}{\partial \xi} = -u'(C_t)P_t + \beta E_t \left[ u'(\tilde{C}_{t+1})\tilde{X}_{t+1} \right].$$ 

(3)

The corresponding first-order condition for a maximum is:

$$0 = -u'(C_t)P_t + \beta E_t \left[ u'(\tilde{C}_{t+1})\tilde{X}_{t+1} \right]$$

$$u'(C_t)P_t = \beta E_t \left[ u'(\tilde{C}_{t+1})\tilde{X}_{t+1} \right].$$ 

(4)

Here, $u'(C_t)P_t$ equals the loss in utility caused by having to pay the asset’s purchase price and $\beta E_t \left[ u'(\tilde{C}_{t+1})\tilde{X}_{t+1} \right]$ is the increase in discounted expected utility generated by its payoff. The investor adjusts $\xi$ until this first-order condition holds, i.e., marginal utility loss must equal marginal utility gain. Rearranging (4) yields the central asset pricing formula of the consumption-based model, which states that the price we ought to expect for the asset is driven by the payoff $\tilde{X}_{t+1}$, the representative investor’s utility function, as well as his time preferences ($\beta$) and consumption choice ($C_t$ and $\tilde{C}_{t+1}$):

$$P_t = E_t \left[ \beta \frac{u'(\tilde{C}_{t+1})}{u'(C_t)} \tilde{X}_{t+1} \right].$$ 

(5)

For stocks, $\tilde{X}_{t+1}$ consists of the price ($\tilde{P}_{t+1}$) and the dividend ($\tilde{D}_{t+1}$): $\tilde{X}_{t+1} = \tilde{P}_{t+1} + \tilde{D}_{t+1}$

Returns are only an intermediate objective. Ultimately, utility is driven by consumption (see, e.g., Cochrane, 2005).
2.2 Risk Corrections and the Stochastic Discount Factor

We define the SDF $\tilde{M}_{t+1}$ (pricing kernel) as the intertemporal marginal rate of substitution:

$$\tilde{M}_{t+1} = \beta \frac{u'(\tilde{C}_{t+1})}{u'(C_t)}.$$  

(6)

Substituting (6) into (5), gives a more convenient expression for the pricing equation:

$$P_t = E_t[\tilde{M}_{t+1} \tilde{X}_{t+1}].$$  

(7)

Due to their well-behaved properties (e.g., stationarity), many empirical applications draw on returns instead of prices. To obtain the gross return $\tilde{R}_{t+1}$, we need to divide the payoff by the price: $\tilde{R}_{t+1} = \tilde{X}_{t+1}/P_t$. Therefore, (7) can be rearranged into an Euler equation that accounts for $\tilde{R}_{t+1}$ as a special payoff with a price of one:

$$1 = E_t[\tilde{M}_{t+1} \tilde{R}_{t+1}].$$  

(8)

Since the (gross) risk-free rate is – per definition – deterministic, it should equal the inverse of the conditional expectation of the SDF:

$$R_f = \frac{1}{E_t[\tilde{M}_{t+1}]}.$$  

(9)

We may now derive an explicit expression for the risk premium by using the definition of covariance

$$cov_t[\tilde{M}_{t+1}, \tilde{X}_{t+1}] = E_t[\tilde{M}_{t+1} \tilde{X}_{t+1}] - E_t[\tilde{M}_{t+1}] \cdot E_t[\tilde{X}_{t+1}]$$

on (7) and exploiting the relationship in (9):

$$P_t = E_t[\tilde{M}_{t+1}] \cdot E_t[\tilde{X}_{t+1}] + cov_t[\tilde{M}_{t+1}, \tilde{X}_{t+1}]$$

$$= \frac{E_t[\tilde{X}_{t+1}]}{R_f} + cov_t[\tilde{M}_{t+1}, \tilde{X}_{t+1}].$$  

(10)

The first term represents the expected payoff discounted at the risk-free rate and the second term is a risk adjustment. Hence, Equation (10) states that it is possible to determine asset-specific risk corrections by means of a unique SDF. Assets whose payoffs exhibit a negative covariance (correlation) with the random component of the SDF are positively correlated with consumption and thus make it more volatile. Since investors prefer a steady consumption stream over time, they will only hold such assets if their price is lower than in a risk-neutral world. Insurance, in contrast, indemnifies people after they have suffered a shock to wealth, which causes their consumption to be low and their marginal utility to be high. Its value therefore exceeds the expected payoff discounted at the risk-free rate. In other words, market participants are prepared to buy insurance policies despite the fact that they expect to

4At this rate the investor is willing to forgoe consumption at time $t$ for additional consumption at time $t+1$.
5An Euler equation represents the necessary condition for optimality in an intertemporal choice problem.
6Recall from (6) that the random part of the SDF equals marginal utility (of $\tilde{C}_{t+1}$), which is high when $\tilde{C}_{t+1}$ is low.
lose money on them, because their payoff is negatively correlated with consumption. The same economic idea can be expressed by means of the return representation (8):

\[ 1 = \mathbb{E}_t[\tilde{M}_{t+1}] \cdot \mathbb{E}_t[\tilde{R}_{t+1}] + \text{cov}_t[\tilde{M}_{t+1}, \tilde{R}_{t+1}] \]

\[ \mathbb{E}_t[\tilde{R}_{t+1}] = R_f - \frac{\text{cov}_t[\tilde{M}_{t+1}, \tilde{R}_{t+1}]}{\mathbb{E}_t[\tilde{M}_{t+1}]]. \]

According to (11), the expected return of an asset is composed of the risk-free rate and a risk premium. The latter is positive, if the asset’s return exhibits a negative covariance with the SDF (marginal utility), i.e., a positive covariance with consumption. This type of asset performs badly in those states of nature where wealth is highly desired by investors. Returns on insurance, in contrast, are negatively correlated with consumption. Due to this hedging property, such contracts may offer expected gross returns below the risk-free rate or even negative expected net returns.

### 2.3 Connecting the Stochastic Discount Factor to Data

#### Excess Returns and Constant Relative Risk Aversion

To study risk premiums separately from interest rates, we look at (gross) excess returns \( \tilde{R}^e_{t+1} = \tilde{R}_{t+1} - R_f \), for which the central pricing equation (7) turns into:

\[ 0 = \mathbb{E}_t[\tilde{M}_{t+1}\tilde{R}^e_{t+1}]. \]

To see this, rearrange (8) as follows: \( 1 = \mathbb{E}_t[\tilde{M}_{t+1}(\tilde{R}_{t+1} - R_f + R_f)] = \mathbb{E}_t[\tilde{M}_{t+1}\tilde{R}^e_{t+1}] + \mathbb{E}_t[\tilde{M}_{t+1}]R_f \)

and recall from (9) that \( \mathbb{E}_t[\tilde{M}_{t+1}]R_f = 1 \). Now let us express (12) analogously to (11):

\[ 0 = \mathbb{E}_t[\tilde{M}_{t+1}] \cdot \mathbb{E}_t[\tilde{R}^e_{t+1}] + \text{cov}_t[\tilde{M}_t, \tilde{R}^e_{t+1}] \]

\[ \mathbb{E}_t[\tilde{R}^e_{t+1}] = -\frac{\text{cov}_t[\tilde{M}_{t+1}, \tilde{R}^e_{t+1}]}{\mathbb{E}_t[\tilde{M}_{t+1}]]. \]

This relationship is commonly used for empirical tests of the consumption-based model in combination with constant relative risk aversion (CRRA) represented by the power utility function:

\[ u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \]

where \( \gamma \) equals the RRA coefficient.\(^7\) The corresponding marginal utility is \( u'(C_t) = C_t^{-\gamma} \), which leads to the SDF:

\[ \tilde{M}_{t+1} = \beta \left( \frac{\tilde{C}_{t+1}}{C_t} \right)^{\gamma}. \]

\(^7\)For \( \gamma \rightarrow 1 \), we have \( u(C_t) = \ln(C_t) \). Note that, since the elasticity of intertemporal substitution \( \psi \) is the reciprocal of the RRA coefficient \( \gamma \), the standard power utility function does not allow for a disentanglement of time and risk preferences.
An important property of these preferences is scale invariance, implying that increases in wealth or the size of the economy do not alter risk premiums as long as asset return distributions remain constant (see, e.g., Campbell, 2003). Hence, we are able to aggregate agents with different wealth levels into a single representative investor, given they all exhibit the same power utility function.

The Extended Stein’s Lemma for Asset Pricing

Through (15), the RRA coefficient $\gamma$ enters the covariance in (13). This complicates an empirical analysis of the risk premium considerably. However, it is possible to analytically access $\gamma$ through (15), the RRA coefficient $\gamma$, by means of the extended version of Stein’s Lemma introduced by Söderlind (2009a):

Assume (a) the joint distribution of $\tilde{x}$ and $\tilde{y}$ is a mixture of $n$ bivariate normal distributions; (b) the mean and variance of $\tilde{y}$ is the same in each of the $n$ components; (c) $h(\tilde{y})$ is a differentiable function such that $E[|h'(\tilde{y})|] < \infty$. Then, $\text{cov}[\tilde{x}, h(\tilde{y})] = E[h'(\tilde{y})] \cdot \text{cov}[\tilde{x}, \tilde{y}]$.

Recognizing that $\tilde{x} = \tilde{R}_t^{e+1}$, $\tilde{y} = \ln(\tilde{M}_{t+1})$, and $h(\cdot) = \exp(\cdot)$ and assuming that the log SDF is Gaussian, we can therefore decompose the covariance $\text{cov}_t[\tilde{M}_{t+1}, \tilde{R}_t^{e+1}]$ in (13) as follows:

$$\text{cov}_t[\exp(\ln(\tilde{M}_{t+1})), \tilde{R}_t^{e+1}] = E_t[\tilde{M}_{t+1}] \cdot \text{cov}_t[\tilde{m}_{t+1}, \tilde{R}_t^{e+1}]$$

with $\tilde{m}_{t+1} = \ln(\tilde{M}_{t+1})$. Denote log consumption growth $\Delta \tilde{c}_{t+1} = \ln(\tilde{C}_{t+1}/C_t)$. Due to the fact that $\tilde{m}_{t+1} = \ln(\beta) - \gamma \Delta \tilde{c}_{t+1}$, we have $\text{cov}_t[\tilde{m}_{t+1}, \tilde{R}_t^{e+1}] = -\gamma \text{cov}_t[\Delta \tilde{c}_{t+1}, \tilde{R}_t^{e+1}]$ and may thus employ (16) to rewrite the risk premium (13) in terms of $\gamma$, the standard deviations $\sigma_t[\Delta \tilde{c}_{t+1}]$ and $\sigma_t[\tilde{R}_t^{e+1}]$, as well the correlation function $\rho_t[\Delta \tilde{c}_{t+1}, \tilde{R}_t^{e+1}]$:

$$E_t[\tilde{R}_t^{e+1}] = -\text{cov}_t[\tilde{m}_{t+1}, \tilde{R}_t^{e+1}]$$

$$= \rho_t[\Delta \tilde{c}_{t+1}, \tilde{R}_t^{e+1}] \cdot \sigma_t[\Delta \tilde{c}_{t+1}] \cdot \sigma_t[\tilde{R}_t^{e+1}] \cdot \gamma.$$ (17)

On the right hand side, we now clearly recognize the four drivers of the risk premium in the model. Drawing on the law of iterated expectations, it can be shown that this expression also holds for unconditional moments (see, e.g., Söderlind, 2009b). An alternative way of assessing the empirical performance of the consumption-based model are the well-known Hansen and Jagannathan (1991) bounds. The binding lower limit for the volatility of a log SDF that prices a given set of assets can be computed from (17) by setting the correlation $\rho_t[\Delta \tilde{c}_{t+1}, \tilde{R}_t^{e+1}]$ to its maximum of one (minimum of minus one), and solving for the Sharpe ratio:

$$\sigma_t[\tilde{m}_{t+1}] = \gamma \sigma_t[\Delta \tilde{c}_{t+1}] \geq \frac{E_t[\tilde{R}_t^{e+1}]}{\sigma_t[\tilde{R}_t^{e+1}]}.$$ (18)

---

8This is consistent with empirical findings showing that, despite the strong economic growth over the past century, interest rates and risk premiums did not exhibit a time trend.
A correlation of one should be used in the context of risky assets such as stocks. Insurance contracts, in contrast, help to smooth consumption and should therefore be evaluated based on a correlation of minus one. Mixture normal distributions can take a wide variety of shapes. Hence, the extended Stein’s Lemma allows us to account for skewness and kurtosis in asset returns. At the same time, the necessity to assume a Gaussian log SDF is not much of a sacrifice, since macro variables such as log consumption growth are typically almost normally distributed (see, e.g., Söderlind, 2009a).

**Jointly Lognormally-Distributed Asset Returns and Consumption Growth**

Apart from the aforementioned solution, we will also follow Hansen and Singleton (1983) in assuming that the joint conditional distribution of gross returns and consumption growth is lognormal as well as homoskedastic. Although this approach is not particularly realistic, it has been widely used in empirical research on the consumption-based model and is thus well-suited as a robustness check for our results. Consider the following definition (see, e.g., Campbell, 2003):

*Any lognormally-distributed random variable \( \tilde{x} \) exhibits the property: \( \ln(E_t[\tilde{x}]) = E_t[\ln(\tilde{x})] + \frac{1}{2} \text{var}_t[\ln(\tilde{x})] \), with \( \text{var}_t[\ln(\tilde{x})] = E_t[\ln(\tilde{x})^2] - E_t[(\ln(\tilde{x}))^2] \). If, moreover, \( \tilde{x} \) is homoskedastic, then \( \text{var}_t[\ln(\tilde{x})] = \sigma^2[\ln(\tilde{x})] \).*

This means that, given joint conditional lognormality and homoskedasticity of \( \tilde{M}_{t+1} \) and \( \tilde{R}_{t+1} \), we may log (8) and drop the time subscript on the variance to obtain the following relationship:

\[
\begin{align*}
\ln(1) & = \ln(E_t[\tilde{M}_{t+1}\tilde{R}_{t+1}]) \\
0 & = E_t[\ln(\tilde{M}_{t+1}\tilde{R}_{t+1})] + \frac{1}{2} \text{var}_t[\ln(\tilde{M}_{t+1}\tilde{R}_{t+1})] \\
& = E_t[\ln(\tilde{M}_{t+1}) + \ln(\tilde{R}_{t+1})] + \frac{1}{2} \text{var}[\ln(\tilde{M}_{t+1}) + \ln(\tilde{R}_{t+1})] \\
& = E_t[\ln(\tilde{M}_{t+1}) + \ln(\tilde{R}_{t+1})] + \frac{1}{2} \text{var}[\ln(\tilde{M}_{t+1})] + \frac{1}{2} \text{var}[\ln(\tilde{R}_{t+1})] \\
& + \frac{1}{2} \text{cov}[\ln(\tilde{M}_{t+1}), \ln(\tilde{R}_{t+1})] \\
& = E_t[\ln(\tilde{M}_{t+1}) + \ln(\tilde{R}_{t+1})] + \frac{1}{2} \sigma^2[\ln(\tilde{M}_{t+1})] \\
& + \frac{1}{2} \sigma^2[\ln(\tilde{R}_{t+1})] + \frac{1}{2} \gamma \sigma^2[\ln(\tilde{M}_{t+1})] + \frac{1}{2} \gamma \sigma^2[\ln(\tilde{R}_{t+1})] + \gamma \text{cov}[\ln(\tilde{M}_{t+1}), \ln(\tilde{R}_{t+1})] \text{ (19)}
\end{align*}
\]

Defining the log return as \( \tilde{r}_{t+1} = \ln(\tilde{R}_{t+1}) \) and rearranging yields:

\[
E_t[\tilde{r}_{t+1}] = E_t[\tilde{r}_{t+1}] - \frac{1}{2} (\sigma^2[\tilde{m}_{t+1}] + \sigma^2[\tilde{r}_{t+1}] + 2\gamma \sigma^2[\tilde{m}_{t+1}, \tilde{r}_{t+1}]) - \gamma \text{cov}[\ln(\tilde{M}_{t+1}), \ln(\tilde{R}_{t+1})] \text{ (20)}
\]

Now note that the log risk-free rate is \( r_f = -E_t[\tilde{m}_{t+1}] - \frac{1}{2} \sigma^2[\tilde{m}_{t+1}] \) because both its own variance and its covariance with the log SDF must be zero.\(^9\) Subtracting \( r_f \) from (20) and substituting \( \sigma[\tilde{r}_{t+1}] = \sigma[\tilde{r}_{t+1}] \) as well as \( \text{cov}[\tilde{m}_{t+1}, \tilde{r}_{t+1}] = \text{cov}[\tilde{m}_{t+1}, \tilde{r}_{t+1}] \) allows us to switch to log excess returns \( \tilde{r}_{t+1} = \tilde{r}_{t+1} - r_f \).

Furthermore, since \( \tilde{m}_{t+1} = \ln(\beta) - \gamma \Delta \tilde{c}_{t+1} \), we have \( \text{cov}[\tilde{m}_{t+1}, \tilde{r}_{t+1}] = -\gamma \text{cov}[\Delta \tilde{c}_{t+1}, \tilde{r}_{t+1}] \). Consequently, as in (17), we are able to express the risk premium in terms of \( \gamma \), the standard deviations \( \sigma[\Delta \tilde{c}_{t+1}] \) and \( \sigma[\tilde{r}_{t+1}] \), as well as the correlation function \( \rho[\Delta \tilde{c}_{t+1}, \tilde{r}_{t+1}] \):\(^{10}\)

\(^9\)This expression can be used to illustrate the risk-free rate puzzle. By inserting the log SDF \( \tilde{m}_{t+1} = \ln(\beta) - \gamma \Delta \tilde{c}_{t+1} \), we get \( r_f = -\ln(\beta) + \gamma E_t[\Delta \tilde{c}_{t+1}] - \frac{1}{2} \gamma^2 \sigma^2[\Delta \tilde{c}_{t+1}] \). Ignoring the variance term, it is easy to see that a high RRA coefficient \( \gamma \) can only be reconciled with low interest rates, if the time discount factor \( \beta \) exceeds one (see Campbell, 2003).

\(^{10}\)The Jensen’s Inequality term \( \frac{1}{2} \sigma^2[\tilde{r}_{t+1}] \) arises due to the fact that we consider the expected value of a log return.
\begin{align*}
E_t[\tilde{r}_{t+1}^e] + \frac{1}{2} \sigma^2[\tilde{r}_{t+1}^e] &= -\text{cov}[\tilde{m}_{t+1}, \tilde{r}_{t+1}^e] \\
&= \rho[\Delta \tilde{c}_{t+1}, \tilde{r}_{t+1}^e] \cdot \sigma[\Delta \tilde{c}_{t+1}] \cdot \sigma[\tilde{r}_{t+1}^e] \cdot \gamma \tag{21}
\end{align*}

Again, the Hansen and Jagannathan (1991) volatility bound for the log SDF can be derived by acknowledging that the absolute value of the correlation \(\rho[\Delta \tilde{c}_{t+1}, \tilde{r}_{t+1}^e]\) may not exceed one. Rearranging for the (logarithmic) Sharpe ratio leads to:

\[\sigma[\tilde{m}_{t+1}] = \gamma \sigma[\Delta \tilde{c}_{t+1}] \geq \left| \frac{E_t[\tilde{r}_{t+1}^e] + \frac{1}{2} \sigma^2[\tilde{r}_{t+1}^e]}{\sigma[\tilde{r}_{t+1}^e]} \right|. \tag{22}\]

### 3 Empirical Analysis

#### 3.1 Data, Sample Selection, and Descriptive Statistics

Our empirical analysis is based on historical time series of aggregate annual premium volumes and claims for fire insurance in Australia (1992–2012), Italy (1973–2011), Japan (1987–2012), and the Netherlands (1986–2011), which have been provided by the global reinsurance company Swiss Re. This data set is complemented by direct premiums written and direct losses paid for U.S. fire insurance in the time period 1989–2013 from the insurance rating agency A.M. Best as well as annual premium and claims data from the German Insurance Association (GDV) during the period 1974–2012. The latter is available for the whole nonlife insurance industry in Germany as well as the following lines of business: motor, casualty, legal, fire, credit, household and homeowners insurance. Since the payoff of a property-casualty insurance contract is the indemnification for a loss suffered by the policyholder, we estimate the gross return on the policy of the representative agent by means of the loss ratio, i.e., aggregate claims divided by the premium volume in a given year.\(^{11}\) To be able to fit the model to real returns, we additionally adjust the claims ratios for inflation.

Furthermore, we draw on an updated version of the macroeconomic data set used in Campbell (1999) and Campbell (2003), which can be downloaded from the International Financial Statistics (IFS) website and comprises time series of consumption, consumer price indices, short-term interest rates, population, and GDP deflator.\(^{12}\) Data on non-durables and services consumption is only available for the United States. Consequently, we need to work with total household consumption expenditure for Australia, Germany, Italy, Japan, and the Netherlands. The time frames of the macroeconomic series are matched to those of the premiums and claims figures for each country. In line with the empirical literature on

\(^{11}\) Hence, the excess return \(\tilde{R}_{t+1}^e\) equals the difference between the loss ratio \(\tilde{X}_{t+1}/P_t\) and the gross risk-free rate \(R_f\). A timing convention is needed as both premiums and claims are flows over the year instead of point-in-time observations. In line with common practice in the insurance industry, we calculate the loss ratios based on contemporaneous premiums and claims, thus implicitly assuming that the former are measured at the beginning and the latter at the end of the year.

\(^{12}\) The pre-Euro consumption figures for Germany, Italy, and the Netherlands have been converted by means of the fixed exchange rates between the Euro and the former domestic currencies of those countries.
the equity premium puzzle, we compute log consumption growth based on real per capita consumption, which is defined as the overall level of consumption divided by the population and the GDP deflator. Finally, we follow Campbell (2003) and draw on stock market returns from Morgan Stanley Capital International (MSCI), which we have downloaded from the Wharton Research Data Services website and inflation-adjusted using the consumer price index of each country. This will enable us to compare our RRA coefficient estimates for the insurance sector with the values that constitute the classical equity premium puzzle. Due to the restricted availability of insurance data, we are in fact looking at shorter time horizons than most of the earlier empirical asset pricing literature. Yet, to illustrate the robustness of our results, we have included an exact replication of the Campbell (2003) results in the Appendix.

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>(r^s)</th>
<th>(\sigma[r^s])</th>
<th>(\Delta c)</th>
<th>(\sigma[\Delta c])</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUL</td>
<td>1992-2012</td>
<td>6.06%</td>
<td>17.83%</td>
<td>52.63%</td>
<td>1.46%</td>
</tr>
<tr>
<td>GER</td>
<td>1974-2011</td>
<td>7.18%</td>
<td>25.77%</td>
<td>47.53%</td>
<td>1.11%</td>
</tr>
<tr>
<td>ITA</td>
<td>1973-2011</td>
<td>1.33%</td>
<td>28.83%</td>
<td>45.73%</td>
<td>1.63%</td>
</tr>
<tr>
<td>JAP</td>
<td>1987-2012</td>
<td>-1.56%</td>
<td>23.92%</td>
<td>99.95%</td>
<td>1.66%</td>
</tr>
<tr>
<td>NL</td>
<td>1986-2011</td>
<td>6.24%</td>
<td>23.57%</td>
<td>57.97%</td>
<td>1.25%</td>
</tr>
<tr>
<td>USA</td>
<td>1989-2012</td>
<td>6.40%</td>
<td>17.61%</td>
<td>76.96%</td>
<td>1.56%</td>
</tr>
</tbody>
</table>

Table 1: International Equity Returns, Insurance Returns, Risk-Free Rates, and Consumption Growth

This table shows the average annual log return of the equity market (column three), fire insurance market (column five), and risk-free asset (column seven) as well as the corresponding standard deviations (columns four, six, and eight) for all countries in our sample. In addition, the last two columns contain the average annual log growth rates of per capita consumption and the standard deviations of log consumption growth. The time period for each country is determined by the availability of insurance premiums and claims data. All figures are reported in real terms.

Table 1 contains some descriptive statistics. We see that the stock markets in all countries apart from Italy and Japan have delivered average log returns (\(r^s\)) in excess of six percent per annum over the considered time periods. The corresponding volatilities (\(\sigma[r^s]\)) range between 17 and 29 percent. This is consistent with the findings of Campbell (2003) who suggests that the relatively poor performance of the Italian stock market is due to its small size in percent of GDP. The figures for Japan, in contrast, are clearly dominated by the burst of the asset price bubble in the late 1980s and the subsequent period of economic stagnation termed “the lost decade”. Moreover, the average annual log returns on fire insurance policies (\(r^i\)) in all of the six countries lie below -20 percent. Hence, as implied by asset pricing theory, individuals are prepared to accept a significant negative return on their insurance contracts because the latter represent a consumption hedge. Interestingly, four of the six insurance market return volatilities (\(\sigma[r^i]\)) are even slightly smaller than the corresponding stock market return volatilities. This could be due to the fact that insurance sector fundamentals and actuarial premium drivers are quite stable over

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13Just as premiums and claims, consumption is a flow measure and thus requires a timing convention. Campbell (2003) advocates that the latter should be determined so as to generate the highest possible contemporaneous correlation between consumption growth and stock returns. Accordingly, we resort to a beginning-of-year timing convention for Germany, Italy, the Netherlands and the United States, meaning that consumption growth in a specific year is calculated as per capita consumption in the next year divided by per capita consumption in the current year. By contrast, an end-of-year timing convention is adopted for Australia and Japan. In these two cases, current consumption growth equals this year’s per capita consumption divided by last year’s per capita consumption.

14The extreme figure for Japan is attributable to very low loss ratios, which, in turn, imply a highly profitable business.
time and insurance prices generally display a relatively low sensitivity to short-term changes in market sentiment. Australia and Japan seem to be exceptions, since the aggregate fire insurance returns in these two countries turned out to be roughly twice as volatile as the stock market returns. Turning to the average log return ($r_f$) and the standard deviation ($\sigma[r_f]$) of the risk-free asset, we notice that short-term government debt yielded less than three percent per annum in all of the six countries. Finally, the last two columns show the average log consumption growth rates ($\Delta c$) as well as their standard deviations ($\sigma[\Delta c]$). With one exception (Italy), both measures lie consistently below two percent. Overall, Table 1 underlines that high single-digit average equity returns, large negative average insurance returns, and a low consumption growth volatility characterize many developed countries, including the United States.

3.2 The Equity Premium Puzzle: A Brief Review

We begin our empirical analysis with a brief review of the classical equity premium puzzle. Estimates of the relevant variables for our sample countries and time periods can be found in Table 2. In line with Table 1, all figures are reported in real terms. Panel A is based on the extended Stein’s Lemma and Panel B relies on the lognormality assumption. Accordingly, the estimated annual equity risk premium in column three is represented by expected excess returns ($E[\tilde{R}_{t+1}]$) in the first case as well as adjusted expected log excess returns ($aE[\tilde{r}_{t+1}] = E_t[\tilde{r}_{t+1}] + \frac{1}{2}\sigma^2[\tilde{r}_{t+1}]$) in the second case. Columns four and five contain the corresponding standard deviations ($\sigma[\tilde{R}_{t+1}]$ and $\sigma[\tilde{r}_{t+1}]$) and the Hansen-Jagannathan bounds for the log SDF ($\sigma[\tilde{m}_{t+1}]$) as defined in Equations (18) and (22). Furthermore, in columns six and seven, we have provided the volatilities of log consumption growth ($\sigma[\Delta \tilde{c}_{t+1}]$) and the correlations of log consumption growth with the equity risk premium ($\rho[\Delta \tilde{c}_{t+1}, \tilde{R}_{t+1}]$ and $\rho[\Delta \tilde{c}_{t+1}, \tilde{r}_{t+1}]$). In order to evaluate the risk aversion of the representative agent, we employ Equations (17) and (21) in combination with two different inputs for the correlation: (a) the empirically estimated correlation coefficient in column seven and (b) a correlation of one. The latter is consistent with the definition of the Hansen-Jagannathan bounds and allows us to differentiate between the impact of the SDF volatility as well as the correlation of the SDF with excess returns. We have included the respective estimates in the last two columns labeled $\gamma(1)$ and $\gamma(2)$. All time periods are determined by the availability of insurance data.

The figures in Panels A and B are very similar. Apart from Japan, the equity risk premiums range from three to around eight percent and exhibit volatilities of between 16 and 30 percent. While our findings for the United States and Germany are fully consistent with earlier research, we observe sampling-related deviations of at least one percentage point for Australia, Italy, the Netherlands and Japan (cf. Table 7 in the Appendix). Turning to the Hansen-Jagannathan bounds and ignoring Japan for the time being, we realize that pricing kernels which are capable of explaining historical asset prices in the other

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15Our values are slightly lower than those reported in Campbell (2003), because the low-interest rate environment after the millennium forms a larger part of the sample. For a comparison please refer to the Appendix.

16The adjustment is Jensen’s Inequality, i.e., half a sample variance of the log excess return (see Equation 21).

17Recall that a perfect positive correlation between excess returns and consumption growth implies a perfect negative correlation between excess returns and the SDF. This is characteristic for portfolios on the mean-variance frontier, which are maximally risky and thus earn the highest expected returns for a given standard deviation (see, e.g., Cochrane, 2005).

18For comparison purposes, an exact replication of the corresponding Campbell (2003) results is provided in the Appendix.

19As mentioned before, the Japanese stock market figures are driven by an extended period of stagnation in the 1990s.
five countries would need to exhibit a minimum volatility of between 11 and 38 percent. Yet, all log consumption growth volatilities are around two percent or lower. Thus, Equations (18) and (22) tell us that the model will only fit, if the RRA coefficient $\gamma$ is large. This can be seen explicitly when considering the values of $\gamma(1)$, which have been calculated based on the empirical correlation coefficients in column seven. Most authors, including Mehra and Prescott (1985), deem RRA values of between one and ten to be acceptable. Our estimates for Australia, Germany, Italy, the Netherlands, and the United States, in contrast, lie all considerably above this range. Again, Japan is an exception due to the very low/slightly negative historical equity risk premium in our sample period. Even after setting the correlation to one, only the $\gamma(2)$ value for Italy drops below the theoretical RRA threshold of ten. Therefore, the empirically observed stock market returns can only be reconciled with the theoretical model on the basis of implausibly high RRA coefficients. This phenomenon, which holds consistently across countries, is the equity premium puzzle.

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|c|c|c|c|}
\hline
\textbf{Panel A: Extended Stein’s Lemma} & & & & & & & \\
\hline
\textbf{Country} & \textbf{Period} & $E[\tilde{R}_{t+1}]$ & $\sigma[\tilde{R}_{t+1}]$ & $\sigma[\tilde{m}_{t+1}]$ & $\sigma[\Delta\tilde{c}_{t+1}]$ & $\rho[\Delta\tilde{c}_{t+1}, \tilde{R}_{t+1}]$ & $\gamma(1)$ & $\gamma(2)$ \\
\hline
AUL & 1992-2012 & 4.91\% & 16.95\% & 28.95\% & 1.84\% & +0.3667 & 43.00 & 15.77 \\
GER & 1974-2011 & 8.38\% & 26.68\% & 31.41\% & 1.46\% & +0.2884 & 74.41 & 21.46 \\
ITA & 1973-2011 & 3.34\% & 29.39\% & 11.38\% & 2.19\% & +0.3702 & 14.02 & 5.19 \\
JAP & 1987-2012 & -0.19\% & 23.22\% & 0.80\% & 1.66\% & +0.0929 & < 0 & < 0 \\
NL & 1986-2011 & 6.62\% & 22.65\% & 29.24\% & 1.84\% & +0.6101 & 25.98 & 15.85 \\
USA & 1989-2012 & 6.47\% & 17.04\% & 37.95\% & 1.56\% & +0.5704 & 42.56 & 24.28 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|c|c|c|c|}
\hline
\textbf{Panel B: Lognormality Assumption} & & & & & & & \\
\hline
\textbf{Country} & \textbf{Period} & $E[\tilde{e}_{t+1}]$ & $\sigma[\tilde{e}_{t+1}]$ & $\sigma[\tilde{m}_{t+1}]$ & $\sigma[\Delta\tilde{c}_{t+1}]$ & $\rho[\Delta\tilde{c}_{t+1}, \tilde{R}_{t+1}]$ & $\gamma(1)$ & $\gamma(2)$ \\
\hline
AUL & 1992-2012 & 4.82\% & 17.57\% & 27.43\% & 1.84\% & +0.4358 & 34.28 & 14.94 \\
GER & 1974-2011 & 8.13\% & 25.74\% & 31.61\% & 1.46\% & +0.3441 & 62.74 & 21.59 \\
ITA & 1973-2011 & 3.19\% & 27.84\% & 11.44\% & 2.19\% & +0.4186 & 12.46 & 5.22 \\
JAP & 1987-2012 & 0.05\% & 23.95\% & 0.19\% & 1.66\% & +0.0608 & 1.90 & 0.12 \\
NL & 1986-2011 & 6.59\% & 23.44\% & 28.13\% & 1.84\% & +0.6249 & 24.40 & 15.25 \\
USA & 1989-2012 & 6.41\% & 17.80\% & 36.04\% & 1.56\% & +0.5966 & 38.64 & 23.05 \\
\hline
\end{tabular}
\end{table}

Table 2: Evidence for the Equity Premium Puzzle

This table illustrates the equity premium puzzle based on the extended Stein’s Lemma (Panel A) and the lognormality assumption (Panel B). It contains the annual equity market risk premiums (column three), the corresponding excess return standard deviations (column four), and the resulting Hansen-Jagannathan bounds (column five) for all countries in our sample. In addition, it shows the volatilities of consumption growth (column six) and its correlations with excess returns (column seven). The last two columns labeled $\gamma(1)$ and $\gamma(2)$ display the RRA coefficient estimates for (a) the empirical correlation in column seven and (b) a correlation of one. The time period for each country is determined by the availability of insurance premiums and claims data. All figures are reported in real terms.

\begin{itemize}
\item[\textsuperscript{20}]It has been documented in the literature that the correlation estimates tend to increase with the interval of the underlying time series (see, e.g., Campbell and Cochrane, 1999). Thus, our results of between 0.3 and 0.6 (excluding Japan) are somewhat higher than those reported in earlier studies, since we work with annual rather than quarterly data (cf. Table 7 in the Appendix). As a corollary, we obtain smaller values for $\gamma(1)$.
\item[\textsuperscript{21}]Brown et al. (1995) suggested that the high ex-post equity premiums in the United States may be caused by a survivor bias of stock exchanges. Their hypothesis, however, is contradicted by the fact that the puzzle also exists in other countries.
\end{itemize}
### 3.3 Consumption-Based Asset Pricing in Insurance Markets

In the following, we fit the consumption-based asset pricing model to inflation-adjusted excess returns on insurance policies. The respective results can be found in Table 3, which exhibits the same structure as Table 2. Once more, the estimates in column three of Panel A are based on the extended Stein’s Lemma and expected excess returns whereas column three of Panel B relates to the lognormality assumption and adjusted expected excess returns. The associated volatilities are reported in column four. Columns five to seven contain the Hansen-Jagannathan bounds for the log SDF, the volatility of consumption growth, and the correlations of log consumption growth with the insurance market risk premium. The last two columns present the RRA coefficients $\gamma(1)$ and $\gamma(2)$, which have been estimated using (a) the empirical correlation and (b) a correlation of minus one. \(^{22}\) In the upper part of each panel, we have summarized the results for the fire insurance markets in Australia, Italy, Japan, the Netherlands, and the United States, while the lower parts are dedicated to the German property-casualty market as a whole as well as seven individual business lines.

In contrast to Table 2, we now observe considerable differences between Panels A and B. This is a clear indication that the widespread lognormality assumption in asset pricing does not fit the typical shape of claims ratio distributions. The extended Stein’s Lemma, on the other hand, is much better suited for the analysis of insurance data, since it accounts for skewness and kurtosis. \(^{23}\) We therefore decide to focus our subsequent interpretation efforts on Panel A. As expected, the insurance risk premiums are highly negative, ranging from around -12 percent for German Motor insurance down to almost -59 percent for Japanese fire insurance. \(^{24}\) Interestingly, only two of the corresponding volatilities exceed 30 percent (Australia and Japan), while the majority lies under ten percent. Hence, insurance risk premiums seem to be generally more stable than equity risk premiums (cf. Table 2). Furthermore, the Hansen-Jagannathan bounds vary between 124 and 891 percent. This means that we need an excessively volatile SDF to explain the average historical excess returns - a condition which is clearly not fulfilled by log consumption growth. Consistent with the model predictions, almost all correlation coefficients between log consumption growth and insurance risk premiums are negative. Yet, their absolute values are quite small, leading to absurdly high $\gamma(1)$ estimates of between 360 and 9500. Although setting the correlation to minus one causes a substantial decrease, the $\gamma(2)$ of between 68 and 583 still remain way beyond any reasonable threshold. Excluding Japan, they equal between 4 and 37 times their equity counterparts. Overall, our results support the theory insofar as individuals are indeed prepared to accept negative excess returns on insurance contracts to smooth consumption over time. However, to fit the insurance data, we need extreme levels of risk aversion, which are even higher than in the stock markets. This phenomenon will be subsequently referred to as the insurance premium puzzle.

\(^{22}\)Since asset pricing theory assumes a negative covariance of insurance returns with consumption growth (refer to the second section), both the Hansen-Jagannathan bounds for the log SDF and $\gamma(2)$ now require a perfect negative correlation.

\(^{23}\)As an example consider Japan and the Netherlands. In the first case, the insurance risk premiums exhibit a skewness of 2.1 and an excess kurtosis of 5.1, whereas for the equity risk premiums these figures amount to 0.4 and 0.4. Similarly, in the second case, the risk premium distribution in the insurance market is characterized by a skewness of 1.1 and an excess kurtosis of 3.6, compared to 0.2 and 0.2 for the distribution of the equity risk premium.

\(^{24}\)The moderately negative excess returns on German motor insurance contracts are driven by the intense competition that has shaped this market throughout the last decade (see, e.g., Eling and Luhnen, 2008).
### Panel A: Extended Stein’s Lemma

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>$E[R_{t+1}]$</th>
<th>$\sigma[R_{t+1}]$</th>
<th>$\sigma[m_{t+1}]$</th>
<th>$\sigma[\Delta c_{t+1}]$</th>
<th>$\rho[\Delta c_{t+1}, R_{t+1}]$</th>
<th>$\gamma(1)$</th>
<th>$\gamma(2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUL (Fire)</td>
<td>1992-2012</td>
<td>-38.27%</td>
<td>30.72%</td>
<td>124.60%</td>
<td>1.84%</td>
<td>-0.1862</td>
<td>364.52</td>
<td>67.86</td>
</tr>
<tr>
<td>ITA (Fire)</td>
<td>1973-2011</td>
<td>-37.89%</td>
<td>9.11%</td>
<td>413.78%</td>
<td>2.19%</td>
<td>-0.0298</td>
<td>6370.89</td>
<td>189.56</td>
</tr>
<tr>
<td>JAP (Fire)</td>
<td>1987-2012</td>
<td>-58.55%</td>
<td>34.41%</td>
<td>170.15%</td>
<td>1.66%</td>
<td>-0.1277</td>
<td>802.84</td>
<td>102.55</td>
</tr>
<tr>
<td>NL (Fire)</td>
<td>1986-2011</td>
<td>-45.87%</td>
<td>8.40%</td>
<td>545.88%</td>
<td>1.84%</td>
<td>-0.0313</td>
<td>9466.93</td>
<td>295.87</td>
</tr>
<tr>
<td>USA (Fire)</td>
<td>1989-2012</td>
<td>-54.70%</td>
<td>6.13%</td>
<td>891.92%</td>
<td>1.56%</td>
<td>+0.2702</td>
<td>&lt; 0</td>
<td>570.59</td>
</tr>
<tr>
<td>GER (Total)</td>
<td>1974-2011</td>
<td>-25.98%</td>
<td>4.34%</td>
<td>598.03%</td>
<td>1.46%</td>
<td>-0.0954</td>
<td>4280.91</td>
<td>408.52</td>
</tr>
<tr>
<td>Motor</td>
<td>1974-2011</td>
<td>-12.31%</td>
<td>6.81%</td>
<td>189.80%</td>
<td>1.46%</td>
<td>-0.0587</td>
<td>2105.58</td>
<td>123.51</td>
</tr>
<tr>
<td>Casualty</td>
<td>1974-2011</td>
<td>-51.34%</td>
<td>6.01%</td>
<td>853.68%</td>
<td>1.46%</td>
<td>+0.0624</td>
<td>&lt; 0</td>
<td>583.16</td>
</tr>
<tr>
<td>Legal</td>
<td>1974-2011</td>
<td>-33.99%</td>
<td>7.86%</td>
<td>432.46%</td>
<td>1.46%</td>
<td>-0.2382</td>
<td>1240.32</td>
<td>295.42</td>
</tr>
<tr>
<td>Fire</td>
<td>1974-2011</td>
<td>-24.23%</td>
<td>14.04%</td>
<td>172.59%</td>
<td>1.46%</td>
<td>-0.1060</td>
<td>1112.77</td>
<td>117.90</td>
</tr>
<tr>
<td>Credit</td>
<td>1974-2011</td>
<td>-28.69%</td>
<td>21.42%</td>
<td>133.96%</td>
<td>1.46%</td>
<td>-0.0791</td>
<td>1156.98</td>
<td>91.51</td>
</tr>
<tr>
<td>Household</td>
<td>1974-2011</td>
<td>-46.54%</td>
<td>6.54%</td>
<td>711.32%</td>
<td>1.46%</td>
<td>+0.3013</td>
<td>&lt; 0</td>
<td>485.91</td>
</tr>
<tr>
<td>Homeowners</td>
<td>1974-2011</td>
<td>-30.32%</td>
<td>20.51%</td>
<td>147.81%</td>
<td>1.46%</td>
<td>-0.0556</td>
<td>1814.63</td>
<td>100.97</td>
</tr>
</tbody>
</table>

### Panel B: Lognormality Assumption

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>$E[\tilde{R}_{t+1}]$</th>
<th>$\sigma[\tilde{R}_{t+1}]$</th>
<th>$\sigma[\tilde{m}_{t+1}]$</th>
<th>$\sigma[\Delta \tilde{c}_{t+1}]$</th>
<th>$\rho[\Delta \tilde{c}<em>{t+1}, \tilde{R}</em>{t+1}]$</th>
<th>$\gamma(1)$</th>
<th>$\gamma(2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUL (Fire)</td>
<td>1992-2012</td>
<td>-46.58%</td>
<td>42.03%</td>
<td>110.82%</td>
<td>1.84%</td>
<td>-0.1984</td>
<td>504.25</td>
<td>60.36</td>
</tr>
<tr>
<td>ITA (Fire)</td>
<td>1973-2011</td>
<td>-46.66%</td>
<td>14.76%</td>
<td>316.00%</td>
<td>2.19%</td>
<td>-0.0270</td>
<td>5332.66</td>
<td>144.06</td>
</tr>
<tr>
<td>JAP (Fire)</td>
<td>1987-2012</td>
<td>-89.83%</td>
<td>47.72%</td>
<td>188.23%</td>
<td>1.46%</td>
<td>-0.1696</td>
<td>576.26</td>
<td>113.45</td>
</tr>
<tr>
<td>NL (Fire)</td>
<td>1986-2011</td>
<td>-59.43%</td>
<td>13.63%</td>
<td>433.90%</td>
<td>1.84%</td>
<td>-0.0321</td>
<td>7363.56</td>
<td>236.26</td>
</tr>
<tr>
<td>USA (Fire)</td>
<td>1989-2012</td>
<td>-77.39%</td>
<td>15.08%</td>
<td>513.22%</td>
<td>1.56%</td>
<td>+0.1095</td>
<td>&lt; 0</td>
<td>296.80</td>
</tr>
<tr>
<td>GER (Total)</td>
<td>1974-2011</td>
<td>-29.22%</td>
<td>5.58%</td>
<td>523.27%</td>
<td>1.46%</td>
<td>-0.1288</td>
<td>2774.93</td>
<td>357.45</td>
</tr>
<tr>
<td>Motor</td>
<td>1974-2011</td>
<td>-12.73%</td>
<td>7.65%</td>
<td>166.35%</td>
<td>1.46%</td>
<td>-0.0878</td>
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<tr>
<td>Casualty</td>
<td>1974-2011</td>
<td>-69.44%</td>
<td>10.38%</td>
<td>668.90%</td>
<td>1.46%</td>
<td>+0.0426</td>
<td>&lt; 0</td>
<td>456.93</td>
</tr>
<tr>
<td>Legal</td>
<td>1974-2011</td>
<td>-40.25%</td>
<td>11.69%</td>
<td>344.21%</td>
<td>1.46%</td>
<td>-0.2726</td>
<td>862.58</td>
<td>235.14</td>
</tr>
<tr>
<td>Fire</td>
<td>1974-2011</td>
<td>-26.85%</td>
<td>17.83%</td>
<td>150.61%</td>
<td>1.46%</td>
<td>-0.1484</td>
<td>693.06</td>
<td>102.88</td>
</tr>
<tr>
<td>Credit</td>
<td>1974-2011</td>
<td>-33.09%</td>
<td>28.66%</td>
<td>115.47%</td>
<td>1.46%</td>
<td>-0.0960</td>
<td>821.29</td>
<td>78.88</td>
</tr>
<tr>
<td>Household</td>
<td>1974-2011</td>
<td>-60.63%</td>
<td>12.35%</td>
<td>490.88%</td>
<td>1.46%</td>
<td>+0.2608</td>
<td>&lt; 0</td>
<td>335.32</td>
</tr>
<tr>
<td>Homeowners</td>
<td>1974-2011</td>
<td>-34.87%</td>
<td>28.89%</td>
<td>120.69%</td>
<td>1.46%</td>
<td>-0.0387</td>
<td>2128.80</td>
<td>82.44</td>
</tr>
</tbody>
</table>

### Table 3: Evidence for the Insurance Premium Puzzle

This table illustrates the insurance premium puzzle based on the extended Stein’s Lemma (Panel A) and the lognormality assumption (Panel B). It contains the annual insurance market risk premiums (column three), the corresponding standard deviations (column four), and the resulting Hansen-Jagannathan bounds (column five) for all countries and insurance business lines in our sample. In addition, it shows the volatilities of consumption growth (column six) and its correlations with excess returns on insurance policies (column seven). The last two columns labeled $\gamma(1)$ and $\gamma(2)$ display the RRA coefficient estimates for (a) the empirical correlation in column seven and (b) a correlation of one. All figures are reported in real terms.
At first glance, the insurance premium puzzle comprises two baffling aspects: the ridiculously high RRA coefficients implied by the consumption-based model and the fact that individuals seem to be more risk averse in the insurance market than for stock investments. The latter can be plausibly explained with the latest insights in economics and psychology. Both disciplines have long been dominated by the notion that attitude towards risk is a stable personality trait over time and across contexts. More recent work, however, calls this assumption into question (see, e.g., Weber et al., 2002; Soane and Chmiel, 2005; Hanoch et al., 2006; Riddel, 2012). Instead of categorizing individuals as being risk taking or risk averse, decision researchers have begun to adopt a domain-specific approach. There are many domains in which people are confronted with choices over risky outcomes, including finance, insurance, sports, leisure, health, career, and the environment. Experiments and surveys show that the preferences of most subjects display a substantial domain heterogeneity. Against this background, we view the observed discrepancy in the RRA coefficients as a contribution to the literature on domain-specific risk aversion and focus our subsequent efforts on understanding the actual level of $\gamma$.

4 Tackling the Puzzles

4.1 A Discussion of Selected Model Extensions

As indicated at the outset of this paper, the literature on the equity premium puzzle is abundant in modifications of the consumption-based model. In this section, we consider a number of the most influential approaches as natural candidates for tackling the insurance premium puzzle. Epstein and Zin (1989, 1991) introduced a recursive utility function, which disentangles risk aversion from the elasticity of intertemporal substitution and thus allows for a high expected excess return and a low risk-free interest rate. While being key in overcoming the risk-free rate puzzle, this refinement on a stand-alone basis is known to capture only a third of the historical equity premium when calibrated with reasonable values for the RRA coefficient (see, e.g., Siegel and Thaler, 1997). Hence, it will be insufficient to match the, in absolute terms, even larger average excess returns on insurance policies. Moreover, we could turn to the habit persistence frameworks of Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999), under which utility depends on the difference between current consumption and a benchmark level. Unfortunately, they come with undesirable side effects regarding consumption volatility and fiscal policy (see, e.g., Lettau and Uhlig, 2000; Ljungqvist and Uhlig, 2000). Another potential remedy is the rare disaster hypothesis as coined by Rietz (1988) and Barro (2006), suggesting that major economic crises or wars are an important driver of asset prices. It seems consequential that the demand for insurance might depend on such event risks, too. More recently, however, several objections have been raised. Julliard and Ghosh (2012), e.g., point out that both the empirical frequency and magnitude of economic disasters are too small to rationalize the equity premium puzzle and that the rare events hypothesis actually deteriorates the model’s ability to explain the cross-sectional variation in risk premiums. The next category of refinements to be considered is based on the work of Mankiw (1986) and Weil (1992), who abandoned

25Note that this class of preference functionals is irreconcilable with expected utility theory (see, e.g., Weil, 1990).

26Also, it lacks time-varying risk aversion to match empirically-observed equity volatilities (see, e.g., Barberis et al., 2001).

27Barro (2009) estimates that society would be prepared to forgo 20 percent of GDP each year to eliminate rare disasters.
the representative-agent economy to account for heterogeneous investors, facing uninsurable idiosyncratic labor income shocks. Consequently, consumption growth, and hence the stochastic discount factor, are more volatile on the individual than on the aggregate (or per capita) level. Meanwhile, however, it has been shown that the historical equity premium can hardly be generated by realistically calibrated model of this type (see, e.g., Lettau, 2002; Heaton and Lucas, 2008).

More plausible explanations for the equity and the insurance premium puzzle include the potential discrepancy between ex ante beliefs and ex post realizations, behavioral economic phenomena, as well as the role of higher-order risk preferences. It is certainly conceivable that distributional moments estimated from historical data do not reflect ex ante expectations of economic agents. In other words, the magnitude of claims ratios (or insurance risk premiums) may be a surprise for most policyholders, since they experience difficulties in accurately assessing loss frequencies and severities before purchasing insurance. Support for this argument is provided by Söderlind (2009b), who finds that the equity premium puzzle can be reduced when calibrating the consumption-based model based on survey answers and option-implied volatilities. A test of this hypothesis, however, is beyond the scope of our paper, since it would require a full-scale psychometric study. The first attempt to overcome the equity premium puzzle with ideas from behavioral economics, particularly prospect theory of Kahneman and Tversky (1979), has been made by Benartzi and Thaler (1995), who employ loss aversion together with a myopic evaluation horizon of one year for gains and losses. Following their article, several studies have established similar concepts in asset pricing (see, e.g., Barberis et al., 2001; Barberis and Huang, 2001; Barberis et al., 2006; Barberis and Huang, 2008a,b, 2009). A major achievement in this strand of the literature is the inclusion of loss aversion and narrow framing into the Euler equations of the consumption-based model. By means of this modification, it is possible to explain several characteristics found in aggregate data with sensible levels of risk aversion. In particular, the approach of Barberis et al. (2001) is able to reconcile a low volatility of consumption growth with a high equity premium, a low risk-free interest rate, a high equity volatility, and a low correlation of excess returns with consumption growth. Another promising solution for the equity premium puzzle has been developed by Dionne et al. (2015), who suggest that, apart from the covariance between consumption growth and excess returns, investors also care about downside risk. Accounting for prudence through an explicit second-order expectation dependence term, their reformulated consumption-based model is able to fit empirical equity premiums with RRA coefficients of less than ten. Due to their proven success in addressing the equity premium puzzle, both the refinements by Barberis et al. (2001) and Dionne et al. (2015) should help to (at least partially) tackle the insurance premium puzzle, too. Hence, they will be at the center of the following analyses.

This approach ties in with the literature on background risk, which examines decision making under uncertainty in incomplete markets. A number of studies in this area have illustrated that the presence of uninsurable exogenous risks can increase the risk aversion of individuals and trigger more cautious behavior elsewhere (see, e.g., Eeckhoudt et al., 1996; Franke et al., 2006; Lee, 2008). A direct consequence of this “risk vulnerability effect” is a reduction of the demand for risky investments (see, e.g., Kimball, 1990; Gollier and Pratt, 1996). Hence, models that do not account for background risks may underestimate expected excess returns. Apart from asset pricing, insights on background risk have been applied in the context of portfolio selection (see, e.g., Heaton and Lucas, 2000; Jiang et al., 2010; Baptista, 2008, 2012) and optimal insurance decisions (see, e.g., Doherty and Schlesinger, 1983; Dana and Scarsini, 2007; Fei and Schlesinger, 2008).
4.2 Loss Aversion and Narrow Framing

Barberis et al. (2001) and Barberis and Huang (2001) enrich the consumption-based model with two key experimental insights from the literature on decision making under risk. Firstly, they argue that loss aversion, a long-standing notion in psychology, which has been established through Kahneman and Tversky’s (1979) prospect theory, should be incorporated in the representative investor’s preferences. Rather than focusing on his or her absolute wealth level, a loss-averse agent evaluates the corresponding changes relative to a reference point, thereby reacting more severely to losses than to similarly-sized gains. Secondly, they draw on the phenomenon of narrow framing as demonstrated by Tversky and Kahneman (1981). In the classical setup, utility is defined over consumption. Accordingly, individuals assess stocks or insurance contracts in combination with other wealth risks that they face, such as the stochasticity of labor income or house prices. Narrow framing, in contrast, means that the decision to engage in a gamble is frequently taken in isolation, i.e., people act as if they receive utility directly from variations in their financial wealth, although it is only one element of their total wealth. Barberis and Huang (2008b) offer two plausible explanations for narrow framing in the stock market. On the one hand, it may occur due to the fact that investors suffer regret about poor investment choices in the past, which constitutes a form of nonconsumption disutility. On the other hand, as pointed out by Kahneman (2003), individuals tend to frame narrowly whenever they rely on intuition instead of consequential reasoning. Intuitive actions are spontaneous and strongly driven by the information that is most accessible in a given situation. Evidently, information on the return distributions of single stocks or portfolios is more readily available and easier to analyze than information on the distribution of outcomes that arises from the combination of the new investment with all other wealth components of the decision maker.

The early framework of Benartzi and Thaler (1995) already assumed that individuals are loss averse and narrowly frame equity returns, which they evaluate at a myopic horizon of one year. Barberis et al. (2001) and Barberis and Huang (2001) ensure that investors additionally receive direct utility from consumption, thus enabling empirical tests of the model’s predictions for the distributional moments of consumption growth and stock market excess returns. We take their reasoning one step further by proposing that both loss aversion and narrow framing are likely to play a central role in the insurance context as well. A negative excess return on a property-casualty policy should have a stronger impact than a gain of the same size caused by an indemnification payment. Similarly, people should also feel regret over the decision to purchase coverage, which they did not need, since they would have been better off not paying the premium. Below, we fit a parsimonious version of the consumption-based model with loss aversion and narrow framing as proposed by Barberis and Huang (2008b) to our insurance market data.

---

29Following the extant literature, we treat loss aversion and narrow framing as separate concepts. Nevertheless, they are closely linked and in many cases they naturally occur together (see, e.g., Kahneman, 2003; Barberis et al., 2006).
30It appears quite natural to assume that investors focus on annual gains and losses, since media coverage, fund performance reports, tax filings etc. are all centered around one-year returns (see, e.g., Barberis and Huang, 2008b).
31Barberis et al. (2001) and Barberis and Huang (2001) additionally allow for time-varying loss aversion in line with prior gains or losses. Under their approach, losses are more painful if they follow earlier losses and less so, if they occur after prior gains. This feature is of subordinate importance to our analysis of asset risk premiums, since it is mainly needed to generate empirically observed equity volatilities and long-term return predictability (see Barberis and Huang, 2008b).
The following extensions of the Euler equations for returns (8) and excess returns (12) apply:\(^\text{32}\)

\[ 1 = E_t[\tilde{M}_{t+1} \tilde{R}_{t+1}] + b_0 \beta E[\tilde{R}_{t+1} - R_f] \]
\[ 0 = E_t[\tilde{M}_{t+1} \tilde{R}_{t+1}^e] + b_0 \beta E[\tilde{R}_{t+1}^e] \]  \hspace{1cm} (23)

We now have a second term reflecting the idea that, apart from consumption, the investor also gets utility directly from changes in his financial wealth, represented by excess returns on the risky asset.\(^\text{33}\)

The constant \(b_0\) controls the degree of narrow framing, i.e., the prominence of utility derived from gains and losses in financial wealth relative to consumption utility. Choosing \(b_0 = 0\) results in the classical model. The additional preference function \(\tau\) captures loss aversion as suggested by prospect theory. It exhibits a piecewise-linear form kinked at the origin and overweighs negative excess returns through the parameter \(\lambda\):

\[ \tau(x) = \begin{cases} 
    x & \text{for } x \geq 0 \\
    \lambda x & \text{for } x < 0, \text{where } \lambda > 1.
\end{cases} \]  \hspace{1cm} (24)

Employing the definition of covariance to (23) and rearranging, we obtain an expression for the risk premium as in (13):

\[ 0 = E_t[\tilde{M}_{t+1}] \cdot E_t[\tilde{R}_{t+1}^e] + \text{cov}_t[\tilde{M}_{t+1}, \tilde{R}_{t+1}^e] + b_0 \beta E[\tilde{R}_{t+1}^e] \\
E_t[\tilde{R}_{t+1}^e] = -\frac{\text{cov}_t[\tilde{M}_{t+1}, \tilde{R}_{t+1}^e]}{E_t[\tilde{M}_{t+1}]} - \frac{b_0 \beta E[\tilde{R}_{t+1}^e]}{E_t[\tilde{M}_{t+1}]} \]  \hspace{1cm} (25)

Furthermore, we resort to the extended version of Stein’s Lemma and break down the covariance in terms of \(\rho_t[\Delta \hat{c}_{t+1}, \tilde{R}_{t+1}^e], \sigma_t[\Delta \hat{c}_{t+1}], \sigma_t[\tilde{R}_{t+1}^e]\), and the RRA coefficient \(\gamma\):

\[ E_t[\tilde{R}_{t+1}^e] = -\frac{\text{cov}_t[\tilde{m}_{t+1}, \tilde{R}_{t+1}^e]}{E_t[\tilde{M}_{t+1}]} - \frac{b_0 \beta E[\tilde{R}_{t+1}^e]}{E_t[\tilde{M}_{t+1}]} \]
\[ = \rho_t[\Delta \hat{c}_{t+1}, \tilde{R}_{t+1}^e] \cdot \sigma_t[\Delta \hat{c}_{t+1}] \cdot \sigma_t[\tilde{R}_{t+1}^e] \cdot \gamma - \frac{b_0 \beta E[\tilde{R}_{t+1}^e]}{E_t[\tilde{M}_{t+1}]} \]  \hspace{1cm} (26)

Finally, recall from (9) that \(R_f = 1/E_t[\tilde{M}_{t+1}]\) to get:

\[ E_t[\tilde{R}_{t+1}^e] = \rho_t[\Delta \hat{c}_{t+1}, \tilde{R}_{t+1}^e] \cdot \sigma_t[\Delta \hat{c}_{t+1}] \cdot \sigma_t[\tilde{R}_{t+1}^e] \cdot \gamma - b_0 \beta E[\tilde{R}_{t+1}^e] \cdot R_f. \]  \hspace{1cm} (27)

In contrast to the original framework, the consumption-based model with loss aversion and narrow framing is only testable once we have fixed values for the three additional parameters \(b_0, \beta, \) and \(\lambda\), that cannot

\(^{32}\)Please refer to Barberis and Huang (2008b) for a detailed representation of the utility maximization problem faced by the representative agent. A proof of optimality can be found in Barberis et al. (2001).

\(^{33}\)Hence, only returns below the risk-free rate are considered to be a loss. Alternatively, one could set the representative agent’s reference point to zero instead of the risk-free rate. In that case, positive and negative net returns would be perceived as “gains” and “losses”, respectively.
be estimated from the data. Consistent with Barberis et al. (2001) and Barberis and Huang (2001), we pick $\beta = 0.98$ and $\lambda = 2.25$. The latter has been proposed by Tversky and Kahneman (1992) based on a comprehensive analysis of human behavior in gambling experiments. Unfortunately, determining a reasonable $b_0$ is not as straightforward as it may seem. At the same time, this parameter has a key impact on the results, as it governs the relative importance of nonconsumption utility. Barberis et al. (2001) stress that there are no strong clues for the choice of $b_0$ and employ a wide range of positive values. Ruling out the possibility of a negative $b_0$, however, implies that the model can, in many cases, no longer accommodate assets with negative risk premiums. To see this, consider (27): unless the second term on the right hand side is smaller than the first one, $E_t[\tilde R_{t+1}] < 0$ requires $\rho_t[\Delta \tilde c_{t+1}, \tilde R_{t+1}] < 0$ and $b_0 < 0$, since loss aversion (24) with $\lambda = 2.25$ almost surely leads to $E[\tilde v(\tilde R_{t+1})] < 0$. Consequently, we suggest that the degree of narrow framing should be represented by the absolute size of $b_0$ and, in line with our insurance context, allow for negative values. Against this background, $b_0$ will be determined separately for each country and business line by decreasing its value in increments of 0.1 until the RRA coefficient drops below the theoretically acceptable value of ten for the first time. In line with many extant studies in the empirical asset pricing literature, we now exclusively focus on $\gamma(2)$, which assumes a perfect negative correlation between consumption growth and excess returns on insurance. Table 4 summarizes our results.

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>$E_t[\tilde R_{t+1}]$</th>
<th>$\sigma_{\tilde R_{t+1}}$</th>
<th>$\sigma[\Delta \tilde c_{t+1}]$</th>
<th>$E[\tilde v(\tilde R_{t+1})]$</th>
<th>$R_f$</th>
<th>$b_0$</th>
<th>$\gamma(2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUL (Fire)</td>
<td>1992–2012</td>
<td>−38.27%</td>
<td>30.72%</td>
<td>1.84%</td>
<td>−0.9054</td>
<td>102.83%</td>
<td>−0.36</td>
<td>9.61</td>
</tr>
<tr>
<td>ITA (Fire)</td>
<td>1973–2011</td>
<td>−37.89%</td>
<td>9.11%</td>
<td>2.19%</td>
<td>−0.8422</td>
<td>102.13%</td>
<td>−0.43</td>
<td>8.25</td>
</tr>
<tr>
<td>JAP (Fire)</td>
<td>1987–2012</td>
<td>−58.55%</td>
<td>34.41%</td>
<td>1.66%</td>
<td>−1.3619</td>
<td>101.28%</td>
<td>−0.40</td>
<td>7.85</td>
</tr>
<tr>
<td>NL (Fire)</td>
<td>1986–2011</td>
<td>−45.87%</td>
<td>8.40%</td>
<td>1.84%</td>
<td>−1.0321</td>
<td>102.45%</td>
<td>−0.43</td>
<td>8.46</td>
</tr>
<tr>
<td>USA (Fire)</td>
<td>1989–2012</td>
<td>−54.70%</td>
<td>6.13%</td>
<td>1.56%</td>
<td>−1.2307</td>
<td>101.60%</td>
<td>−0.44</td>
<td>8.12</td>
</tr>
<tr>
<td>GER (Total)</td>
<td>1974–2011</td>
<td>−25.98%</td>
<td>4.34%</td>
<td>1.46%</td>
<td>−0.5845</td>
<td>102.41%</td>
<td>−0.44</td>
<td>2.63</td>
</tr>
<tr>
<td>Motor</td>
<td>1974–2011</td>
<td>−12.31%</td>
<td>6.81%</td>
<td>1.46%</td>
<td>−0.2776</td>
<td>102.41%</td>
<td>−0.41</td>
<td>8.95</td>
</tr>
<tr>
<td>Casualty</td>
<td>1974–2011</td>
<td>−51.34%</td>
<td>6.01%</td>
<td>1.46%</td>
<td>−1.1551</td>
<td>102.41%</td>
<td>−0.44</td>
<td>3.76</td>
</tr>
<tr>
<td>Legal</td>
<td>1974–2011</td>
<td>−33.99%</td>
<td>7.86%</td>
<td>1.46%</td>
<td>−0.7649</td>
<td>102.41%</td>
<td>−0.43</td>
<td>8.58</td>
</tr>
<tr>
<td>Fire</td>
<td>1974–2011</td>
<td>−24.23%</td>
<td>14.04%</td>
<td>1.46%</td>
<td>−0.5467</td>
<td>102.41%</td>
<td>−0.41</td>
<td>8.46</td>
</tr>
<tr>
<td>Credit</td>
<td>1974–2011</td>
<td>−28.69%</td>
<td>21.42%</td>
<td>1.46%</td>
<td>−0.6643</td>
<td>102.41%</td>
<td>−0.39</td>
<td>8.59</td>
</tr>
<tr>
<td>Household</td>
<td>1974–2011</td>
<td>−46.54%</td>
<td>6.54%</td>
<td>1.46%</td>
<td>−1.0472</td>
<td>102.41%</td>
<td>−0.44</td>
<td>3.13</td>
</tr>
<tr>
<td>Homeowners</td>
<td>1974–2011</td>
<td>−30.32%</td>
<td>20.51%</td>
<td>1.46%</td>
<td>−0.6921</td>
<td>102.41%</td>
<td>−0.40</td>
<td>8.43</td>
</tr>
</tbody>
</table>

Table 4: Consumption-Based Insurance Pricing with Loss Aversion and Narrow Framing
This table illustrates the effect of the loss aversion and narrow framing modification with $\beta = 0.98$ and $\lambda = 2.25$ on the consumption-based model’s ability to explain our insurance market data. It contains the annual risk premiums on insurance policies (column three) and the corresponding excess return standard deviations (column four) for all countries and insurance business lines in our sample. In addition, it shows the volatilities of consumption growth (column five), the expected prospect (nonconsumption) utility (column six), and the average gross risk-free interest rate (column seven). The last two columns labeled $b_0$ and $\gamma(2)$ display the narrow framing coefficient and the RRA coefficient, which have been estimated using the extended version of Stein’s Lemma and a perfect negative correlation between consumption growth and excess returns. All figures are reported in real terms.

34 Apart from insurance contracts, such assets are plentiful in modern financial markets. Obvious examples include exchange-traded funds (ETF) that offer the buyer a short position in a stock market index. If the risk premium of the long ETF is positive, that of the short ETF must be negative.
Columns three to seven comprise the estimates for the insurance market risk premiums, excess return volatilities, consumption growth volatilities, expected nonconsumption utilities, and the average gross risk-free rate over the considered time horizons. The last two columns present the narrow framing coefficient $b_0$ and the RRA coefficient $\gamma(2)$, which have been estimated based on (27) in combination with the aforementioned assumptions. As targeted, all values for $\gamma(2)$ now lie between two and ten. This massive reduction compared to Table 3 has been achieved with narrow framing parameters of between -0.44 and -0.36. To assess whether these values are reasonable, we turn to Barberis and Huang (2001), who note that “one way to think about $b_0$ is to compare the disutility of losing a dollar in the stock market with the disutility of having to consume a dollar less”. Moreover, they argue that the ratio of these two magnitudes in equilibrium can be expressed as $b_0 \beta \lambda$. Thus, Barberis and Huang (2001) set $b_0 = 0.45$ so that the loss of one dollar on a stock investment causes approximately the same reduction in utility as foregoing one dollar of consumption ($0.45 \times 0.98 \times 2.25 = 0.99$). When interpreting our $b_0$ estimates in absolute terms, we range between $|−0.36| \times 0.98 \times 2.25 = 0.79$ and $|−0.44| \times 0.98 \times 2.25 = 0.97$. These figures imply that the representative agent puts a little more importance on consumption utility than on psychological (prospect theory) utility. This is an economically reasonable outcome, which raises our confidence in the results and constitutes strong evidence that the loss aversion and narrow framing approach represents a viable solution for both the equity and the insurance premium puzzle.

### 4.3 Higher-Order Risk Preferences

The second promising refinement of the consumption-based model that we consider has been brought forward by Dionne et al. (2015). Motivated by the work of Kraus and Litzenberger (1976), Harvey and Siddique (2000), Ang et al. (2006) and others, they assume that a rational investor will exhibit higher-order risk attitudes. Therefore, he not only cares about the covariance of an asset’s (excess) returns with consumption, but also about downside risk. Dionne et al. (2015) capture downside risk through second-degree expectation dependence (SED), which they define as follows:

$$SED(\tilde{R}^e_{t+1}|C^*_t) \geq 0 \iff -cov \left(\tilde{R}^e_{t+1}, (C^*_t - \tilde{C}_{t+1})_+\right) \geq 0,$$

with $C^*_t$ denoting some shortfall threshold for the individual’s random consumption level $\tilde{C}_{t+1}$. Hence, positive SED is equivalent to the negative covariance of excess returns with the payoff of a European put option on consumption, struck at $C^*_t$. They then show that the risk premium in a consumption-based framework can be approximated by means of two components, namely a covariance effect and an integrated SED effect.\(^{35}\)

$$E_t[\tilde{R}^e_{t+1}] \approx \lambda cov_1[\Delta \tilde{C}_{t+1}, \tilde{R}^e_{t+1}] + \lambda^2 \int_{-\infty}^\infty SED[\tilde{R}^e_{t+1}|C^*_t] \, dC_{t+1}$$

$$\approx \lambda \left(\rho_1[\Delta \tilde{C}_{t+1}] \cdot \sigma_t[\Delta \tilde{C}_{t+1}] \cdot \sigma_t[\tilde{R}^e_{t+1}]\right) + \lambda^2 \int_{-\infty}^\infty SED[\tilde{R}^e_{t+1}|C^*_t] \, dC^*_t. \quad (29)$$

\(^{35}\)For an extensive derivation, including proofs, refer to Dionne et al. (2015).
It is important to point out that, in contrast to the classical setup, the pricing kernel is now based on absolute changes in consumption $\Delta \tilde{C}_{t+1} = \tilde{C}_{t+1} - C_t$ instead of log consumption growth $\Delta \tilde{\epsilon}_{t+1}$. Apart from that, the RRA coefficient does no longer appear directly. The risk premium is now a function of the Arrow-Pratt measure of constant absolute risk aversion (CARA) $\lambda$ as well as a coefficient of constant absolute prudence (CAP) $\lambda^2$, introduced by Modica and Scarsini (2005), Crainich and Eeckhoudt (2008), and Denuit and Eeckhoudt (2010):

$$
\lambda = \frac{u''(C_t)}{u'(C_t)} \quad \lambda^2 = \frac{u'''(C_t)}{u'(C_t)}
$$

Correspondingly, Dionne et al. (2015) abandon the power utility function in favor of exponential utility:

$$
u(C_t) = -\exp(-\lambda C_t) \quad (30)
$$

To estimate the model, they rely on an approximation for the integrated SED term in (29):\(^{36}\)

$$
\int_{\tilde{C}} SED[\tilde{R}_{t+1}^*|C_{t+1}^*] \ dC_{t+1}^* \approx \sum_{i=2}^{n} \text{cov}_t[\tilde{R}_{t+1}^*, \Delta \tilde{C}_{t+1}^*|\tilde{C}_{t+1}^* \leq C^{(i)}] \cdot \left( C^{(i)} - C^{(i-1)} \right), \quad (31)
$$

which can be computed based on the following algorithm (see Dionne et al. (2015)):

(i) Sort the elements of the consumption level time series in ascending order ($C_1 = C^{(1)} \leq \ldots \leq C_{i} = C^{(i)} \leq \ldots \leq C_n = \tilde{C}$) and find the corresponding excess returns as well as differenced consumption levels.

(ii) Calculate $n - 1$ successive lower partial covariances between the sorted sequences of excess returns and differenced consumption levels (starting with the ones that pertain to $C^{(1)}$ and $C^{(2)}$).

(iii) Now, integrated consumption SED can be evaluated as the sum of the products of the $n - 1$ lower partial covariances and the differences between the sorted consumption levels.

(iv) Finally, solve (29) for $\lambda$ and compute the RRA coefficient as follows: $\gamma = \lambda \cdot \mathbb{E}[\tilde{C}_{t+1}]$.

Once more, the model is fitted to our insurance market data. As in the previous section, we exclusively report $\gamma(2)$, thus assuming a perfect negative correlation between the absolute consumption changes and excess returns on property-casualty insurance policies. The results can be found in Table 5. Columns three to seven comprise the estimates for the insurance market risk premium, excess return volatility, average per capita consumption level, volatility of consumption changes, and integrated SED of excess returns and consumption. The last two columns present the CARA coefficient $\lambda$ and the associated RRA coefficient $\gamma(2)$, which have been estimated based on (29) as well as the methodology outlined by Dionne et al. (2015). Although, the $\gamma(2)$ are now substantially smaller than in Table 3, their sizes still remain between 13 and 52. Hence, embracing higher-order risk preferences does alleviate but not eliminate the insurance premium puzzle. This leads us to conclude that the SED approach may ultimately be more suitable to explain asset prices in the stock market.

\(^{36}\)It should be noted that Dionne et al. (2015) derive their model in terms of net returns, while, for comparison purposes, our presentation is based on excess returns.
<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>$E[R_{t+1}^c]$</th>
<th>$\sigma[R_{t+1}^c]$</th>
<th>$E[\Delta C_{t+1}]$</th>
<th>$\sigma[\Delta C_{t+1}]$</th>
<th>$f SED dC$</th>
<th>$\lambda$</th>
<th>$\gamma(2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUL (Fire)</td>
<td>1992–2012</td>
<td>-38.27%</td>
<td>30.72%</td>
<td>304.15</td>
<td>581.32</td>
<td>+0.0025</td>
<td>0.07</td>
<td>21.66</td>
</tr>
<tr>
<td>ITA (Fire)</td>
<td>1973–2011</td>
<td>-37.89%</td>
<td>9.11%</td>
<td>128.78</td>
<td>282.21</td>
<td>-0.5127</td>
<td>0.17</td>
<td>21.25</td>
</tr>
<tr>
<td>JAP (Fire)</td>
<td>1987–2011</td>
<td>-58.55%</td>
<td>34.41%</td>
<td>198.77</td>
<td>295.87</td>
<td>-8.7307</td>
<td>0.12</td>
<td>24.78</td>
</tr>
<tr>
<td>NL (Fire)</td>
<td>1986–2011</td>
<td>-45.87%</td>
<td>8.40%</td>
<td>145.88</td>
<td>270.97</td>
<td>-1.5339</td>
<td>0.22</td>
<td>32.53</td>
</tr>
<tr>
<td>USA (Fire)</td>
<td>1989–2012</td>
<td>-54.70%</td>
<td>6.13%</td>
<td>283.21</td>
<td>447.72</td>
<td>-0.3877</td>
<td>0.18</td>
<td>51.99</td>
</tr>
<tr>
<td>GER (Total)</td>
<td>1974–2011</td>
<td>-25.98%</td>
<td>4.34%</td>
<td>142.77</td>
<td>196.48</td>
<td>-0.4691</td>
<td>0.21</td>
<td>29.38</td>
</tr>
<tr>
<td>Motor</td>
<td>1974–2011</td>
<td>-12.31%</td>
<td>6.81%</td>
<td>142.77</td>
<td>196.48</td>
<td>-0.0406</td>
<td>0.11</td>
<td>15.96</td>
</tr>
<tr>
<td>Casualty</td>
<td>1974–2011</td>
<td>-51.34%</td>
<td>6.01%</td>
<td>142.77</td>
<td>196.48</td>
<td>+0.6008</td>
<td>0.34</td>
<td>48.69</td>
</tr>
<tr>
<td>Legal</td>
<td>1974–2011</td>
<td>-33.99%</td>
<td>7.86%</td>
<td>142.77</td>
<td>196.48</td>
<td>-1.1369</td>
<td>0.19</td>
<td>26.55</td>
</tr>
<tr>
<td>Fire</td>
<td>1974–2011</td>
<td>-24.23%</td>
<td>14.04%</td>
<td>142.77</td>
<td>196.48</td>
<td>-1.9923</td>
<td>0.12</td>
<td>16.74</td>
</tr>
<tr>
<td>Credit</td>
<td>1974–2011</td>
<td>-28.69%</td>
<td>21.42%</td>
<td>142.77</td>
<td>196.48</td>
<td>+0.1049</td>
<td>0.10</td>
<td>13.93</td>
</tr>
<tr>
<td>Household</td>
<td>1974–2011</td>
<td>-46.54%</td>
<td>6.54%</td>
<td>142.77</td>
<td>196.48</td>
<td>+1.2233</td>
<td>0.27</td>
<td>37.85</td>
</tr>
<tr>
<td>Homeowners</td>
<td>1974–2011</td>
<td>-30.32%</td>
<td>20.51%</td>
<td>142.77</td>
<td>196.48</td>
<td>-1.6111</td>
<td>0.11</td>
<td>15.08</td>
</tr>
</tbody>
</table>

Table 5: Consumption-Based Insurance Pricing with Second-Degree Expectation Dependence

This table illustrates the ability of the consumption-based model's SED modification to explain our insurance market data. It contains the annual risk premiums on insurance policies (column three) and the corresponding excess return volatilities (column four) for all countries and insurance business lines in our sample. In addition, it shows estimates of the average per capita consumption level (column five), the volatility of absolute consumption changes (column six), and the integrated SED of excess returns and consumption. The last two columns labeled $\lambda$ and $\gamma(2)$ display the Arrow-Pratt measure of CARA as well as the RRA coefficient. The latter has been derived assuming a perfect negative correlation between excess returns and absolute consumption changes. All figures are reported in real terms.

5 Conclusions

Motivated by the fact that insurance is the typical textbook example for an asset whose payoff negatively correlates with consumption, we fit the classical consumption-based model with CRRA utility to an international property-casualty market data set. In doing so, we are able to provide evidence for another asset pricing anomaly, which we dub the insurance premium puzzle. More specifically, due to the low volatility of consumption growth, the highly-negative empirically-observed excess returns on the representative agents’ policies can only be explained with absurdly large RRA coefficients. Those even exceed their counterparts implied by stock market data from the analyzed countries by far. We attribute this difference to the fact that agents are known to exhibit domain-specific risk aversion and continue our analysis of the insurance setting with a focus on model refinements, which promise to generate more reasonable RRA estimates. In particular, we implement the loss aversion and narrow framing approach brought forward by Barberis et al. (2001) as well as the SED framework of Dionne et al. (2015). Our results indicate that the former largely and the latter at least partially solves the insurance premium puzzle. However, a substantial limitation of these analyses is sampling uncertainty. The absence of more comprehensive premiums and claims data prevents us from performing additional robustness checks with regard to aggregation level, geographical scope, and time frame. This issue needs to be overcome by future research.
6 Appendix

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>( r^* )</th>
<th>( \sigma[r^*] )</th>
<th>( r_f )</th>
<th>( \sigma[r_f] )</th>
<th>( \Delta )</th>
<th>( \sigma[\Delta c] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUL</td>
<td>1970.1–1999.1</td>
<td>3.540%</td>
<td>22.699%</td>
<td>2.054%</td>
<td>2.528%</td>
<td>2.099%</td>
<td>2.056%</td>
</tr>
<tr>
<td>GER</td>
<td>1978.4–1997.4</td>
<td>9.838%</td>
<td>20.097%</td>
<td>3.219%</td>
<td>1.152%</td>
<td>1.681%</td>
<td>2.431%</td>
</tr>
<tr>
<td>ITA</td>
<td>1971.2–1998.2</td>
<td>3.168%</td>
<td>27.039%</td>
<td>2.371%</td>
<td>2.847%</td>
<td>2.200%</td>
<td>1.700%</td>
</tr>
<tr>
<td>JAP</td>
<td>1970.2–1999.1</td>
<td>4.715%</td>
<td>21.909%</td>
<td>1.388%</td>
<td>2.298%</td>
<td>2.305%</td>
<td>2.554%</td>
</tr>
<tr>
<td>NL</td>
<td>1977.2–1998.4</td>
<td>14.070%</td>
<td>15.645%</td>
<td>0.891%</td>
<td>1.746%</td>
<td>1.964%</td>
<td>1.073%</td>
</tr>
<tr>
<td>USA</td>
<td>1947.2–1998.4</td>
<td>8.085%</td>
<td>15.263%</td>
<td>0.891%</td>
<td>1.746%</td>
<td>1.964%</td>
<td>1.073%</td>
</tr>
</tbody>
</table>

Table 6: Equity Returns, Risk-Free Rates, and Consumption Growth as Employed by Campbell (2003)

This table shows the average annual log return of the equity market (column three) and risk-free asset (column seven) as well as the corresponding standard deviations (columns four and six) for all countries in our sample. In addition, the last two columns contain the average annual log growth rates of per capita consumption and the standard deviations of log consumption growth. The time period for each country has been chosen in line with Campbell (2003). All figures are reported in real terms.

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>( aE[r^*_t+1] )</th>
<th>( \sigma[r^*_t+1] )</th>
<th>( \sigma[m_{t+1}] )</th>
<th>( \sigma[\Delta c_{t+1}] )</th>
<th>( \rho[\Delta c_{t+1}, r^*_t+1] )</th>
<th>( \gamma(1) )</th>
<th>( \gamma(2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUL</td>
<td>1970.1–1998.4</td>
<td>3.885%</td>
<td>22.403%</td>
<td>17.342%</td>
<td>2.059%</td>
<td>+0.1439</td>
<td>58.51</td>
<td>8.42</td>
</tr>
<tr>
<td>GER</td>
<td>1978.4–1997.3</td>
<td>8.669%</td>
<td>21.909%</td>
<td>42.922%</td>
<td>2.447%</td>
<td>+0.0293</td>
<td>59.60</td>
<td>17.54</td>
</tr>
<tr>
<td>ITA</td>
<td>1971.2–1998.1</td>
<td>4.687%</td>
<td>27.068%</td>
<td>17.315%</td>
<td>1.665%</td>
<td>–0.0056</td>
<td>&lt; 0</td>
<td>10.40</td>
</tr>
<tr>
<td>JAP</td>
<td>1970.2–1998.4</td>
<td>5.098%</td>
<td>21.497%</td>
<td>23.716%</td>
<td>2.561%</td>
<td>+0.1116</td>
<td>82.62</td>
<td>9.26</td>
</tr>
<tr>
<td>NL</td>
<td>1977.2–1998.3</td>
<td>11.421%</td>
<td>16.901%</td>
<td>67.576%</td>
<td>2.510%</td>
<td>+0.0317</td>
<td>84.98</td>
<td>26.92</td>
</tr>
<tr>
<td>USA</td>
<td>1947.2–1998.4</td>
<td>8.074%</td>
<td>15.272%</td>
<td>52.867%</td>
<td>1.071%</td>
<td>+0.2050</td>
<td>240.67</td>
<td>49.34</td>
</tr>
</tbody>
</table>

Table 7: Evidence for the Equity Premium Puzzle as Provided by Campbell (2003)

This table illustrates the equity premium puzzle based on the joint lognormality assumption of Hansen and Singleton (1983). It contains the annual equity market risk premiums (column three), the corresponding excess return standard deviations (column four), and the resulting Hansen-Jagannathan bounds (column five) for all countries in our sample. In addition, it shows the volatilities of consumption growth (column six) and its correlations with excess returns (column seven). The last two columns labeled \( \gamma(1) \) and \( \gamma(2) \) display the RRA coefficient estimates for (a) the empirical correlation in column seven and (b) a correlation of one. The time period for each country has been chosen in line with Campbell (2003). All figures are reported in real terms.
References


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