ENHANCED ANNUITIES AND THE IMPACT OF INDIVIDUAL UNDERWRITING ON AN INSURER'S PROFIT SITUATION

GUDRUN HOERMANN
JOCHEN RUSS

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Gudrun Hoermann
Jochen Ruß

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ABSTRACT

We analyze the effect of enhanced annuities on an insurer engaging in individual underwriting. We use a frailty model for the heterogeneity of the insured population and model the individual underwriting by a random variable that positively correlates with the corresponding frailty factor. For a given annuity portfolio, we analyze the effect of the quality of the underwriting on the insurer’s profit/loss situation and the impact of adverse selection effects.

1. INTRODUCTION

In many countries, the United Kingdom and Germany among them, there are tax incentives that encourage owners of old age provision contracts to receive their benefits in the form of a lifelong annuity as opposed to a lump sum. In some places, there are state-subsidized or tax-sheltered product lines for which annuitization is compulsory. However, in a market where only so-called standard annuities are offered, such regulations result in significant disadvantages for insured persons whose life expectancy is below average at the time of annuitization.

* Gudrun Hoermann is with the University of St. Gallen, Institute of Insurance Economics, Kirchlistrasse 2, 9010 St. Gallen, Switzerland; Internet: www.ivw.unisg.ch, Email: gudrun.hoermann@unisg.ch.
Jochen Ruß is Managing Director of the Institute for Financial and Actuarial Science and a lecturer at the University of Ulm, Helmholtzstrasse 22, 89081 Ulm, Germany; Internet: www.ifa-ulm.de, Email: j.russ@ifa-ulm.de.

An earlier version of the paper was titled "Enhanced Annuities, Individual Underwriting, and Adverse Selection - A Solution for the Annuity Puzzle?".
With standard annuities, the annuity paid depends only on the amount of money that is annuitized, the insured’s age at the time of annuitization, and the insured’s gender. Thus, the value for money of a standard annuity is higher the longer the life expectancy of the insured. If certain tax incentives favor annuitization, a person with a reduced life expectancy has the choice only between an annuitization at “unfair” rates or a lump-sum benefit that triggers some sort of tax disadvantage. In product lines with compulsory annuitization, impaired people are, practically speaking, forced to annuitize at unfavorable rates, that is, the present value of the annuity benefits may be significantly lower than the amount to be annuitized.

This “unfair” situation could be avoided if so-called enhanced annuities were offered—products where the annuity paid is larger the lower the insured’s life expectancy. In other words, with enhanced annuities, annuity rates are adjusted to reflect the individual health status of an insured.

In this paper, we develop a model to specify the individual underwriting of enhanced annuities and, based on that model, quantitatively analyze the effect of individual underwriting, in particular of the underwriting quality, on an insurer’s profit/loss situation.

It is common practice in many term-life and disability insurance markets to offer so-called preferred life products, where the premium is lower for “good risks,” i.e., insureds with low mortality or morbidity rates. In the annuity business, impaired persons are obviously good risks from the insurer’s perspective. Therefore, enhanced annuities are sometimes also referred to as impaired annuities.

For immediate enhanced annuities, the relation of premium and annuity depends on the insured’s health at the time the contract is signed. For deferred annuities, however, the insurer needs to perform some kind of underwriting at the end of the deferment period. If the insured person does not submit to the underwriting proceeding, he or she would receive the annuity corresponding to the healthiest class of insureds.

\[1\] Cf., e.g., Ainslie (2000, p. 6), Hamdan/Rinke (1998, p. 5), and Jones/Richards (2004, p. 20).
The fact that enhanced annuities are still rare in many insurance markets could explain the so-called annuity puzzle. For example, Dushi and Webb (2004) found that only 10.2% of seniors in the United States have annuitized a portion of their wealth. Academics are surprised by this low figure because Brown et al. (2005), for example, showed that under usual assumptions, (partial) annuitization increases expected utility. However, this analysis assumes that future mortality rates are known and depend only on the insured’s age and gender. In other words, it is assumed that the value for money of annuities is essentially the same for all potential insureds. Yet, in reality, strong selection effects can be observed. Persons who elected to annuitize part of their wealth have significantly lower than average mortality rates, i.e., higher than average life expectancies. We can conclude that out of those individuals who receive a good value for money when purchasing an annuity, significantly more than 10.2% annuitize, whereas only a very small portion of people with low life expectancy do so. If enhanced annuities based on the insured’s individual health information were offered, everybody could get a “fair deal” when purchasing an annuity. In this situation, the purchase of annuities should increase and, at the same time, the degree of adverse selection in the annuity market should decrease.

The extant literature on enhanced annuities primarily concentrates on practical issues of the enhanced annuity market, mainly in the United Kingdom, and covers topics including this market’s development, size, or potential, different types of enhanced annuities, underwriting methods and challenges, tax considerations, distribution channels, and reinsurance. Ainslie (2000) provides a quantitative analysis of potential adverse selection effects on the standard annuity market by determining some critical size of the enhanced annuity market. He considers a hypothetical portfolio of males aged 65. The heterogeneity of their health is modeled using a normal distribution for the mortality. For different parameter combinations for this normal distribution he determines the portion of pensioners buying enhanced annuities (instead of standard annuities). Levantesi and Menzietti (2007) focus on the evaluation of biometric risk in enhanced annuity prod-

2 To the authors’ knowledge, in the German insurance market, e.g., there are only two enhanced annuity products (LV1871 “Extra-Rente” and “DSP-Vorzugsrente”).
4 According to Weinert (2006), in the third quarter of 2005 the market share by premium of enhanced annuities amounted to nearly 20% of the entire annuity market in the United Kingdom.
ucts including long-term care coverage. Jones and Richards (2004) discuss the risk of underwriting enhanced annuities, but do not perform quantitative analyses.

To date, there have been no attempts to develop a model that describes the individual underwriting of enhanced annuities, the quality of such underwriting, or that quantifies the effects of such underwriting on the insurer’s profit/loss situation. The impact of adverse selection resulting from competition induced by an enhanced annuities market is another topic that has received no investigation as of yet.

The aim of the present paper is to fill these gaps. We present quantitative analyses of the effect enhanced annuities have on an insurer engaging in individual underwriting. First, the heterogeneity of insured persons is specified in Section 2.1 by modeling the distribution of the degree of impairment within a population using a frailty model for individual mortality rates. In Section 2.2, we present our model for individual underwriting. The result of the underwriting is a stochastic frailty factor that correlates with the actual frailty factor of the insured person. The correlation coefficient is our measure of the quality of the underwriting. In Section 2.3, we detail the considered insurance product and the community of insureds, and in Section 2.4, we explain how adverse selection effects can be analyzed within our model framework. Numerical results derived using Monte Carlo methods are presented in Section 3. After specifying the parameters for our analyses in Section 3.1, results for three model companies are given in Section 3.2. By calculating the empirical profit distribution of each of the three companies, we analyze the effect of enhanced annuities and of the quality of the underwriting on the insurer’s profit/loss situation. We also assess the impact of adverse selection effects on companies who do not offer enhanced annuities when other insurers in the market do. We summarize our results in Section 4.

\[^{6}\text{Cf. Jones/Richards (2004, p. 20).}\]
2. Model Framework

2.1. Individual Mortality Rates

We define \( (x) \) and \( (y) \) as a male or female person age \( x \in \mathbb{N}_0 \) or \( y \in \mathbb{N}_0 \), respectively. In the following, however, we consider only a male insured. Age at death is modeled by the random variable \( X \geq 0 \). The random variable \( K(x) = X - x, \) \( X > x \) describes the remaining lifetime of \( (x) \). Its distribution function \( k_x q_x \) at a point \( k \in \mathbb{N}_0 \) is denoted by

\[
k_x q_x = F_{k(x)} (k) = P(K(x) \leq k | X > x) = 1 - k p_x,
\]

where \( k p_x \) is the \( k \)-year survival probability of \( (x) \).

To specify heterogeneity in the insurance portfolio we use a frailty model, i.e., an individual factor\(^7\) (also referred to as a mortality multiplier) by which the actual mortality of each person differs from a given standard mortality table.\(^8\) Probabilities given in the standard mortality table will be denoted with a prime (’) mark.\(^9\) Thus, the one-year individual mortality rate for a given insured with mortality multiplier \( d \) is given by

\[
q_x = \begin{cases} 
    d \cdot q'_x, & \text{if } d \cdot q'_x \leq 1 \\
    1, & \text{otherwise}
\end{cases} \quad \text{with } x \in \{0, \ldots, \omega\}.
\]

The individual mortality rates \( q_x \) determine the distribution of the annuity payments, i.e., the insurer’s liabilities.

The parameter \( d \) describes the individual’s state of health as follows:

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\(^7\) Such a factor (often analogously applied to the continuous force of mortality) is usually called a “frailty factor.” Cf. Vaupel et al. (1979, p. 440) and Jones (1998, p. 81).

\(^8\) Cf., e.g., Pitacco (2003, p. 13). This is a reasonable modeling assumption, since many impairments generally increase mortality over a longer period of time, as, e.g., cardiovascular disease, which, according to WHO Europe (2004, p. 12) and PAN American Health Organization (2006, p. 7), is the most common cause of death. However, in practical implementations, one might prefer to use a different “shape” of extra mortality for different impairments, e.g., so-called flat extras, that is, an additive extra mortality over several years only.

\(^9\) Cf., e.g., Haberman (1982, p. 211).
For $0 < d < 1$, we have $q_x < q'_x$. The individual has an above average life expectancy.

For $d = 1$, we have $q_x = q'_x$. The individual’s life expectancy matches that given in the standard mortality table.

For $1 < d$, we have $q_x > q'_x$. The individual is impaired.

For $d < 1$, we let $q_\omega := 1$, where $\omega$ is the so-called limiting age of the standard mortality table—the age that, according to that table, will not be exceeded. Thus, the remaining probability mass is assigned to the mortality rate of the last year, and we obtain a truncated distribution.

If we randomly select an individual from some population, the corresponding $d$ is modeled as a realization of a random variable $D$.\footnote{Cf. Czernicki et al. (2003). In the continuous context, the model framework of Jones (1998, pp. 80–83) and Pitacco (2003, p. 14) traces back to Vaupel et al. (1979, p. 440).}

Slightly simplified, the distribution $F_D$ of $D$ describes which portion of the general population is in which state of health. More precisely, it specifies the portion of individuals whose mortality is lower or higher than a certain percentage of standard mortality. For this distribution $F_D$, we make the following assumptions:\footnote{Cf. Ainslie (2000, p. 44), Butt/Haberman (2002, p. 5), Czernicki et al. (2003, p. 5), Hougaard (1984, pp. 75, 79), and Pitacco (2003, p. 14).}

- The distribution $F_D$ is continuous, making possible very fine nuances in state of health and remaining life expectancy.
- Its domain is positive ($d \geq 0$).
- The probability density function is “flat” at zero and equal to zero for $d = 0$, since mortality rates near zero are unrealistic.
- The distribution is right-skewed, i.e., very high values of $d$ can occur; however, they are bounded below by zero.
- Across the population, the expected value $E(D) = 1$, i.e., the standard mortality table describes an “average individual.”

### 2.2. Individual Underwriting

The purpose of individual underwriting is to assign each insured an estimate $\hat{d}$ for the frailty factor $d$,\footnote{This is a “numerical rating system” (cf. Pitacco, 2003, p. 13).} i.e., to determine the so-called pricing mortality rates $\hat{q}_x$ used by the insurer for premium calculation:
\[ \hat{q}_x = \begin{cases} \hat{d} \cdot q_x', & \hat{d} \cdot q_x' \leq 1 \\ 1, & \text{otherwise} \end{cases} \quad \text{with } x \in \{0, \ldots, \omega\}. \]

Again, we let \( \hat{q}_\omega := 1 \) for \( \hat{d} < 1 \).

We model the parameter \( \hat{d} \) as a realization of a random variable \( \hat{D} \) and assume \( D \) and \( \hat{D} \) to be identically distributed: \( D \sim \hat{D} \). This means that there is no systematic underwriting error, i.e., the mortality estimation of the underwriting across the whole population is not only correct on average but also with respect to the portion of people identified as belonging to a certain impairment range. Note that because we wish to focus on the pure effect of introducing individual underwriting, we do not consider any safety loadings.\(^{13}\) Finally, the random variables \( D \) and \( \hat{D} \) are expected to be positively correlated with a correlation coefficient \( 0 \leq \rho_{D,\hat{D}} \leq 1 \). This correlation coefficient is a measure of the quality of the individual underwriting: the larger \( \rho_{D,\hat{D}} \), the smaller the mean deviation between \( d \) and \( \hat{d} \). The condition \( \rho_{D,\hat{D}} = 1 \) implies \( d = \hat{d} \), i.e., hypothetically “perfect” underwriting.

Note that this individual underwriting model is continuous in the sense that all values for \( \hat{d} \) are possible, and symmetric in the sense that insureds with increased and reduced life expectancy are treated analogously. In practice, for the sake of tractability, one might prefer a discrete model, where, e.g., only integer multiples of 0.25 are admissible values for \( \hat{d} \). Alternatively or additionally, an asymmetric model might be favored, where all insureds with above-average life expectancy are clustered in one group and health-adjusted annuity amounts are paid to impaired persons only.

### 2.3. The Considered Annuity Contract and the Portfolio of Insureds

To simplify notation, in what follows we concentrate on a simple immediate lifelong annuity where the annual annuity paid to the insured is calculated from the single premium using pricing mortality rates. We disregard any fees or charges. Our findings also apply to a deferred annuity where the underwriting takes place at the end of the deferment period.

\(^{13}\) Safety loadings can, however, be considered in the model, e.g., by assuming that the pricing rates are the mortality rates that result from the underwriting multiplied by a certain factor.
We distinguish between three model companies. Company A does not engage in individual underwriting. The annuity is always calculated using the standard mortality table (i.e., the mortality rates \( q_x' \)). Company B does engage in individual underwriting as described in Section 2.2, the quality of which is characterized by the correlation coefficient \( \rho_{D,\hat{D}} \). For the sake of comparison, we also consider Company C, an insurer with “perfect” individual underwriting, meaning that \( d \) and \( \hat{d} \) coincide and, thus, so do actual and estimated mortality rates.\(^{14}\)

Below, formulas for calculating benefits and analyzing the insurer’s profit and loss are derived for all three companies. For an immediate lifelong annuity paying annual amount \( A_{[i]} \) to a male insured aged \( x \), the present value of future benefits\(^{15}\) is given by:

\[
B_{[i]} = A_{[i]} \sum_{k=0}^{k(x)} v^k, \quad [i] = A, B \text{ or } C,
\]

with \( v = \frac{1}{1+r} \), where \( r \) denotes the guaranteed rate of return,\(^{16}\) and

\[
K(x) \sim F_{k(x)}(k) = q_x = 1 - p_x = 1 - \prod_{l=0}^{k-1} (1 - D \cdot q_{x+l}'), \quad k \in \mathbb{N}_0, \text{ (cf. Equation (1)).}
\]

For a single premium \( P \), Company A would pay an annual annuity of

\[
A_A = \frac{P}{\sum_{k=0}^{q_x} v^k p_x'} \cdot \frac{P}{\sum_{k=0}^{q_x} v^k \prod_{l=0}^{k-1} (1 - 1 \cdot q_{x+l}')} \]

and this annuity will be the same for all insured persons of equal age and gender because it is based on the standard mortality rates only.

Company B determines the annuity amount \( A_B \) using the estimated mortality:

\(^{14}\) This is, obviously, exactly what happens in Company B when \( \rho_{D,\hat{D}} = 1 \).


\[ A_B = \frac{P}{\sum_{k=0}^{q-2} v^k \hat{D}_x} = \frac{P}{\sum_{k=0}^{q-2} v^k \prod_{l=0}^{k-1} (1 - \hat{D} \cdot q_{x+l}')} \].

Therefore, \( A_B \) is a random variable.

Finally, for Company C, we obtain an annuity amount of

\[ A_C = \frac{P}{\sum_{k=0}^{q-2} v^k P_c} = \frac{P}{\sum_{k=0}^{q-2} v^k \prod_{l=0}^{k-1} (1 - D \cdot q_{x+l}')}. \]

We analyze a portfolio of \( n \) insureds, \( i, \ i=1,\ldots,n \), all of the same gender and age, who are randomly selected from some general population, and implicitly assume that the standard mortality table is appropriate for an average individual of this population. We denote all figures that refer to a specific insured or contract with a corresponding index \( i \): \( D_i \) denotes the mortality multiplier of person \( i \), and \( \hat{D}_i \) denotes the mortality multiplier estimated by the insurer’s underwriting for that person, etc. The present value of future profits of person \( i \)’s policy in the observed portfolio is given by the difference between the single premium and the present value of future benefits:

\[ \Pi_{[\cdot]} = P - B_{[\cdot]}, \quad [\cdot] = A, B \text{ or } C, \ i=1,\ldots,n. \]

Thus, the cumulated present value of future profits of Company A, B, or C (\( \Pi(A), \Pi(B) \) and \( \Pi(C) \), respectively) is

\[ \Pi(i) = \sum_{j=1}^n \Pi_{[j]} \].

We denote the corresponding distribution function by \( F_{\Pi(i)} \). In Section 3, properties of this distribution are analyzed using Monte Carlo simulation methods.

2.4. The Impact of Adverse Selection

If some market players engaged in rate classification by offering enhanced annuities, then, in theory, impaired persons (low risks) would prefer such a product
over a standard annuity. Consequently, no impaired person would buy a contract from an insurer offering only standard annuities. Thus, the nondiscriminating Company A would end up with a portfolio of insureds with increased average life expectancy (high risks) and suffer loss. Realistically, however, because of market imperfections, only a portion of impaired persons would purchase enhanced annuity products. We assume that merely $s\%$ of all persons with a mortality multiplier $d$ exceeding a threshold value $d^*$ (the selection barrier) avoid Company A, the standard insurer. The resulting modified distribution of the frailty factors $\tilde{D}$ in Company A’s portfolio is denoted by $F_{\tilde{D}}$, and the quantities calculated above change as follows:

$$B_{A_{\text{sel}}} = A_A \cdot \sum_{k=0}^{\bar{K}(x)} r^k \quad \text{with} \quad \bar{K}(x) \sim F_{\bar{k}(x)},$$

where

$$F_{\bar{k}(x)}(k) = \bar{q}_x k = 1 - \bar{p}_x k = 1 - \prod_{i=0}^{k-1} \left(1 - \tilde{D} \cdot q_{x+i}^* \right), \quad k \in \mathbb{N}_0,$$

and

$$\Pi(A_{\text{sel}}) = \sum_{i=1}^{n} \Pi A_{i,j} \quad \text{with} \quad \Pi A_{i,j} = P - B_{A_{i,j}}, \quad i = 1, \ldots, n.$$

Again, the corresponding distribution $F_{\Pi(A_{\text{sel}})}$ can be approximated using simulation techniques (cf. Appendix A).

3. Numerical Analyses

3.1. Specification of Parameters and Simulation Details

We use the DAV2004R table as the standard mortality table throughout our analyses.\cite{DAV2004R}

\cite{Akerlof1970}

\cite{DAV2005}

\cite{DAV2004R}

\cite{DAV2005}
We are not aware of any data that can be linked to the distribution $F_D$ in the population, especially with regard to higher age groups. Furthermore, based on information from direct insurers regarding the proportion of different insurance ratings, at the most rough inferences could be made as to the health distribution of younger applicants, whereby an assumption for senior citizens would have to be made. Therefore, in our analyses, we choose a distribution, which has – for a suitable choice of parameters – the characteristics listed in Section 2.1, and which is commonly used in the literature to describe the distribution of the mortality multiplier in the general population. We let $D$ follow a gamma-distribution, $D \sim \Gamma(\alpha, \beta, \gamma)$. Density function, expected value, and variance are then given by:

$$f_{\alpha, \beta, \gamma}(d) = \frac{1}{\Gamma(\alpha)\beta^\alpha(d-\gamma)^{\alpha-1}}e^{-\frac{d-\gamma}{\beta}}, \quad E(D) = \alpha \beta + \gamma, \quad \text{and} \quad \text{Var}(D) = \alpha \beta^2$$

for $d \geq \gamma, \gamma \in \mathbb{R}, \alpha, \beta > 0$.

The fact that certain accidents are inevitable, and thus mortality rates close to zero are unrealistic, supports a positive third parameter $\gamma$. Taking into account the properties mentioned in Section 2.1, the following results are based on the parameter combination $\alpha = 2$, $\beta = \frac{1}{4}$, and $\gamma = \frac{1}{2}$.

Again, please remember that our distribution, as well as its parameterization, is not based on “real-life” epidemiological or medical data but is, instead, motivated by the desired properties listed above. Therefore, we considered a variety of different reasonable parameter values, i.e., alternative parameter values, that also lead to a distribution fulfilling the requirements demanded by Section 2.1; all the alternatives produced similar outcomes. In Section 3.2, we comment on stability and differences.

We look at an insurance portfolio consisting of 1,000 65-year-old male insureds (cf. Table 1). The premium paid by each insured is €100,000, and we use the current guaranteed interest rate for German annuity insurance products of 2.25%.

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We also assume that the insurer always earns exactly this rate on the assets backing the annuity, i.e., any profit or loss considered stems only from a deviation of the actual from the based on pricing rates expected number of deaths. Such a deviation can occur due to pure random effects or due to differences between actual individual and expected mortality. For Company C, of course, only random effects are observed since in this company pricing mortality rates and actual rates coincide.

Unless stated otherwise, the parameters have the values set out in Table 1.

**Table 1**

<table>
<thead>
<tr>
<th>Specification of simulation parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Age (and gender)</td>
</tr>
<tr>
<td>Guaranteed interest rate</td>
</tr>
<tr>
<td>Single premium</td>
</tr>
<tr>
<td>Number of insureds</td>
</tr>
<tr>
<td>Number of simulations</td>
</tr>
<tr>
<td>Selection barrier</td>
</tr>
<tr>
<td>Correlation coefficient (Company B only)</td>
</tr>
<tr>
<td>Selection intensity</td>
</tr>
</tbody>
</table>

We use Monte Carlo methods to analyze the distribution of the insurer’s profit. For each insured, we need to generate three random numbers: $d$, which specifies the individual mortality (i.e., the probability distribution of the remaining lifetime and thus the benefits); $\hat{d}$, which is the mortality multiplier resulting from individual underwriting that determines the pricing rate and, therefore, the annuity amount; and $u$, which specifies time of death.

The random numbers $d$, $\hat{d}$, and $u$ are realizations of the random variables $D$, $\hat{D}$, and $U$, where the first two are $F_D$-distributed and correlated with $\rho_{D,\hat{D}}$ and the third follows a continuous uniform distribution $U(0,1)$. From $u$, the year of death $\kappa$ is calculated as the first year in which the value of the probability distribution of the remaining lifetime exceeds $u$, i.e.

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\[ \kappa = \min \left\{ k \in \{1, \ldots, \omega - x + 1\} : k \cdot q_x > u \right\}. \]

From \( \kappa \) for each insured, realizations \( b_i \) of the benefits \( B_i \) can be calculated for all three model companies \( [\cdot] = A, B, \) or \( C \), respectively.

To derive the empirical distribution of future profits in a portfolio, we conduct \( m \) simulations \( j = 1, \ldots, m \). In each simulation, the random numbers \( (d_{i,j}, \ldots, d_{n,j}), (\hat{d}_{i,j}, \ldots, \hat{d}_{n,j}), \) and \( (u_{i,j}, \ldots, u_{n,j}) \) are generated and the annuity amount, the duration of the annuity payment, and thus the insurer’s profit can be calculated per policy and cumulated over all insured persons \( i = 1, \ldots, n \). We denote the realizations of the insurer’s profit for policy \( i \) in simulation run \( j \) by

\[
\pi_{[\cdot],i,j} = P - b_{[\cdot],i,j}, \quad i = 1, \ldots, n; \quad j = 1, \ldots, m; \quad [\cdot] = A, B, C,
\]

and the cumulated profit over all policies in simulation run \( j \) by

\[
\pi(\cdot)_j = \sum_{i=1}^{n} \pi_{[\cdot],i,j}, \quad j = 1, \ldots, m \quad \text{for } [\cdot] \text{ or } (\cdot) \text{ equal to A, B, C and } A_{Sel}.
\]

The Monte Carlo estimate for the distribution of \( \Pi(\cdot) \) is then given by

\[
\hat{F}_{\Pi(\cdot)}(\nu) = \hat{F}(\Pi(\cdot) \leq \nu) \approx \frac{1}{m} \sum_{j=1}^{m} I_{\pi(\cdot,j) \leq \nu},
\]

where \( I \) is the indicator function. In the following section, we take a closer look at this distribution and at certain of its functions in our analysis of the effect of enhanced annuities on the insurer’s risk profile.

### 3.2. Results

We first examine a simulated distribution of the insurance companies’ profit/loss, with special attention to expected value, standard deviation, and specific quantiles, which allows us to draw conclusions about the impact of enhanced annuities on insurer’s profit/loss situation and risk structure. In particular, we can analyze the effect of the quality of the underwriting. We compare an insurer that offers enhanced annuities with a standard insurer that does not engage in individual underwriting. Adverse selection effects are also investigated.
We “calibrate” our model in such a way that, allowing for heterogeneity in the insurance portfolio as implied by the gamma-distribution for the frailty factors introduced above, the expected profit for Company A is zero (cf. Appendix B). We then calculate the expected profit for Company B as a function of the correlation coefficient $\rho_{D,\hat{D}}$. The result is displayed in Figure 1, where the expected profits of Companies A and C, which do not depend on $\rho_{D,\hat{D}}$, are displayed as a straight line.

**Figure 1**
Expected value of future profits as a function of the correlation coefficient

Surprisingly, even though the pricing of both Company A and Company B is correct on average, Company B’s practice of taking into account the actual mortality distribution has a substantial effect on its profit. Even for a rather poor quality of the underwriting, i.e., a correlation coefficient of zero, Company B’s expected profit clearly exceeds the expected profit of the standard insurer, Company A. When Company B improves the quality of its underwriting, the expected profit roughly doubles from slightly more than 0.8% of the premium volume (for $\rho_{D,\hat{D}} = 0$) to somewhat less than 1.6% (for $\rho_{D,\hat{D}} = 1$), which by definition coincides with expected profit of Company C. In other words, even under the quite weak assumption that the insurer is able to assess the mortality distribution (“on average” over the population) correctly (i.e., $D \sim \hat{D}$), individual underwriting has
a beneficial effect on the insurer’s expected profit. This positive effect and thereby the profit increases with the quality of the underwriting.

The stronger the right tail of the underlying mortality distribution (e.g., $\Gamma(1.25, 0.4, 0.5)$), the greater the benefit potential of the insurer offering enhanced annuities. Not only is the initial difference between the expected profit of Company A and Company B higher, the increase in the correlation coefficient is stronger.

We now investigate whether this increase in expected profit will have to be “paid for” with an increase in the insurer’s risk. We find that for low values of $\rho_{D,\hat{D}}$, this is indeed the case (cf. Figure 2): the standard deviation of Company B’s profit starts at 1.04% of the premium volume for $\rho_{D,\hat{D}} = 0$, exceeding the value of Company A (1.01%; black line). However, for increasing $\rho_{D,\hat{D}}$, the standard deviation decreases. At a correlation of 0.4, the standard deviation of Company B falls below that of Company A and approaches the level of Company C (at about 0.96%) as $\rho_{D,\hat{D}}$ approaches 1. Hence, with increasing quality of individual underwriting, expected profit increases and volatility decreases. For a correlation of 0.4 or higher, Company B has both higher expected profit and lower volatility of the profit than does Company A.

**Figure 2**

Standard deviation of future profits as a function of the correlation coefficient

![Figure 2](image-url)
The intersection at the value 0.4 is stable, i.e., the standard deviation of Company B’s future profits exceeds that of Company A until a correlation coefficient of about 0.4 for all considered mortality distributions. The parameterization of the mortality distribution has only marginal impact on the volatility of future profits. A stronger right tail of the mortality distribution leads to a slightly wider spread of possible standard deviation values (from Company B with a correlation coefficient of zero to Company C).

Figure 3 illustrates the empirical distribution of the insurer’s profit/loss corresponding to the results displayed above. From the figure, we see that as we move from no underwriting (Company A) to imperfect underwriting (Company B (here with $\rho_{D,D} = 0.4$)) to perfect underwriting (Company C), the distributions move to the right (i.e., expected profit increases) and become denser (i.e., volatility decreases).

**FIGURE 3**
Probability density function of future profits for 1,000 insured persons

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Different parametrizations of the gamma-distribution that fulfil the requirements demanded in Section 2.1 are, e.g., $\Gamma(1.25, 0.4, 0.5), \Gamma(3, 0.2, 0.4), \Gamma(2, 0.35, 0.3), \Gamma(8, 0.125, 0), \Gamma(4, 0.15, 0.4)$, or $\Gamma(1.5, 0.4, 0.4)$. 
The probability of a positive profit increases from 50% for Company A\(^{24}\) to over 87% for Company B (\(\rho_{D,D} = 0.4\)) to 95% for Company C. Even with a correlation coefficient of zero, the probability of a positive profit for Company B increases to 79%. There is a 5% probability that Company A’s losses will exceed 1.7% of its premium volume. For Company B with \(\rho_{D,D} = 0.4\), this 5%-quantile is reduced to 0.5%, and for Company C, it only amounts to 0.0% of the total premium income, i.e. the loss probability is less than 5%.

We conducted sensitivity analyses with respect to the portfolio size. When we decreased the number of insureds from 1,000 to 100, the risk of random fluctuations increased due to less diversification. Figure 4 shows the resulting distribution. In comparison with Figure 3, we can see that, as expected, the range of random fluctuations roughly triples if we reduce the portfolio size by factor of 10. Additionally, the densities lie closer to each other, i.e., the rate classification effect is less distinctive. The expected value of future profits as a percentage of premium volume is stable.

**Figure 4**
Probability density function of future profits for 100 insured persons

\(^{24}\) These values would be even higher for all three companies if we considered safety loadings of any kind.
For Company C, the probability of positive profit is reduced from 95% to 69% for the smaller portfolio. It is decreased from 87% to 63% for Company B if $\rho_{D,0} = 0.4$; from 79% to 59% if $\rho_{D,0} = 0$. For Company A, the 5% quantile described above is raised to 5.2%, for Company B ($\rho_{D,0} = 0.4$) it is increased from 0.5% to 4.1%, and for Company C it reaches 3.4% of the premium volume (compared to 0.0% in the portfolio with 1,000 insureds). Altogether, we observe strong diversification effects; however, the advantages of individual underwriting persist.

If we instead double the number of insureds (i.e., 2,000 instead of 1,000), the variance and thus the risk of random fluctuation is reduced. Again, the expected value of future profits is always the same percentage of the total premium income.

Finally, Figure 5 illustrates the adverse selection effects discussed in Section 2.4. To assess the impact of adverse selection, we assume that $s\%$ of all insured persons with a mortality multiplier exceeding a threshold of $d^* = 1.25$, $d^* = 1.5$, or $d^* = 1.75$, respectively, prefer an insurer that offers individual underwriting over Company A, which does not. Under our distribution assumption for the frailty factor in the population, about 20%, 9%, and 4% of the general population have a mortality multiplier above the selection barrier $d^* = 1.25$, $d^* = 1.5$, and $d^* = 1.75$, respectively.

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25 See Appendix A for a description of how this is taken into account in our Monte Carlo algorithm.
Figure 5 shows the expected profit of Company A as a function of the selection intensity $s$. The expected profit decreases as $s$ increases up to an expected loss of about 1.7% for $s = 1$ and $d^* = 1.5$. The fact that for a lower selection barrier more insureds are affected (in the sense that their mortality multiplier $d$ lies above $d^*$ and that they avoid the nondiscriminating insurer) leads to a higher expected loss for the standard insurer, Company A. Analogously, the stronger the right tail of the mortality distribution, the greater the number of insureds affected, and the stronger the negative impact in terms of potential losses for Company A.

In short, our analyses show that standard insurers will suffer significant disadvantages if competitors begin to offer enhanced annuities, even if only a small percentage of insureds are aware of the new product.

4. Summary

In this paper, we analyzed the impact of enhanced annuities on an insurer’s risk profile. A heterogeneous portfolio (with regard to the insureds’ mortality) of enhanced annuities was considered using a frailty model. We modeled the result of the insurer’s underwriting by means of a distribution for pricing mortality rates that correlates with the distribution of individual mortalities. By employing
Monte Carlo simulation techniques, we determined the future cash flows of an annuity portfolio and compared the results for insurers engaging or not in individual underwriting of different qualities. Additionally, selection effects were considered.

Our results show that offering enhanced annuities would be beneficial for both the insurer and the insured. If the insurer can correctly assess the distribution of excess and lower mortality in the population, the practice of individual underwriting will always increase the company’s profitability. In particular, the expected profit clearly exceeds the expected profit of an insurer that is just right on average (e.g., by using an on average correct mortality table) but does not appraise the actual mortality distribution in the population. The higher the quality of the individual underwriting, the higher the profit. Higher quality underwriting along with a larger number of insureds has the additional beneficial effect of decreasing the volatility of profits. Or, put a different way, not very many insureds accompanied by poor underwriting quality (below a correlation coefficient of 0.4) will increase the risk of random fluctuations in profit. We quantified the negative effect of adverse selection on insurers that do not offer enhanced annuities in a market where competitors do. The impact is significant, even if only a small percentage of impaired insureds prefer enhanced annuities over standard annuities. We thus conclude that offering enhanced annuities would have a beneficial effect on an insurer’s risk profile.

On the other hand, enhanced annuities will pay significantly increased benefits to impaired persons. Consequently, the value for money is the same for all insureds and, unlike traditional annuity products, enhanced annuities will be attractive to persons with a below-average life expectancy. Hence, enhanced annuities would increase the acceptance of annuities in the general population, which can be shown to be beneficial for the annuitant under certain assumptions.

If several insurers started offering enhanced annuities, the ensuing competition would result in rate discriminating annuity products, and could finally attract persons who otherwise would not buy annuities. Besides, with individual risk assessment and individual underwriting the extent of adverse selection in the annuity market could be reduced.
**APPENDIX A. GENERATION OF THE DISTRIBUTION OF** $\Pi(A_{sel})$

As a consequence of the selection effect described in Section 2.4, the frailty factors in Company A’s portfolio no longer have a gamma-distribution. This situation is dealt with by using the following algorithm to generate a random number as a realization of the mortality multiplier of an insured person:

i. Generate an $F_{\tilde{d}}$-distributed random number $\tilde{d}$.

ii. If $\tilde{d} \leq d^*$, accept $\tilde{d}$. (Interpretation: Persons with lower mortality continue to purchase insurance from Company A.)

iii. Otherwise, generate an additional $U(0,1)$-distributed random number $z$.

iv. If $z \leq 1 - s$, accept $\tilde{d}$. (Interpretation: $1 - s\%$ of those with increased mortality continue to purchase insurance from Company A).

v. Otherwise, do not accept $\tilde{d}$ and go back to i. (Interpretation: $s\%$ of the insureds choose an insurer who offers individual underwriting and therefore do not purchase insurance from Company A.)

By producing a random frailty factor for each insured as described above, we create a portfolio where the distribution of mortality multipliers reflects the selection effects mentioned in the paper. The rest of the analysis is similar to the case without selection.

**APPENDIX B. ADJUSTMENT OF PROFIT FOR COMPANY A**

As a basis for comparison, we calibrate our model such that the expected value of Company A’s future profits is zero. Prior to this calibration, Company A’s expected gain is given by

$$\Pi(A) = P - A_k \sum_{k=0}^{m-x} v^k p_x = P - B_A$$

with expected value
\[ E(\Pi(A)) = P - E(B_x) = P - E \left( \sum_{k=0}^{\infty} \frac{P}{v^k \prod_{l=0}^{k-1} (1 - q_{x+k}')} \cdot \sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - D \cdot q_{x+l}') \right) \]

\[ := P - \frac{P}{f(1)} \cdot E\left(f(D)\right) \leq 0 \]

This value is \( \leq 0 \) since \( f(d) = \sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - d \cdot q_{x+l}') \), \( d \geq 0 \) is a convex function as \( (1 - d \cdot q_{x+l}') \geq 0 \ \forall d, l \).

Thus, Jensen’s inequality\(^{26}\) yields \( E\left(f(D)\right) \geq f\left(E(D)\right) = f(1) \).

Therefore, we determine a factor \( \mu \) to modify the pricing mortality rates such that

\[ E(\Pi(A)) = P - E \left( \frac{P}{\sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - q_{x+k}') \cdot \sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - D \cdot q_{x+l}')} \right) = 0. \]

For the sake of comparability, we assume that Companies B and C also use these modified rates in their annuity calculations, i.e.

\[ A_B = \sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - D \cdot \mu \cdot q_{x+k}'), \]

\[ A_C = \sum_{k=0}^{\infty} v^k \prod_{l=0}^{k-1} (1 - D \cdot \mu \cdot q_{x+k}'). \]

REFERENCES


