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A DECISION-THEORETIC FOUNDATION FOR TWO-PARAMETER PERFORMANCE MEASURES

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A decision-theoretic foundation for two-parameter performance measures

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Abstract

In this paper we prove that partial-moments-based performance measures (e.g., Omega, Kappa, Upside-potential ratio, Sortino-Satchell ratio, Farinelli-Tibiletti ratio), value-at-risk-based performance measures (e.g., VaR ratio, CVaR ratio, Rachev ratio, Generalized Rachev ratio), and other admissible performance measures are a strictly increasing function in the Sharpe ratio. The theoretical basis of this result is the locations-scale- and two other plausible and mild conditions. Our result provides a decision-theoretic foundation for all these frequently used performance measures. Moreover, it explains the empirical finding that all these measures typically lead to very similar rankings.

Keywords: Asset management; Performance measurement; Sharpe ratio; Location and scale condition; Risk and reward measurement

JEL classification: D81; G10; G11; G23; G29

1. Introduction

The most popular two-parameter performance measure is the Sharpe ratio (see, e.g., Alexander and Baptista, 2010; Darolles and Gourieroux, 2010; Ding et al., 2009; Eling and Faust, 2010; Szakmary et al., 2010; Serban, 2010). The least restrictive sufficient condition for expected utility to imply Sharpe ratio rankings is the location and scale (LS) property (see Sinn, 1983; Meyer, 1987). This property requires that the random returns from the investment funds in the choice set differ from one another only by location and scale parameters. Schuhmacher and Eling (2011) argue that the LS property is

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also sufficient for expected utility to imply drawdown-based performance measure rankings. Hence, the same conditions that provide an expected utility foundation for the Sharpe ratio also provide a foundation for drawdown-based performance measures. Their result shows that drawdown-based performance measures will lead to the same ranking as the Sharpe ratio if the random returns can be described by the LS property.

Against this background, the question arises as to whether the LS property is sufficient to ensure consistency between expected utility and other alternative two-parameter performance measures that differ from the Sharpe ratio by the risk and reward measure employed. To answer this question, we argue that any admissible risk measure should satisfy two minimal conditions: first, it should satisfy positive homogeneity, which is an important axiom in most axiomatic systems (see Kijima and Ohnishi, 1993; Pedersen and Satchell, 1998; Artzner et al., 1999; Rockafellar et al., 2006); second, adding a positive constant to an investment fund's random rate of return should not increase the investment fund's risk measure. This condition contains the mutually incompatible axioms of translation invariance (see Artzner et al., 1999) and shift invariance (see Kijima and Ohnishi, 1993; Pedersen and Satchell, 1998; Rockafellar et al., 2006) as special cases.

Similarly, any admissible reward measure should also satisfy two minimal conditions: positive homogeneity and that adding a positive constant to an investment fund's random rate of return does increase the investment fund's reward. The main result is that under the location and scale property, any admissible performance measure is a strictly increasing function in the Sharpe ratio. An admissible performance measure is a two-parameter measure using admissible risk and reward measures.

This finding has two important implications. First, it provides a decision-theoretic foundation for lower-partial-moments, value-at-risk, and other admissible performance measures that differ from the Sharpe ratio by the risk and reward measure employed. Second, since the normal, the extreme value, and many other distributions commonly used in finance satisfy the LS property (see Schuhmacher and Eling, 2011), the finding may explain the empirical observation that rank correlations between the

Sharpe ratio and alternative performance measures are extremely high (see Eling and Schuhmacher, 2007).

This paper is structured as follows. In Section 2 we present our main result. In Section 3 we illustrate the main result using well-known risk and reward measures. We conclude in Section 4.

2. The Main Result

Let X_i (i=1, …, n) denote a random variable that represents investment fund i's excess rate of return (over the risk-free rate) at the end of a period, e.g., a month. An investor who invests the proportion p in investment fund i and the remaining proportion 1–p in the risk-free asset realizes an excess rate of return of $p \cdot X_i$. The expected excess rate of return is $\mu(X_i)$, the standard deviation $\sigma(X_i)$, and the Sharpe ratio is defined as

$$S(X_i) := \mu(X_i)/\sigma(X_i). \quad (1)$$

Sinn (1983) and Meyer (1987) show that if the investment fund returns are equal in distribution to one another except for location and scale (LS), a Sharpe ratio ranking can be derived based on expected utility. According to Feller (1966), Meyer (1987), and Levy (1989), a set of random variables X_i described by probability density functions $f_i(\cdot)$ satisfies the LS property if there exists some random variable Y with a probability density function $g(\cdot)$ such that $f_i(\cdot)$ differs from $g(\cdot)$ only by location parameter a_i and scale parameter $b_i>0$. That is, there exist location and scale parameters a_i and $b_i>0$ such that $b_i \cdot f_i(a_i+b_i \cdot y)=g(y)$, which means that $X_i \sim a_i+b_i \cdot Y$, whereby "~" stands for "is equal in distribution to." Furthermore, the random variable Y can be standardized such that $\mu(Y)=0$ and $\sigma(Y)=1$.

We consider an alternative risk measure denoted by $\rho(X_i)$ and an alternative reward measure denoted by $\pi(X_i)$. The corresponding performance measure $P(X_i)$ is then defined for $\rho(X_i)>0$ as

$$P(X_i)= \pi(X_i)/\rho(X_i). \quad (2)$$

In the literature, there are a number of important axiomatic systems for risk measures (see Kijima and Ohnishi, 1993; Pedersen and Satchell, 1998; Artzner et al., 1999; Rockafellar et al., 2006). One axiom that is common to all four axiomatic systems[1] is positive homogeneity, which requires

$\rho(k \cdot X_i) = k \cdot \rho(X_i)$ for all $k>0$ and $\rho(0)=0$. (3)

Positive homogeneity is the first minimal condition every admissible risk measure needs to satisfy.[2] Note that the first minimal condition concerns the influence of *multiplying* the random variable by a constant. The second minimal condition concerns the influence of *adding* a constant to the random variable. According to Kijima and Ohnishi (1993), Pedersen and Satchell (1998), and Rockafellar et al. (2006), adding a constant should not change the risk measure. Kijima and Ohnishi (1993) call this axiom “shift invariance”:

$\rho(X_i+s)=\rho(X_i)$. (4)

According to Artzner et al. (1999), adding a constant should decrease the risk measure by the same amount. Artzner et al. (1999) call this axiom “translation invariance”:

$\rho(X_i+s)=\rho(X_i)–s$. (5)

Obviously, shift invariance and translation invariance are mutually incompatible. According to these (mutually incompatible) axioms, risk measures can be broadly categorized as deviation risk measures, in the sense of Rockafellar et al. (2006), when they satisfy shift invariance, or as coherent risk measures, in the sense of Artzner et al. (1999), when they satisfy translation invariance. This leads

[1] See Axiom R1 in Kijima and Ohnishi (1993), Axiom BP2 in Pedersen and Satchell (1998), Axiom PH in Artzner et al. (1999), and Axiom D2 in Rockafellar et al. (2006). Note that the first three papers require $\rho(k \cdot X_i)=k \cdot \rho(X_i)$ for all $k \geq 0$, while the last paper requires $\rho(k \cdot X_i)=k \cdot \rho(X_i)$ for all $k>0$ and $\rho(0)=0$. Both formulations are equivalent.

[2] A performance measure is invariant to leverage ($P(k \cdot X_i)=P(X_i)$ for all $k>0$) if and only if the corresponding risk measure satisfies positive homogeneity. Invariance to leverage is a minimal requirement for performance measures, reflecting the well-known fact that moving along the capital allocation line should not influence the performance. Note that positive homogeneity is a sufficient (but not necessary) condition to add to convexity in order to obtain sublinearity (= coherence) à la Artzner et al. (1999). See also Szegö (2002) and Frittelli and Gianin (2002).

to the second minimal condition every risk measure needs to satisfy, which is that adding a strictly positive constant to the random variable should not increase risk, i.e.

$\rho(X_i+s)\leq\rho(X_i)$ for $s>0$. (6)

Obviously, this condition contains shift invariance and translation invariance as special cases. Coombs and Lehner (1981, p. 1116) call this a universal property of risk (see Sarin, 1987).

The alternative reward measure should satisfy two minimal conditions as well. First, it should satisfy positive positive homogeneity, i.e.

$\pi(k\cdot X_i)=k\cdot\pi(X_i)$. (7)

Second, adding a strictly positive constant to the random variable should increase the reward measure, i.e.

$\pi(X_i+s)>\pi(X_i)$ for $s>0$. (8)

Before stating our main result, we need the following lemma, which describes the consequences of positive homogeneity for both the risk and reward measure under the LS property.

Lemma: Assume two funds with the LS property and identical Sharpe ratios, $S(X_1)=S(X_2)$. If both the alternative risk measure $\rho(X)$ and the reward measure $\pi(X_i)$ satisfy positive homogeneity, then the corresponding performance measures for the two funds are also identical, $P(X_1)=P(X_2)$. Proof: See Appendix.

This lemma is important because it implies that a positive homogenous performance measure is a function of the Sharpe ratio. The central question, which will be answered next, is under what conditions is it an increasing function of the Sharpe ratio.

Theorem: Assume two funds with the LS property and different Sharpe ratios, $S(X_1)>S(X_2)$. If both the risk measure $\rho(X_i)$ and the reward measure $\pi(X_i)$ satisfy their two minimal conditions, then fund 1's corresponding performance measure is greater than fund 2's, i.e., $P(X_1)>P(X_2)$. Proof: See Appendix.

This theorem shows that for investment funds with the LS property, admissible performance measures are strictly increasing functions of the Sharpe ratio. This means that performance rankings will be the same, regardless of whether the Sharpe ratio or an alternative performance measure is used.

3. Examples

The following table is identical to Table 1 in Farinelli et al. (2008). Note that $X_- := \max\{-X, 0\}$, $X_+ := \max\{x, 0\}$, VaR(X): = {x: prob(X≤–x)=1–α).[3]

Performance ratio	Risk measure	Reward measure
Sharpe	$E[(X - E[X])^2]^{1/2}$	$E(X)$
MiniMax[4]	$-\text{Inf } X$	$E(X)$
Stable$^\alpha$	$A(p,\alpha)^{1/p} \cdot E(\lvert X\rvert^p)^{1/p}$	$E(X)$
MAD	$E[\lvert X - E(X)\rvert]$	$E(X)$
VaR	$VaR(X - E[X]); \alpha)$	$E(X)$
CVaR	$E(-X \mid X \leq -VaR(X; \alpha)$	$E(X)$
Rachev	$E(-X \mid X \leq -VaR(X; \alpha)$	$E(X \mid X \geq -VaR(X; 1-\beta)$
Generalized Rachev	$E((-X)^\delta \mid X \leq -VaR(X; \alpha)^{1/\delta}$	$E(X^\gamma \mid X \geq -VaR(X; 1-\beta)^{1/\gamma}$
Farinelli-Tibiletti	$E[(X_-)^q]^{1/q}$	$E[(X_+)^p]^{1/p}$

Table 1: Performance, risk, and reward measures

Proposition: The risk and reward measures in Table 1 satisfy the two minimal conditions. Hence, the performance measures in Table 1 are strictly increasing functions in the Sharpe ratio when the funds' random returns satisfy the LS property. Proof: See Appendix.[5]

In addition to the performance measures listed in Table 1, there are a number of other admissible performance measures that satisfy the two minimal conditions, specifically, partial-moments-based

[3] For simplicity we have chosen this value-at-risk definition, which differs from the one used in Farinelli et al. (2008).

[4] We use this definition, instead of the one in Farinelli et al. (2008), so as to have a positive risk measure.

[5] *Note to the referee*: It is easy to show that the risk and reward measures in Table 1 satisfy the two minimal conditions. Hence the proof of the proposition could be omitted or be made available from the authors upon request.

measures such as Omega, Kappa, Upside-potential ratio, and the Sortino-Satchell ratio. The risk measures for these performance ratios are special cases of the Farinelli-Tibiletti risk measure; thus the proof of the last performance ratio in Table 1 is applicable.

4. Conclusion

The main result is that under the location and scale property, any admissible performance measure is a strictly increasing function in the Sharpe ratio. This means that for these distributions, performance ranking will be the same regardless of whether it is conducted via an admissible performance measure or via the Sharpe ratio. This finding has an important implication with regard to the use of admissible performance measures: using admissible performance measures is theoretically justified under the same conditions as is the Sharpe ratio. In other words, the same conditions that provide a decision-theoretic foundation for the Sharpe ratio also provide a decision-theoretic foundation for admissible performance measures.

Appendix

Proof of Lemma:

The assumption that the investment funds' excess rates of return, X_1 and X_2, satisfy the LS condition means that there exists a random variable Y with $\mu(Y)=0$ [L.1] and $\sigma(Y)=1$ [L.2] and parameters a_i and $b_i>0$ such that the excess rates of return satisfy $X_i \sim a_i+b_i \cdot Y$ [L.3]. Hence

$\mu(X_i)=\mu(a_i+b_i \cdot Y)$		(Because L.3)
$\Leftrightarrow \mu(X_i)=a_i+b_i \cdot \mu(Y)$		(Simple calculation rules for μ)
$\Leftrightarrow \mu(X_i)=a_i$	[L.4]	(Because L.1)
$\sigma(X_i)=\sigma(a_i+b_i \cdot Y)$		(Because L.3)
$\Leftrightarrow \sigma(X_i)=b_i \cdot \sigma(Y)$		(Simple calculation rules for σ)
$\Leftrightarrow \sigma(X_i)=b_i$	[L.5]	(Because L.2)
$S(X_i)=a_i/b_i$	[L.6]	(Because L.4 and L.5)
$S(X_1)=S(X_2)$		(Assumption)
$\Leftrightarrow a_1/b_1=a_2/b_2$		(Because L.6)
$\Leftrightarrow a_1=c \cdot a_2$ with $c:=a_1/a_2=b_1/b_2$	[L.7]	(Simple calculation rules)
$\Leftrightarrow b_1=c \cdot b_2$ with $c:= a_1/a_2 = b_1/b_2$	[L.8]	(Simple calculation rules)
$X_1 \sim a_1+b_1 \cdot Y$		(Because L.3 for i = 1)
$\Leftrightarrow X_1 \sim c \cdot a_2+c \cdot b_2 \cdot Y$		(Because L.7 and L.8)
$\Leftrightarrow X_1 \sim c \cdot (a_2+b_2 \cdot Y)$		(Simple calculation rules)
$\Leftrightarrow X_1 \sim c \cdot X_2$	[L.9]	(Because L.3 for i = 2)
$\rho(X_1)=\rho(c \cdot X_2)$		(Because L.9)
$\Leftrightarrow \rho(X_1)=c \cdot \rho(X_2)$	[L.10]	(Positive Homogeneity of ρ)
$\pi(X_1)=\pi(c \cdot X_2)$		(Because L.9)
$\Leftrightarrow \pi(X_1)=c \cdot \pi(X_2)$	[L.11]	(Positive Homogeneity of π)
$P(X_1)=\pi(X_1)/\rho(X_1)$		(Definition of P)

⇔ $P(X_1)=c\cdot\pi(X_2)/[c\cdot\rho(X_2)]$ (Because L.10 and L.11)

⇔ $P(X_1)=\pi(X_2)/\rho(X_2)$ (Simple calculation rules)

⇔ $P(X_1)=P(X_2)$ (Definition of P)

QED

Proof of Theorem:

The assumption that the investment funds' excess rates of return, X_1 and X_2, satisfy the LS condition means that there exists a random variable Y with $\mu(Y)=0$ [T.1] and $\sigma(Y)=1$ [T.2] and parameters a_i and b_i with $b_i>0$ such that the excess rates of return satisfy $X_i \sim a_i+b_i\cdot Y$ [T.3]. Hence, as in the proof of the lemma we have $\mu(X_i)=a_i$ [T.4], $\sigma(X_i)=b_i$ [T.5] and $S(X_i)=a_i/b_i$ [T.6].

For i=1,2 define $d_i=1/b_i$ [T.7] and $X_{ii}=d_i\cdot X_i$ [T.8]. Then

$X_{ii} \sim d_i\cdot(a_i+b_i\cdot Y)$ (Because T.8 and T.3)

⇔ $X_{ii} \sim a_i/b_i+Y$ (Because T.7)

⇔ $X_{ii} \sim S_i+Y$ [T.9] (Because T.6)

Define $Y_2=S_2+Y$ [T.10] and $s=S_1-S_2>0$ [T.11]. Then

$X_{22} \sim S_2+Y$ (Because T.9 for i = 2)

⇔ $X_{22} \sim Y_2$ [T.13] (Because T.10)

$X_{11} \sim S_1+Y$ (Because T.9 for i = 1)

⇔ $X_{11} \sim S_2+s+Y$ (Because T.11)

⇔ $X_{11} \sim Y_2+s$ (Because T.10)

⇔ $X_{11} \sim X_{22}+s$ [T.14] (Because T.13)

$\rho(X_{11})\leq\rho(X_{22})$ [T.15] (Second minimal condition for ρ)

$\pi(X_{11})<\pi(X_{22})$ [T.16] (Second minimal condition for π)

$P(X_{11})=\pi(X_{11})/\rho(X_{11})$ (Definition of P)

$P(X_{11})<\pi(X_{22})/\rho(X_{22})$ (Because T.15 and T.16)

$P(X_{11})>P(X_{22})$ [T.17] (Definition of P)

$S(X_{ii})=\mu(X_{ii})/\sigma(X_{ii})$		(Definition of S)
$\Leftrightarrow S(X_{ii})=\mu(d_i \cdot X_i)/\sigma(d_i \cdot X_i)$		(Because T.8)
$\Leftrightarrow S(X_{ii})=d_i \cdot \mu(X_i)/[d_i \cdot \sigma(X_i)]$		(Positive Homogeneity of μ and σ)
$\Leftrightarrow S(X_{ii})=\mu(X_i)/\sigma(X_i)$		(Simple calculation rules
$\Leftrightarrow S(X_{ii})=S(X_i)$	[T.18]	(Definition of P)
$P(X_{ii})=P(X_i)$	[T.19]	(Because T.18 and lemma)
$P(X_1)>P(X_2)$	[T.17]	(Definition of P)

QED

Proof of Proposition:

For the following proofs we define $\mu = E(X)$.

MiniMax: For the risk measure $\rho(X) = -\inf(X)$ the first minimal requirement is $-\inf(k \cdot X) = -k \cdot \inf(X)$ and the second minimal requirement is $-\inf(X+s) \leq -\inf(X)$ for $s > 0$. Both minimal requirements are obviously satisfied.

MAD: For the risk measure $\rho(X) = E[\,|X - \mu|]$ the first minimal requirement is $E[\,|k \cdot X - E(k \cdot X)|\,] = k \cdot E[\,|X - \mu|\,]$ and the second minimal requirement is $E[\,|X+s - E(X+s)|\,] \leq E[\,|X - E(X)|\,]$ for $s > 0$. Both minimal requirements are obviously satisfied.

VaR: For the risk measure $\rho(X) = VaR(X - \mu)$ the first minimal requirement is $VaR[k \cdot X - E(k \cdot X)] = k \cdot VaR(X - \mu)$ and the second minimal requirement is $VaR[X+s - E(X+s)] \leq VaR[X - \mu]$ for $s > 0$. Thr first requirement is satisfied because $prob[X - \mu < -VaR(X - \mu)] = prob[k \cdot X - E(k \cdot X) < -k \cdot VaR(X - \mu)] = \alpha$. The second requirement is obviously satisfied.

VaR: For the risk measure $\rho(X) = VaR(X)$ the first minimal requirement is $VaR(k \cdot X) = k \cdot VaR(X)$ and the second minimal requirement is $VaR(X+s) \leq VaR(X)$ for $s > 0$. The first requirement is satisfied because $prob[X < -VaR(X)] = prob[k \cdot X < -k \cdot VaR(X)] = \alpha$. The second requirement is obviously satisfied.

CVaR: For the risk measure $\rho(X) = E[-X \mid X \leq -VaR(X)]$ the first minimal requirement is $E[-k \cdot X \mid k \cdot X \leq -VaR(k \cdot X)] = k \cdot E[-X \mid X \leq -VaR(X)]$ and the second minimal requirement is $E[-(X+s) \mid X+s \leq -VaR(X+s)]$ for $s > 0$. The first requirement is satisfied because $VaR(k \cdot X) = k \cdot VaR(X)$ implies $E[-k \cdot X \mid k \cdot X \leq -VaR(k \cdot X)] = E[-k \cdot X \mid k \cdot X \leq -k \cdot VaR(X)] = E[-k \cdot X \mid X \leq -VaR(X)] = k \cdot E[-X \mid X \leq -VaR(X)]$. The second requirement is obviously satisfied.

Generalized Rachev: For the risk measure $\rho(X) = E[(-X)^{\delta} \mid X \leq -VaR(X)]^{1/\delta}$ the first minimal requirement is $E[(-k \cdot X)^{\delta} \mid k \cdot X \leq -VaR(k \cdot X)]^{1/\delta} = k \cdot E[(-X)^{\delta} \mid X \leq -VaR(X)]^{1/\delta}$ and the second minimal requirement is $E[(-X+s)^{\delta} \mid X+s \leq -VaR(X+s)]^{1/\delta} \leq E[(-X)^{\delta} \mid X \leq -VaR(X)]^{1/\delta}$. The first requirement is satisfied because $VaR(k \cdot X) = k \cdot VaR(X)$ implies $E[(-k \cdot X)^{\delta} \mid k \cdot X \leq -VaR(k \cdot X)]^{1/\delta} = E[(-k \cdot X)^{\delta} \mid k \cdot X \leq -k \cdot VaR(X)]^{1/\delta} = E[(-k \cdot X)^{\delta} \mid X \leq -VaR(X)]^{1/\delta} = k \cdot E[(-X)^{\delta} \mid X \leq -VaR(X)]^{1/\delta}$. The second requirement is obviously satisfied.

Farinelli-Tibiletti: For the risk measure $\rho(X) = E[(\max\{-X, 0\})^{q}]^{1/q}$ the first minimal requirement is $E[(\max\{-k \cdot X, 0\})^{q}]^{1/q} = k \cdot E[(\max\{-X, 0\})^{q}]^{1/q}$ and the second minimal requirement is $E[(\max\{-(X+s), 0\})^{q}]^{1/q} \leq E[(\max\{-X, 0\})^{q}]^{1/q}$. The first requirement is satisfied because $\max\{-k \cdot X, 0\} = k$. The second requirement is obviously satisfied.

QED

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